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BINGE/PURGE OSCILLATIONS OF THE LAURENTIDE ICE SHEET AS A CAUSE OF THE NORTH ATLANTIC'S HEINRICH EVENTS

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Abstract. Ice-rafted debris in sediment cores from the North Atlantic suggests that the Laurentide ice sheet (LIS) periodically disgorged icebergs in brief but violent episodes which occurred approximately every 7,000 years. Here, I propose that Heinrich events (i.e., what these episodes are called) were caused by free oscillations in the flow of the Laurentide ice sheet which arose because the floor of Hudson Bay and Hudson Strait is covered with soft, unconsolidated sediment that forms a slippery lubricant when thawed. The proposed Heinrich event cycle has two phases. The growth phase occurs when the sediment is frozen and the LIS is stranded (immobile) on a rigid bed. The volume of the LIS slowly grows during this phase at a rate dictated by snow accumulation. The purge phase occurs when the basal sediment thaws and a basally lubricated discharge pathway (i.e., an ice stream such as those which occur in West Antarctica today) developes through Hudson Strait. The volume of the LIS rapidly equilibrates to the reduced basal friction during this phase by dumping icebergs into the Labrador Sea. The periodicity $T = \pi/\kappa(-k\ \theta_{sl}/2\bar{G})^2 \approx$ 7000 years of the proposed Heinrich event cycle is a function of the thermal conductivity and diffusivity of ice, k and κ, respectively, the atmospheric sea level temperature θ_{sl} (in degrees Celsius), and the excess geothermal heat flux defined by $\bar{G} = G - k\Gamma$, where Γ is the atmospheric lapse rate, and G is the geothermal heat flux. Agreement between the predicted T and the apparent periodicity implied by the marine record is the main virtue of the free oscillation mechanism I propose. An alternative mechanism in which Heinrich events are forced by periodic variations in external climate is implausible, because periodic atmospheric temperature perturbations are strongly attenuated with depth in an ice sheet.

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1.0. INTRODUCTION

Glacial geologists have long been prepared to explain periodic episodes of extreme iceberg discharge from the Laurentide ice sheet (LIS). Only recently, however, has the marine record offered the motivation to do so [Heinrich, 1988; Broecker et al., 1992; Bond et al., 1992; Grousset et al., 1993]. The explanation concerns the North American bedrock geology and its effect on ice sheet flow dynamics [Andrews, 1987; Andrews and Tedesco, 1992]. Except for portions of Hudson Bay and Hudson Strait, the central core of the LIS sat on the hard, crystalline bedrock of the Canadian shield. This type of rock offers strong resistance to ice flowing across it. In Hudson Bay and Hudson Strait, however, the LIS encountered soft Paleozoic carbonates and Cretaceous mudstone which are easily eroded into a slippery lubricant.

During the past two decades, attempts to reconstruct the shape and size of the LIS at the last glacial maximum were stymied by the question of how to deal with this soft, slippery lubricant. One approach was to ignore the deformable bed and the rapid flow of the Hudson Strait ice stream it would facilitate. Reconstructions resulting from this approach [e.g., Denton and Hughes, 1981] are characterized by a single domed ice sheet with a spreading center located over Hudson Bay. The reconstructed surface elevation over Hudson Bay is in excess of 3000 m.

The other prevailing approach to LIS reconstruction accounted for the lubricated basal conditions by reducing the maximum basal shear stresses that could be supported by the bed in Hudson Bay and Hudson Strait. Under this constraint, the ice sheet over the Hudson lowlands was necessarily thinner and less steeply sloped. The result of this reconstruction presented a multidomed ice sheet with a low, flat ice plain over Hudson Bay [e.g., Boulton et al., 1985; Fisher et al., 1985]. The reconstructed surface elevation over Hudson Bay was 2000 m or less.

As can be expected when two reconstructions of the same ice sheet differ, a controversy developed. Proponents of both approaches were able to take advantage of the limited and somewhat ambiguous continental geology to find support (see discussion by Andrews [1987]). With the recent revelations from the marine record and, in particular, the realization that ice-rafted debris (IRD) layers imply significant variations in LIS volume, the original controversy has been resolved. The two approaches to reconstructing the LIS are no longer in conflict. The single-domed reconstruction represents the state of the LIS just prior to an episode of extreme iceberg discharge. The great ice thickness over Hudson Bay in this case would be achieved while the soft subglacial sediments are frozen. The multidomed reconstruction represents the state of the LIS just after an episode of extreme iceberg discharge. The shallow surface elevation over Hudson Bay in this case reflects a melted bed and the activation of a ice stream through Hudson Strait.

Heinrich events may thus be interpreted as manifestations of the LIS purging excess ice volume when it transits between the two plausible states of basal lubrication [e.g., Alley, 1990; Hughes, 1992]. I adopt this interpretation as the point of view of the analysis to come. It is thus important to warn of possible oversimplifications introduced at the outset. The geologic record, in particular the isotopic signatures (e.g., Nd-Sr ratios) of the various IRD layers [Grousset et al., 1993], does not rule out the possibility of multiple ice sheet sources for the various Heinrich events. If there were multiple sources and if they operated independently, then the apparent periodicity of any single ice sheet discharge cycle might be longer than the approximately 7,000-year spacing of the IRD layers in the North Atlantic sediments. If the multiple sources operated simuletaneously, then the Heinrich event phenomena and, in particular, the external climate or sea level mechanism by which behavior of separate ice sheets is synchronized becomes too complicated for the analysis which follows.

Accepting for the sake of argument the interpretation that all Heinrich events result from purges of the LIS, the critical question becomes What causes the bed of the LIS in Hudson Bay and Hudson Strait to transit from a frozen to a thawed state? For the explanation to be plausible, the bed must remain frozen for a lengthy growth period, while the surface elevation over Hudson Bay builds to as much as 3000 m. To trigger the subsequent purge of excess ice into the Labrador Sea, this bed must suddenly melt or, more realistically, the proportion of melted bed in this area must exceed some threshold value. The transition, the sudden purge of icebergs to the North Atlantic, must thus involve the processes which determine the temperature field at the bed of the LIS. This involvement is the focus of the analysis presented in this paper.

I develop a simple conceptual model of ice sheet dynamics and thermodynamics that suggests how the amplitude and apparent periodicity of iceberg discharge events depend on the major environmental factors which affect ice sheet development (i.e., annual average sea level temperature and atmospheric lapse rate). At no point in this study will I appeal to time dependent variations of atmospheric or oceanic climate as a forcing or trigger of Heinrich events. In fact, I will begin with a discussion of why Heinrich events cannot be manifestations of external climate variations. I will close this paper with a discussion of external climate variables which appear to vary synchronously with Heinrich events (e.g., the curious "super interstadials" that appear to immediately follow the deposition of IRD in the North Atlantic [Bond et al., 1993]). I will argue that these variations reflect the response of the atmosphere/ocean system to the consequences of Heinrich events involving the freshwater flux to the North Atlantic and the topographically forced waves in the winds over the North Atlantic.

2.0. EXTERNALLY FORCED?

I believe that Heinrich events cannot be caused by external forcing such as Milankovitch insolation variation or short-term swings in surface temperature and accumulation rate such as those witnessed in the Greenland ice cores [e.g., Johnsen et al., 1992; Alley et al., 1993]. (The climate system is interconnected; thus I cannot argue that external climate does not have an important role in establishing the details of Heinrich events. It may in fact be the case that external climate variations are crucial to the explanation of why the apparent periodicity of Heinrich events is somewhat irregular. The opinion I profess, however, is that Heinrich events would still occur in an imaginary world with steady external climate.) While it is notable that the 7,000-year periodicity of Heinrich events fails to match those of climate variations expected to affect the LIS, this failure is not a crucial argument needed to support my opinion. (As suggested by the observations reported by Grousset et al. [1993], Heinrich events may have an irregular period that can range from 6,000 to 12,000 years. None of the arguments offered in support of the free oscillation mechanism are particularly sensitive to the exact figure of the periodicity. What is important is that the apparent period of Heinrich events is shorter than the shortest component of Milankovitch insolation forcing.) Even if external forcing at the 7,000-year period did exist, ice sheet heat transfer physics prohibits its having a significant influence on the basal temperature of the LIS.

It is well known that temperature oscillations at the surface of an ice sheet are strongly attenuated and phase shifted with increasing depth below the surface. As I will show, 7,000-year variations in surface conditions are unlikely to cause cyclic freeze/thaw transition at the bed of the LIS. (In addition, the timing of iceberg discharge would significantly lag the surface climate. This point should not be forgotten in the event future research reveals 7,000-year variations in external climate conditions which might be investigated as a possible cause of Heinrich events.)

The implausibility of an external cause is readily appreciated by considering the properties of temperature fields in a semi-infinite domain. To exploit the familiar analytical expressions available for this geometry, I temporarily disregard the finite thickness of the ice sheet. This simplification has little bearing on the outcome of the following analysis other than to simplify the mathematics. (I justify a posteriori disregarding of the ice sheet's finite depth by the fact that the thermal effects of harmonic surface forcing are so strongly attenuated with depth.) Consider the temperature field $\theta(z, t)$ in the domain $z < 0$ subject to harmonic temperature variation at $z = 0$:

$$\theta(0, t) = \Delta\theta \cos(\omega t) \tag{1}$$

where $\Delta\theta$ is the amplitude of an external climate forcing and $\omega = 2.84 \times 10^{-11}\ \mathrm{s}^{-1}$ is the frequency associated with a 7,000-year oscillation. The temperature field at depth $z < 0$ is the solution of the heat equation subject to equation (1) as one boundary condition and $\theta_z \to 0$ as $z \to -\infty$. The heat equation appropriate for ice sheet considerations is

$$\theta_t + w \cdot \theta_z = \kappa\theta_{zz} \tag{2}$$

where t is time, z is the vertical coordinate (negative downward), $\kappa = 1.4 \times 10^{-6}\ \mathrm{m}^2\ \mathrm{s}^{-1}$ is the thermal diffusivity of ice (its slight variation with ice density and temperature is disregarded), $w(z, t)$ is the vertical ice velocity (taken to be negative downward) associated with the effects of ice flow and snow accumulation, and subscripts t and z denote partial differentiation with respect to the subscript variable. To solve equation (2) analytically, it is necessary to simplify the vertical velocity field. For the purposes of evaluating an external cause of Heinrich events, it is adequate to take $w(z, t) = W_o < 0$ as a constant downward moving flow equal to the snow accumulation rate (in meters of ice equivalent per year).

The solution of equation (2), subject to the periodic boundary condition represented by equation (1) and the simplified vertical velocity, is

$$\theta(z, t) = \Delta\theta\ \exp\left\{\frac{z\sqrt{\omega}}{\sqrt{2\kappa}}\right\} \cdot \cos\left(\omega t + \frac{z\sqrt{\omega}}{\sqrt{2\kappa}}\right) \tag{3}$$

for $W_o = 0$ [Carslaw and Jaeger, 1988, section 2.6] and

$$\theta(z, t) = \Delta\theta\ \exp\left\{\frac{W_o z}{2\kappa} + z\sqrt{a}\cos\frac{\phi}{2}\right\} \cdot \cos\left(\omega t + z\sqrt{a}\sin\frac{\phi}{2}\right) \tag{4}$$

for $W_o \neq 0$ [Carslaw and Jaeger, 1988, section 15.2], where

$$ae^{i\phi} = \left(\frac{W_o^2}{4\kappa^2} + \frac{i\omega}{\kappa}\right) \tag{5}$$

The first expression, equation (3), represents the thermal conditions that would exist in the absence of snow accumulation. The second expression, equation (4) represents the more realistic conditions in which downward ice movement in response to snow accumulation facilitates the vertical penetration of surface temperature variations.

In both solutions, the $\theta(z, t)$ is characterized by an exponentially damped sinusoid. The e-fold decay scale for the motionless ice column, equation (3), is $\sqrt{2\kappa/\omega} = 314$ m and is only a small fraction of the thickness of ice expected over Hudson Bay. In this circumstance, a harmonic surface temperature variation with an amplitude $\Delta\theta$ of 5° yields a mere 0.00035° temperature oscillation at a depth of $z = -3000$ m, the approximate thickness of the Hudson Bay ice cover (e.g., Denton and Hughes [1981], not counting an additional 1000-m thickness resulting from isostatic depression of the bed).

For a constant vertical velocity field comparable to the accumulation rate at the summit of the Greenland ice sheet [e.g., Alley et al., 1993], $W_o \approx -0.25\ \mathrm{m\ yr}^{-1}$ (ice equivalent), the e-fold decay scale is $[\sqrt{a}\sin(\phi/2) + (W_o/2\kappa)]^{-1} =$ 970 m. Vertical velocity permits deeper penetration of the surface temperature signal, but the attenuation at $z = -3000$ m is still significant. A surface temperature variation with $\Delta\theta = 5°$ is damped to 0.23° at $z = -3000$ m. If more realistic z and t variation of $w(z, t)$ were to be considered, the e-fold decay scale of $\theta(z, t)$ would not be so simple to express. It is likely, however, to lie between the 314-m and 970-m values estimated for the two extremes of W_o discussed earlier.

While it is possible that the exceedingly attenuated cycle of basal temperature associated with surface temperature variation could be a cause of Heinrich events, I believe this to be unlikely. The free oscillation mechanism to be developed next provides a simpler and, in my view, more plausible explanation.

3.0. A KITCHEN-BUILT BINGE/PURGE OSCILLATOR

Before delving into the dynamics and thermodynamics of the LIS, it is instructive to describe a simple kitchen-built experimental device that captures the behavior qualities needed to explain Heinrich events. Consider the axle-mounted container sketched in Figure 1. Initially, this container sits upright on the axle, because its center of mass is assumed to lie between the bottom of the container and the axle. As water drips slowly into the container, the center of mass slowly rises to the point where it exceeds the level of the axle. At this point, the container becomes unstable and flips upside down to purge its contents onto the floor of the kitchen. Once the container has emptied, it flips back to the upright position and begins again to fill slowly with water.

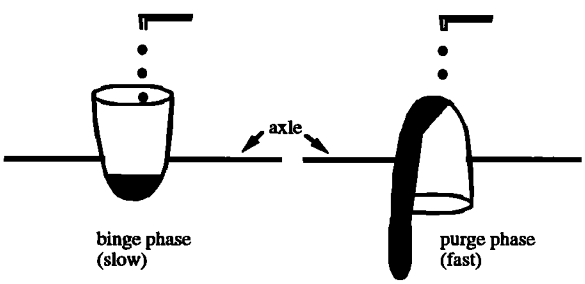

Fig. 1. A simple kitchen-built oscillator which captures the basic idea of the Heinrich event cycle of the Laurentide ice sheet.

As long as the slow trickle of water is maintained, this simple device will continue to cycle between binge (filling) and purge (emptying) behavior.

The simple oscillatory action of this kitchen-built oscillator captures the essence of the behavior required of the LIS to produce Heinrich events. The points of compariso.. are listed as follows: (1) The axle-mounted container represents Hudson Bay. (2) The slow filling of the container represents the accumulation of ice over Hudson Bay when the subglacial bed is frozen. (3) The vertical climb of the container's center of mass represents the slow warming of the basal ice temperature in response to the geothermal flux. (4) The flip and purge of the container represents the fast ice stream discharge that ensues once the subglacial bed in Hudson Bay and Hudson Strait has developed a thawed connection to the Labrador Sea. Finally, (5) the recovery of the axle-mounted container to an upright orientation represents the refreezing of the subglacial bed brought on by the increased vertical heat flux associated with ice stream flow.

The purpose of introducing this simple kitchen-built oscillator is to reinforce the idea that systems can oscillate and display violent changes in state even under steady, time independent forcing. To produce the binge/purge oscillations of the axle-mounted container, all that was necessary was the slow trickle of water. By analogy, I will show below that a steady, time independent snow accumulation rate is all that is required to ellicit an oscillatory behavior from the LIS. Another purpose served by introducing the kitchen-built oscillator is its demonstration that the periodicity of binge/purge oscillations is determined primarily by the length of time required to fill the container to the point of instability. Likewise, the 7,000-year periodicity of Heinrich events will be explained in terms of the time required to warm the bed of an ice sheet to the melting point.

4.0. A CONCEPTUAL MODEL OF ICE SHEET OSCILLATIONS

To explain the ice sheet behavior needed to produce Heinrich events, I display in Figure 2 the sequence of events involved in binge/purge cycles of the LIS. The pictures in Figure 2 surrounding the central circle display the evolution of the temperature-depth profile and thickness of a typical ice column situated in the Hudson lowland (i.e., in Hudson Bay or Hudson Strait). The horizontal and vertical axes for all graphs in Figure 2 are temperature (degrees Celsius) and distance from the bed (meters), respectively. The vertically oriented rectangle to the left of each graph represents the vertical dimension of the ice column. There are three curves plotted in each graph. Two of the curves are straight lines which bracket the temperature depth profile (curved line). The two straight lines intersect at the ice sheet surface and represent (on the left) the adiabatic temperature profile of the atmosphere, with lapse rate $\Gamma \approx 9°$ C km^{-1} and (on the right) the glaciological thermal equilibrium profile that would be required to conduct the geothermal heat flux through the ice, with a vertical gradient of $-G/k \approx -25°$ C km^{-1}.

The adiabatic temperature profile of the atmosphere displayed in Figure 2 is significant, because it represents the temperature profile that the ice would have if the heat diffusivity of ice were zero. The glaciological equilibrium profile displayed in Figure 2 is significant, because it represents the desired end state of the temperature-depth profile, if the ice column were held at a fixed thickness and if temperature were allowed to reach steady state. The temperature-depth profile and the glaciological equilibrium profile are both constrained to intersect the atmospheric profile at the ice column surface.

The sequence of graphs shown in Figure 2, starting at the uppermost graph and proceeding around the left side of the circle to the bottommost graph, represents a process which is analagous to a dog race: the actual temperature-depth profile is always "chasing" the glaciological equilibrium profile, as a dog in a dog race chases a mechanical rabbit. Eventually, despite never having caught up to the glaciological equilibrium profile (just as in a dog race, the lead dog will cross the finish line despite never having caught the mechanical rabbit), the temperature-depth profile crosses the melting point at its base (denoted by the black triangle in Figure 2). This triggers the end of the binge phase and the start of the purge phase.

The right half of the circle shown in Figure 2 describes the purge phase of the cycle. This phase occurs over a very short time interval, perhaps as short as 250 years. Temperature in all but an upper diffusive boundary layer of the ice column (the part of the profile above the temperature minimum near the ice column surface) is thus conserved (unchanged with time). The form of the temperature profile simply compresses with the shrinking vertical dimension of the ice column. As the ice column shrinks, the magnitude of the vertical temperature gradient at the bed increases. The bed remains melted despite the increased temperature gradient because of frictional heat dissipation at the bed (and possibly because of latent heat stored at the bed in the form of water). Eventually, the vertical gradient exceeds the available heat flux, and the bed re-freezes. This signals the end of the purge phase of the cycle and the beginning of a new growth phase.

The conceptual model of the Heinrich event oscillator is described by physical rules which govern the behavior of the temperature and thickness at each step in the cycle. These rules fall in three categories: (1) rules that govern the rate of growth or decay of ice thickness, (2) rules which govern the modification of the temperature-depth profile during the growth or purge cycles, and most importantly, (3) rules that tell us what conditions trigger the switch between the growth and the purge phases of the cycle. To avoid a tedious mathematical analysis, I simplify these mechanical and thermodynamical rules to the utmost. I also treat the external climate system (e.g., snow accumulation rate) as fixed in time both to simplify and to emphasize the point that variation in external climate is not what is needed to produce the basic Heinrich event phenomena.

4.1. *Rules Pertaining to the Slow Growth (Binge) Phase of the Cycle*

(1) When the basal ice temperature $\theta_b(t)$ is less than 0° C (frozen), the ice sheet is assumed to be stagnant. A

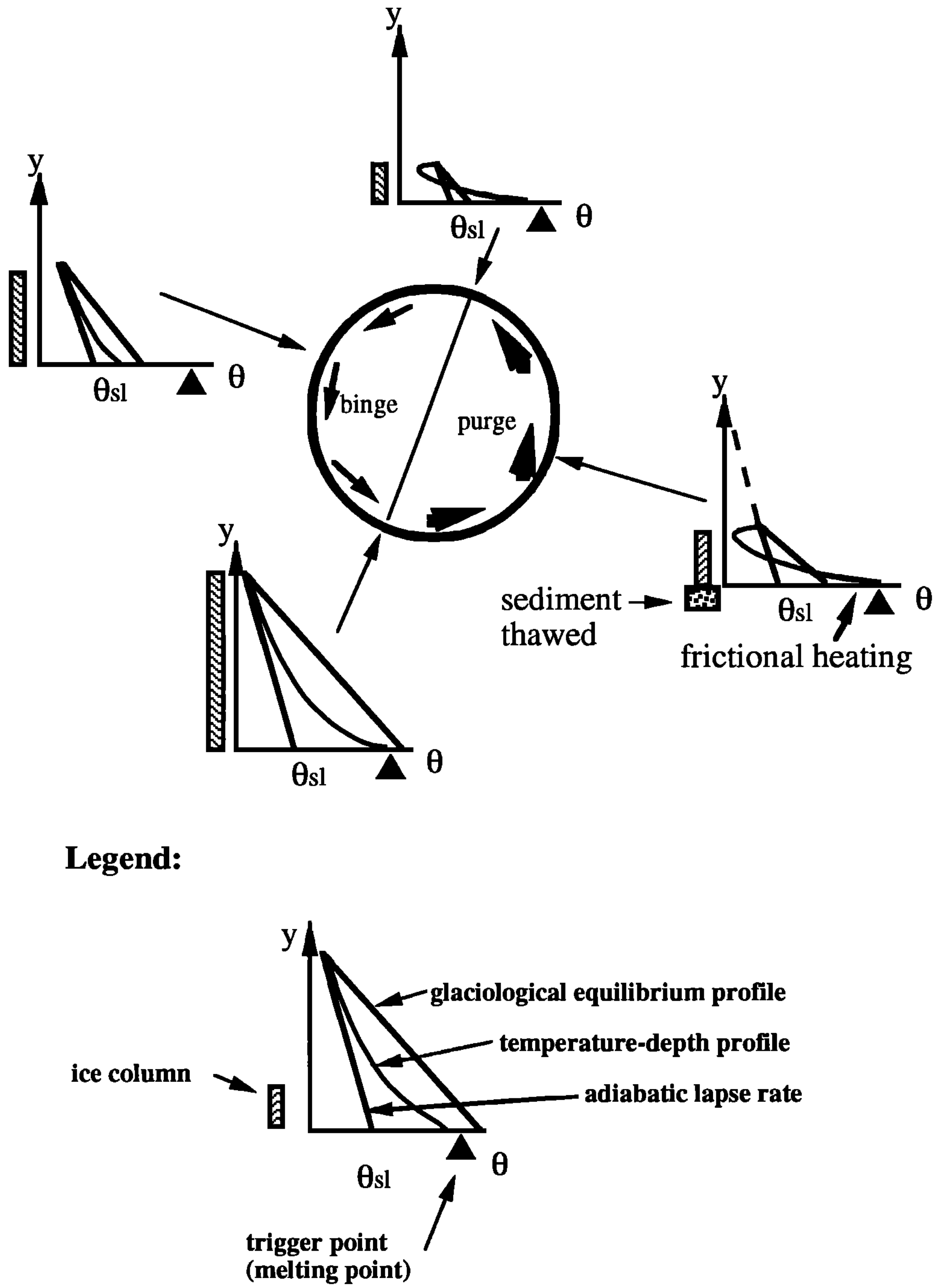

Fig. 2. A conceptual view of the temperature-depth profile $\theta(y)$ in an ice column during the binge/purge cycle of the Laurentide ice sheet. Vertical elevation from the base of the ice column is denoted by y and θ represents temperature. The annual average sea level atmospheric temperature is denoted by θ_{sl}. The melting temperature of ice is represented by the black triangles. The four graphs surrounding the central circle display the sequence of states through which the ice column evolves during a complete cycle. Time passage is represented by counterclockwise progression through the sequence of graphs.

typical ice column over the Hudson lowlands will simply thicken as a result of snow accumulation when $\theta_b < 0°$. (This is not a bad idealization. As witnessed in West Antarctica, ice streams and inter-ice-stream ridges which have frozen beds are virtually motionless.) The rate of thickening is controlled by the atmospheric precipitation rate A (in units of ice equivalent per year per unit surface area) which, as a result of the atmospheric lapse rate and the Clausius-Clapyron relationship, is a function of ice column surface elevation $H(t)$. In other words, the rate of change of ice thickness, $H(t)$, while $\theta_b < 0$ is simply

$$H_t = A(H) \tag{6}$$

Equation (6) disregards the effects of slow, creeping ice flow down the gradient of surface elevation that occurs at varying rates during times when the bed of the LIS is rigid. The effect of this disregard is not substantial until the thickness of the ice sheet approaches a steady state equilibrium in which discharge by creeping ice flow and snow accumulation are in balance. For the Heinrich event cycles discussed here, it is unlikely that the error introduced by this disregard is greater than that introduced by the disregard of time-varying external climate which, as remarked before, is also disregarded.

(2) The atmosphere above the ice sheet has a uniform lapse rate Γ and a uniform annual average sea level temperature that is colder than the melting temperature of ice, $\theta_{sl} < 0°$ C. In other words, the temperature, θ_s, at which snow is deposited on the ice column (i.e., the surface temperature of the ice sheet) is a function of ice column thickness $H(t)$

$$\theta_s = \theta_{sl} - \Gamma \cdot H(t) \tag{7}$$

(3) As a result of equation (7), the ice column is assembled by atmospheric precipitation with a temperature-depth profile that tends to mimic the atmospheric lapse rate. In other words, if the thermal diffusivity of ice were zero (infinite heat capacity or zero heat conductivity), the temperature-depth profile in the ice column, $\theta(y, t)$, would satisfy

$$\theta(y, t) = \theta_{sl} - \Gamma \cdot y \tag{8}$$

for $y > 0$, where y is the vertical coordinate (positive upward but, unlike z, is zero at the ice/ground interface).

(4) A constant geothermal heat flux G is applied to the base of the ice column. (The fact that I take this flux to be steady means that I disregard the important feedback between ice conditions and the temperature field within the upper crust of the Earth.) This flux is sufficiently large (e.g., 0.05 W m^{-2}) to require a thermal gradient within the ice column that is substantially larger than the atmospheric lapse rate. In other words, when the bed is frozen, the effect of the geothermal flux is to force $\theta(y, t)$ to tend toward a conductive equilibrium in which

$$|\theta_y| > |\Gamma| \tag{9}$$

For geothermal heat flow expected from the Hudson lowlands, a conductive equilibrium temperature gradient of approximately 25° C km^{-1} is expected to develop within a stagnant ice column. This gradient is substantially larger than the 9°-15° C km^{-1} lapse rate expected in the atmosphere above the LIS.

(5) As a result of the geothermal flux, the basal temperature of the ice column is expected to warm. The rate of warming is complicated by the fact that the ice column is changing its thickness with time; however, an estimate may be developed (see section 4.2 and Figure 3) using the result for a semi-infinite ice column occupying the region $y > 0$ [Carslaw and Jaeger, 1988, section 2.9]

$$\theta_b(t) = \theta_{sl} + \frac{2\bar{G}}{k}\left(\frac{\kappa t}{\pi}\right)^{\frac{1}{2}} \tag{10}$$

where θ_{sl} is the presumed initial temperature of the base when the first layers of snow are deposited, $\bar{G} = G - k\Gamma$ is the "excess" geothermal flux beyond that needed to support the heat conduction associated with an atmospheric lapse-rate gradient in the ice column, and $k \approx 2$ W m^{-1} °C^{-1} is the thermal conductivity of ice.

(6) The length of time taken to build up the ice column from an initial state of zero thickness to the point when $\theta_b =$ 0° C is defined to be T_L. This length of time determines the maximum ice thickness achieved by the ice column during the growth phase.

4.2. *Idealizations Pertaining to the Fast (Purge) phase of the Cycle*

(1) Once the basal temperature reaches the melting point, the ice sheet begins to move. Ice discharge through Hudson Strait leads to thinning of the ice column within the Hudson lowlands. As expected for ice stream flow [MacAyeal, 1989], the ice column will thin with a vertically uniform vertical strain rate corresponding to a vertically uniform divergence in the horizontal ice velocity. The thinning rate of the ice column will be a function of its thickness, as is the case for ice streams and ice shelves observed in Antarctica. A simple equation which captures this relationship is

$$H_t = \frac{-H}{\tau_{is}} \tag{11}$$

where τ_{is} is an e-fold decay time constant. It is adequate to define τ_{is} only in as much detail as to emphasize that it is much smaller than the time taken to build up the ice column during the growth phase of the cycle:

$$\tau_{is} \ll \frac{H_o}{A_o} \tag{12}$$

where H_o is an ice thickness scale and A_o is an accumulation rate scale (meters of ice equivalent per unit of time). A reasonable value for τ_{is} derived from numerical modeling experiments [MacAyeal and Wang, 1992] is less than 500 years. A reasonable value for H_o/A_o is 4000 years.

(2) Heinrich events result from the iceberg discharge implied by the thinning of the ice column. A simple estimate

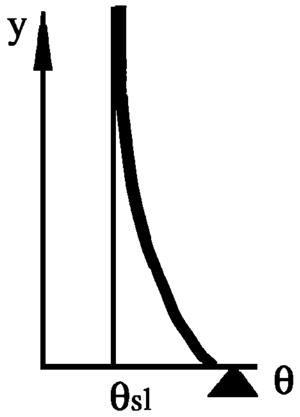

Fig. 3. Warm-up of a semi-infinite ice domain resulting from an "excess" geothermal heat flux $\tilde{G} = G - k \cdot \Gamma$. The solution to the heat equation in this domain gives the time taken for the temperature-depth profile (thick curve) to warm up from an initial isothermal state (thin vertical line) to the point when the temperature at the bed reaches the melting temperature (black triangle). This warm-up time gives the periodicity of Heinrich events expressed in equation (25).

of the iceberg discharge flux to the North Atlantic may be formulated by multiplying equation (11) by the net area of the Hudson lowlands, which is approximately 1×10^{12} m^2.
(3) The timescale τ_{is} is also substantially smaller than the diffusive timescale associated with changes to the temperature within the ice column:

$$\tau_{is} \ll \frac{H_o^2}{\kappa} \tag{13}$$

This implies that during the purge phase of the cycle, the temperature-depth profile may be regarded as a conserved "tracer field," e.g.,

$$\theta(y,t) \approx \Theta(\zeta) \tag{14}$$

where the vertical coordinate ζ is a stretched vertical coordinate designating only the relative position within the ice column,

$$\zeta = \frac{y}{H(t)} \tag{15}$$

with $\zeta = 0$ at the ice/ground contact and $\zeta = 1$ at the surface of the ice column.
(4) Equations (14) and (15) imply that the vertical gradient of θ at the bed, $\theta_y(0,t)$, increases in magnitude as the ice sheet thins. The increased vertical heat flux associated with this change is balanced temporarily by frictional heating at the bed. Eventually, frictional heating falls off and the bed refreezes (see rule 2 in section 4.3).
(5) While the ice column is thinning, it loses gravitational potential energy E at a rate given by:

$$E_t = \frac{-\rho g}{\tau} H^2 \tag{16}$$

where the subscript t denotes time differentiation, $\rho \approx 917$ kg m^{-3} is the density of ice, and $g = 9.8$ m s^{-2} is the gravitational acceleration.
(6) A fraction of the gravitational potential energy dissipation (frictional energy dissipation at the bed of the ice sheet) is liberated as heat at the bed of the rapidly thinning ice column. This heat augments the geothermal flux and keeps the bed from refreezing when the vertical temperature gradient exceeds the value implied by conductive equilibrium. (This heat additionally melts basal ice and thus provides water needed by the basal sediment to form the slippery lubricant at the bed of the Hudson Strait ice stream.)
(7) The time, T_S, taken for the ice column to thin to its minimum value is short compared to the time taken to complete the growth phase of the cycle. In other words,

$$T_S \approx \tau_{is} \ll T_L \tag{17}$$

The period of the Heinrich event oscillation, T, may thus be identified with the time taken to complete the growth phase of the oscillation, $T = T_L + T_S \approx T_L$.

4.3. *Rules Which Govern the Switch From Binge to Purge Behavior*

(1) The growth phase ends when the basal temperature reaches the melting point of ice according to equation (10).
(2) The purge phase ends when the magnitude of the basal temperature gradient exceeds the value that can be maintained (without freezing) by the combined effects of geothermal heating and fricitional energy dissipation. (I disregard latent heat stored in the subglacial till. This will ultimately act to prolong the purge phase and thereby increase the period and amplitude of the Heinrich event oscillation.)

The complete Heinrich event oscillation is determined by the sequential application of the idealized rules listed in section 4 in a self-consistent manner. (A quantitative treatment of the idealizations is provided in the companion paper [MacAyeal, this issue].) I leave it to the imagination of the reader to see how the working of this cycle produces brief episodes when the ice column thins rapidly (see Figure 2). These episodes are identified with Heinrich events, because ice column thinning would imply horizontal ice flow divergence leading to greater ice discharge through the Hudson Strait. The most important aspect of the rules for the purpose of this study is the fact that they predict (approximately) the periodicity of the Heinrich event cycle.

5.0. THE 7,000-YEAR PERIODICITY

A periodicity, $T = T_L + T_S \approx T_L$, can be estimated from the conceptual model detailed in section 4 without lengthy mathematical analysis by again considering the evolution of temperature in a semi-infinite domain, as shown in Figure 3 (i.e., the analysis which is used to derive equation (10)). Here the goal is to determine the time required for the temperature at the boundary $y = 0$ of a semi-infinite domain $y > 0$ to warm from an initial value dictated by atmospheric conditions, θ_{sl}, to the melting point of ice, $\theta = 0$, in response to a heat flux at $y = 0$:

$$\theta_y(y=0,t) = \frac{-G}{k} \tag{18}$$

where the subscript y denotes the partial derivative with respect to y. To simplify the initial condition, I assume the semi-infinite domain $y > 0$ to be the ice sheet which is assembled instantaneously at $t = 0$. (In reality, the ice sheet is assembled by the gradual accumulation of new snow. Its initial geometry is thus finite, and its initial temperature profile may depart significantly from the atmospheric lapse rate.) Because of the presumed assembly of the ice sheet from snow deposited at the prevailing atmospheric temperature, the initial temperature profile is assumed to mimic the atmospheric lapse rate (e.g., equation (8))

$$\theta_o(y) = \theta_{sl} - \Gamma y \tag{19}$$

where θ_{sl} is the sea level temperature of the atmosphere assumed to exist in the absense of the ice sheet (the ice sheet is assumed to rest on a sea level bedrock platform). The boundary condition to be applied at $y \rightarrow \infty$ is $\theta_y \rightarrow -\Gamma$.

The boundary conditions and the heat equation given by equation (2) with $y = z + H$ and $w = 0$ are linear. The temperature-depth profile may thus be decomposed into the sum of a steady profile, $S(y)$, and a transient profile which satisfies a y-independent initial condition, $\tilde{\theta}(y,t)$, for example, $\theta(y,t) = S(y) + \tilde{\theta}(y,t)$. The steady component of the solution is

$$S(y) = 0 - \Gamma y \tag{20}$$

This is steady in the sense of time-independent, not a steady state solution of the equation

The transient component of the solution satisfies the following modified initial value problem in the semi-infinite domain

$$\tilde{\theta}_t = \kappa \tilde{\theta}_{yy} \tag{21}$$

one initial condition

$$\tilde{\theta}(y, t = 0) = \theta_{sl} \tag{22}$$

two boundary conditions

$$\tilde{\theta}(0, t) = \frac{-(G - k\Gamma)}{k} = \frac{-\tilde{G}}{k} \tag{23}$$

should be \tilde\theta_y on LHS

$$\tilde{\theta}_y(y \rightarrow \infty, t) \rightarrow 0 \tag{24}$$

As mentioned previously, the heat flux used to specify the boundary condition, $\tilde{G} = G - k\Gamma$, is referred to as the "excess" geothermal heat flux, because it represents the extra heat flux beyond that required to balance the upward heat conduction associated with the initial temperature gradient (i.e., the atmospheric lapse rate Γ).

A solution to equations (21)-(24) is readily obtained [Carslaw and Jaeger, 1988, section 2.9]. The time T_L taken for the temperature at $y = 0$ to warm from its initial value of θ_{sl} to the melting point of ice (0° C) is

$$T_L = \frac{\pi}{\kappa}\left(\frac{-k\theta_{sl}}{2\tilde{G}}\right)^2 \tag{25}$$

For $G = 0.05$ W m^{-2}, $\Gamma = 9 \times 10^{-3}$ °C m^{-1}, and $\theta_{sl} = -10°$ C, a value of $T_L = 6944$ years is obtained from equation (25) and shown in Figure 4. This estimate of the Heinrich event period is within observational uncertainty of that as-

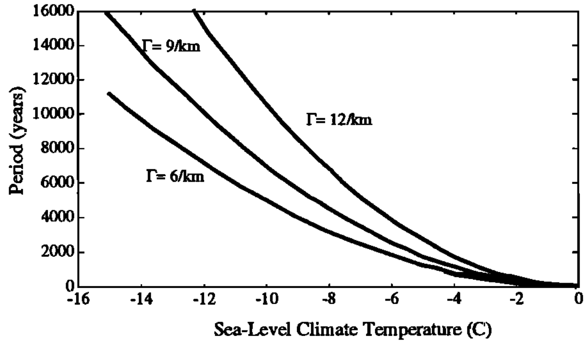

Fig. 4. Heinrich event periodicity as a function of θ_{sl} for three representative lapse rates of $\Gamma = 6°, 9°, 12°$ C km^{-1}.

sociated with the three most recent Heinrich events. (See Grousset et al. [1993] for a possible revision of this figure.) This agreement is reassuring, because it implies that no special qualities of the atmospheric climate are needed to explain the 7,000-year periodicity of Heinrich events. The parameter values used to evaluate T_L in equation (25) describe a plausible state for the atmosphere over Hudson Bay at glacial times.

The dependence of the periodicity of the Heinrich event cycle on atmospheric and geothermal parameters is expressed by equation (25). As anticipated, the periodicity decreases with the square of the "excess" geothermal heat flux (which, in turn, is a function of the atmospheric lapse rate and the geothermal flux) and increases with the square of the initial temperature. In other words, colder sea level climate and a larger, more superadiabatic lapse rate is expected to increase the period of the Heinrich event cycle. (A referee has suggested that paleotemperature data from the North Atlantic and Northern Europe indicate a warmer climate prior to 28,000 years ago. If conditions were also warmer over North America, then Heinrich events should have occurred more frequently prior to 28,000 years ago. Such a change in periodicity does not seem to be supported by the geologic record; so it is possible that the conceptual model presented here fails an important test.) A plot of T versus θ_{sl} for a range of lapse rates Γ is provided in Figure 4. Adjustments to the 7,000-year periodicity attributed to the Heinrich event cycle here [e.g., Grousset et al., 1993] can be accommodated by slight modifications of the atmospheric parameters θ_{sl} and Γ.

6.0. CONCLUSION: DO ICE-SHEET OSCILLATIONS FORCE NORTH ATLANTIC CLIMATE?

The above glaciological analysis has offered a mechanism for the free (unforced) oscillation of an idealized LIS that leads to sudden, violent episodes of iceberg discharge into the North Atlantic. The main virtue of this mechanism is that it predicts the 7,000-year periodicity of Heinrich events. While I believe the existence of Heinrich events is not primarily a consequence of external climate variations, the influence of such variations is undoubtedly important in establishing the fine structure in the Heinrich event chronology.

Having suggested a mechanism by which ice sheet dynamics can drive Heinrich event cycles, I close by listing three ways in which these cycles can influence climate elsewhere in the ocean/atmosphere system.

(1) The purge phase of the Heinrich event cycle represents a brief but intense discharge of icebergs to the North Atlantic. Assuming that an ice sheet area of 1×10^{12} m^2 suffers from the drawdown triggered during a purge phase and that the total drawdown averaged over this area is 1250 m (approximately the amplitude of the thickness variation deduced from a computer model described in a companion paper [MacAyeal, this issue]), the product of these two numbers implies that approximately 12.5×10^{14} m^3 of fresh water is introduced to the North Atlantic over a time span that could be as short as 250-500 years. Dividing the volume of discharged ice by the area of the world ocean, 3.61×10^{14} m^2, sea level rise of approximately 3.5 m over 250 years can be anticipated. It is not clear whether rapid sea level variations of this magnitude can be measured in the U/Th-dated coral terraces of Barbados and elsewhere [Fairbanks, 1989].

(2) Freshwater flux to the North Atlantic associated with the iceberg discharge estimated above is approximately 0.16 Sv (1 Sv = 10^6 m^3 s^{-1}). The magnitude of this flux is comparable to what is required to shut down the North Atlantic Deep Water (NADW) circulation and cause cooling in Greenland and Europe [e.g., Birchfield and Broecker, 1990; Stocker and Wright, 1991; Broecker et al., 1989; MacAyeal and Wang, 1992].

(3) The marine record suggests that quiescent NADW circulation and cold Greenland surface temperatures may have indeed accompanied the iceberg discharge as suggested above. What is mystifying, however, is that interstadials (warm periods) seem to follow immediately after Heinrich events [Bond et al., 1993]. One possible explanation for this pattern comes from the fact that the LIS affects the atmospheric circulation. In particular, the high surface elevation of the LIS sets up stationary wind eddies in the westerlies at all levels that extend eastward over the North Atlantic [Manabe and Broccoli, 1985; Cook and Held, 1988]. Just prior to a Heinrich event, the LIS surface over Hudson Bay is high, perhaps as much as 3000 m. This topography diverts the westerly winds, "splits" the jet stream, and yields a standing atmospheric wave at low elevations downstream of the ice sheet. The standing wave yields a northerly wind anomaly over the Labrador Sea and the western part of the North Atlantic [Cook, 1990]. Cooling associated with this anomaly may supress sea surface evaporation either by the thermodynamic effect or by maintenance of a sea ice cover. If salt rejection associated with evaporation in the North Atlantic is needed to maintain an active NADW circulation, then the pre-Heinrich event state of the LIS should correspond with particularly cold and stable stadial (cold) climates. Immediately following a Heinrich event, the LIS surface elevation is reduced, and the resumption of enhance sea surface evaporation in response to the modified atmospheric standing wave may lead to an active NADW circulation. This, perhaps, explains why particularly strong interstadial (warm) climates seems to follow each Heinrich event [Bond et al., 1993]. Following each such strong interstadial, the climate system becomes prone to rapid oscillations between stadials and less-prolonged interstadials [e.g., Johnsen et al., 1992]. This rapid oscillation may represent a simple "ringing" of the North Atlantic "salt oscillator" described by Birchfield and Broecker [1990]. If true, then the abrupt short-term climate swings heralded by the analysis of ice core geochemistry may owe their origin to the same ice sheet instability as do the IRD layers in the North Atlantic sediments.

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REFERENCES

Alley, R. B., Multiple steady states in ice-water-till systems, *Ann. Glaciol.*, *14*, 1-5, 1990.

Alley, R. B., D. A. Meese, C. A. Shuman, A. J. Gow, K. C. Taylor, P. M. Grootes, J. W. C. White, M. Ram, E. D. Waddington, P. A. Mayewski and G. A. Zielinski, Abrupt increase in snow accumulation at the end of the Younger Dryas event, *Nature*, 362, 527-529, 1993.

Andrews, J. T., The late Wisconsin glaciation and deglaciation of the Laurentide ice sheet, in *North America and Adjacent Oceans During the Last Deglaciation, vol. K-3, The Geology of North America*, edited by W. F. Ruddiman and J. E. Wright, Jr., pp. 13-37, Geological Society of America, Boulder, Colo., 1987.

Andrews, J. T., and K. Tedesco, Detrital carbonate-rich sediments, northwestern Labrador Sea: Implicationms for ice-sheet dynamics and iceberg rafting (Heinrich) events in the North Atlantic, *Geology*, *20*, 1087-1090, 1992.

Birchfield, G. E., and W. S. Broecker, A salt oscillator in the glacial Atlantic? 2, A "scale analysis" model, *Paleoceanography*, *5*, 835-843, 1990.

Bond, G., H. Heinrich, S. Huon, W. Broecker, L. Labeyrie, J. Andrews, J. McManus, S. Clasen, K. Tedesco, R. Jantschik, C. Simet, and M. Klas, Evidence for massive discharges of icebergs into the glacial Northern Atlantic, *Nature*, *360*, 245-250, 1992.

Bond, G., W. Broecker, S. Johnsen, J. Jouzel, L. Labeyrie, J. McManus, and G. Bonani, The North Atlantic-Greenland climate connection during the last glacial, *Nature*, *365*, 143-147, 1993.

Boulton, G. S., G. D. Smith, A. S. Jones, and J. Newsome, Glacial geology and glaciology of the last mid-latitude ice sheets, *J. of the Geol. Soc. London*, *142*, 447-474, 1985.

Broecker, W. S., J. P. Kennett, B. P. Flower, J. Teller, S. Trumbore, G. Bonani, and W. Wolfli, The routing of Laurentide ice-sheet meltwater during the Younger Dryas cold event, *Nature*, *341*, 318-321, 1989.

Broecker, W., G. Bond, M. Klas, E. Clark, and J, McManus, Origin of the northern Atlantic's Heinrich events, *Clim. Dyn.*, *6*, 265-273, 1992.

Carslaw, H. S., and J. C. Jeager, *Conduction of Heat in Solids*, 510 pp. Clarendon, Oxford, 1988.

Cook, K. H., The atmosphere's response to the ice sheets of the last glacial maximum, *Ann. Glaciol.*, *14*, 32-38, 1990.

Cook, K. H., and I. M. Held, Stationary waves of the ice age climate, *J. Clim.*, *1*, 807-819, 1988.

Denton, G. H., and T. J. Hughes, *The Last Great Ice Sheets*, John Wiley, New York, 1981.

Fairbanks, R. G., A 17,000-year glacio-eustatic sea level record: Influence of glacial melting rates on the Younger Dryas event and deep-ocean circulation, *Nature*, *342*, 637-642, 1989.

Fisher, D. A., N. Reeh, and K. Langley, Objective reconstructions of the late Wisconsin Laurentide ice sheet and the significance of deformable beds, *Geogr. Phys. Quat.*, *39*, 229-238, 1985.

Grousset, F. E., L. Labeyrie, J. A. Sinko, M. Cremer, G. Bond, J. Duprat, E. Cortijo, and S. Huon, Patterns of ice-rafted detritus in the glacial North Atlantic (40°-55° N), *Paleoceanography*, *8*, 175-192, 1993.

Heinrich, H., Origin and consequences of cyclic ice rafting in the northeast Atlantic Ocean during the past 130,000 years, *Quat. Res. N. Y.*, *29*, 143-152, 1988.

Hughes, T. J., Abrupt climatic change related to unstable ice-sheet dynamics: Toward a new paradigm, *Paleogeog. Paleoclimatol. Paleoecol.*, *97*, 203-234, 1992.

Johnsen, S. J., H. B. Clausen, W. Dansgaard, K. Fuhrer, N. Gundestrup, C. U. Hammer, P. Iversen, J. Jouzel, B. Stauffer, and J. P. Steffensen, Irregular glacial interstadials recorded in a new Greenland ice core, *Nature*, *359*, 311-313, 1992.

MacAyeal, D. R., Large-scale ice flow over a viscous basal sediment: Theory and application to ice stream B, Antarctica, *J. of Geophy. Res.*, *94*, 4071-4087, 1989.

MacAyeal, D. R., A low-order model of the Heinrich event cycle, *Paleoceanography*, this issue.

MacAyeal, D. R., and H.-L. Wang, A glaciological throttle on fresh water input to the North Atlantic Ocean during glacial climates, *Eos Trans. AGU*, *73*(43), Fall Meeting suppl.,158, 1992.

Manabe, S., and A. J. Broccoli, The influence of continental ice sheets on the climate of an ice age, *J. of Geophys. Res.*, *90*, 2167-2190, 1985.

Stocker, T. F., and D. G. Wright, Rapid transitions of the ocean's deep circulation induced by changes in surface water fluxes, *Nature*, *351*, 729-732, 1991.

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