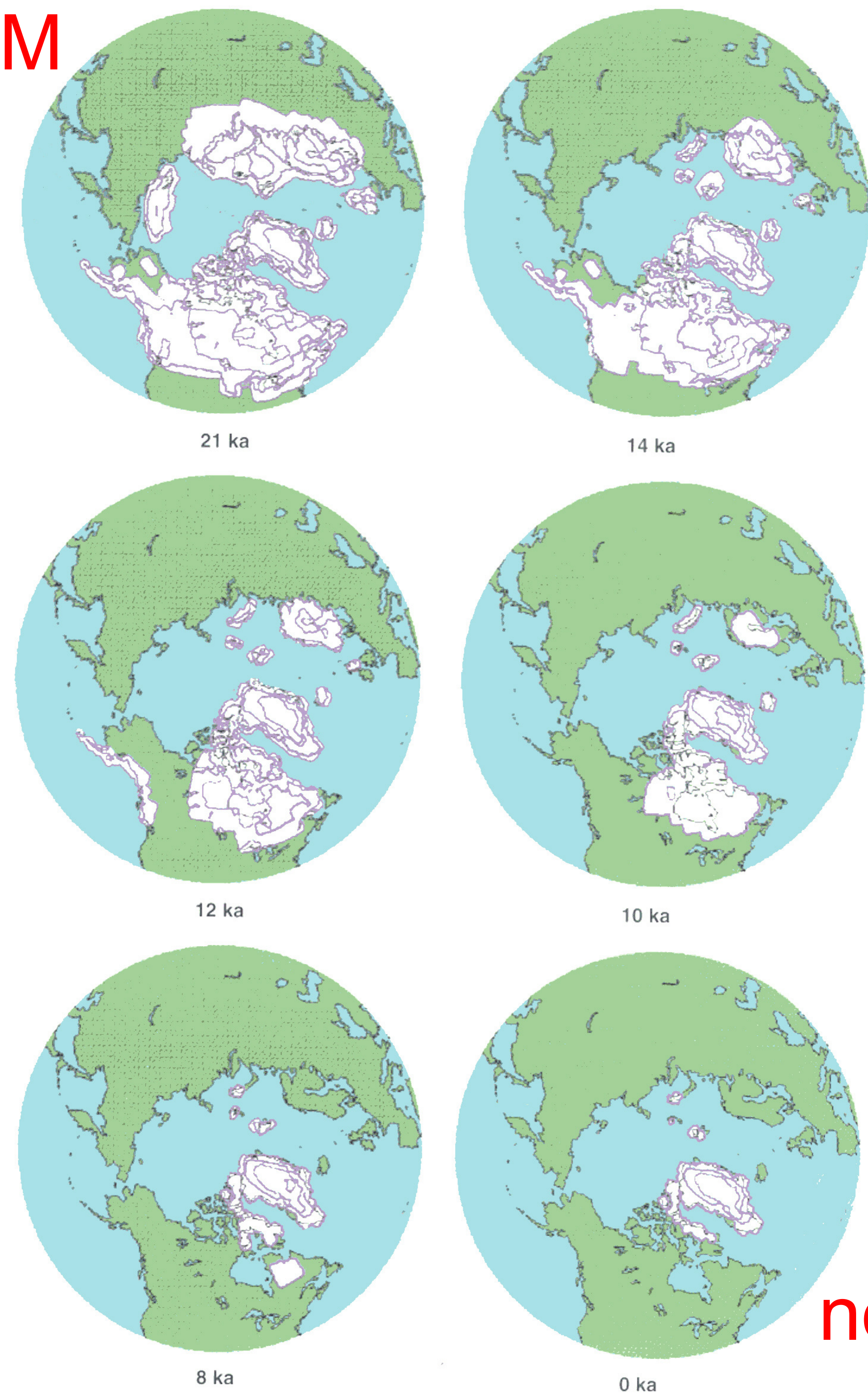
Glacial cycles

EPS 231 Climate dynamics

Eli Tziperman

Observed glacial cycle characteristics

LGM



now

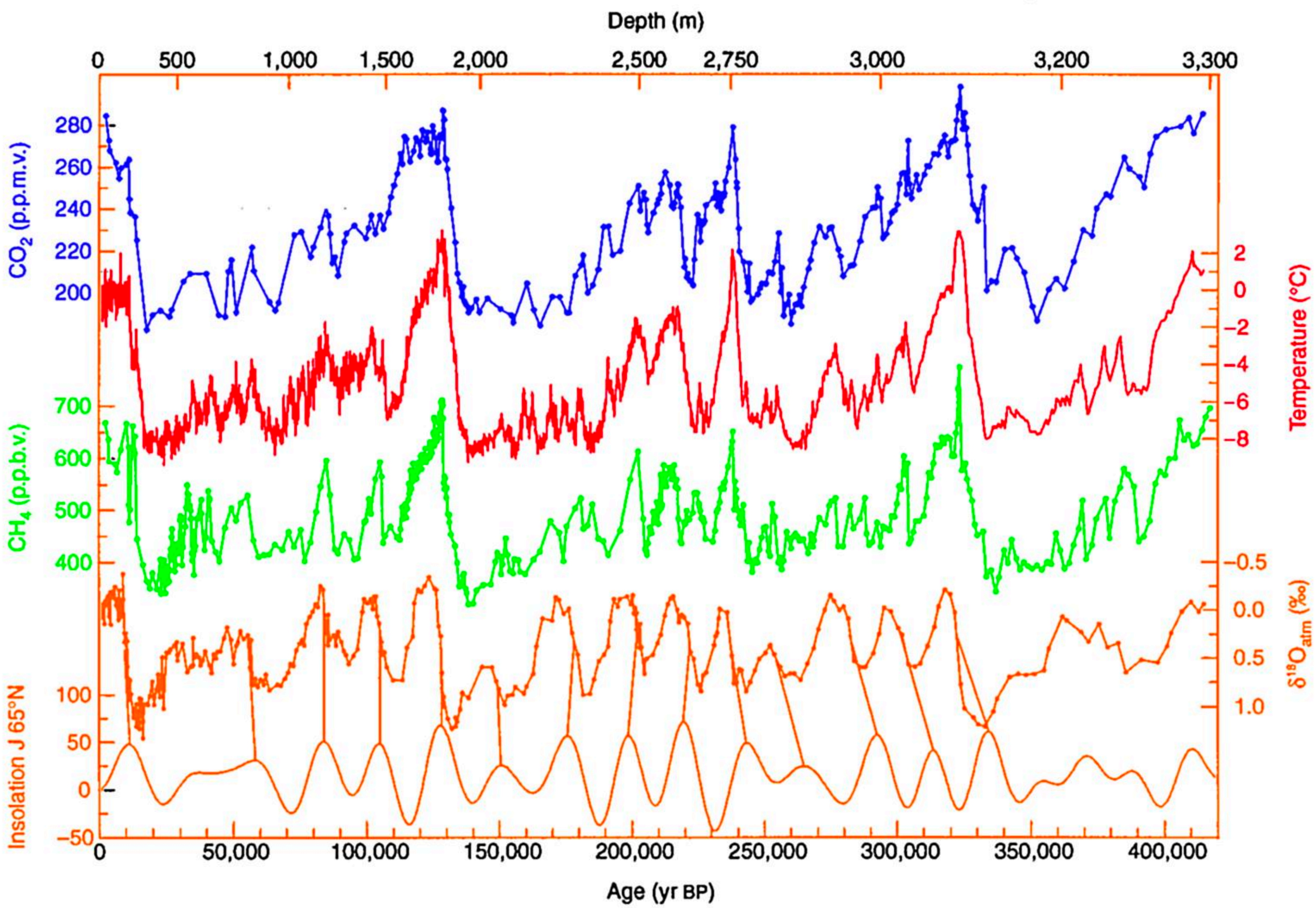
Fig. 4. Thickness isopachs for the ICE-4G model for a sequence of times beginning at Last Glacial Maximum at 21 ka and ending at the present. The contour interval is 1 km.

SCIENCE • VOL. 265 • 8 JULY 1994

Peltier 1994, Science

Last Glacial Maximum: 21 kyr
ice sheet elevation 2–3 km
sea level lower by 130 m

Observed glacial cycle characteristics



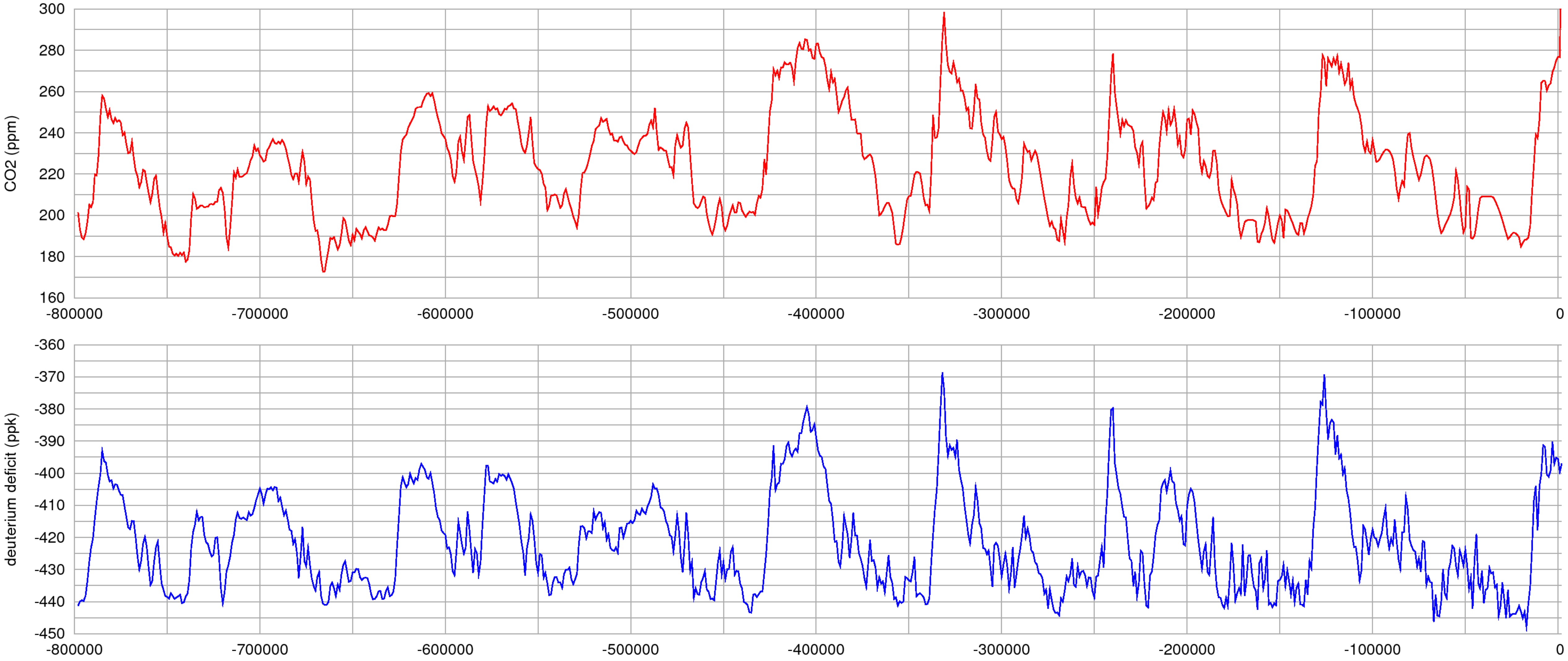
Ice core climate proxy record, Vostok (78S 106E), Antarctica; depth of drilling: 3623 meter; age range: 420,000 years

https://en.wikipedia.org/wiki/Vostok_Station

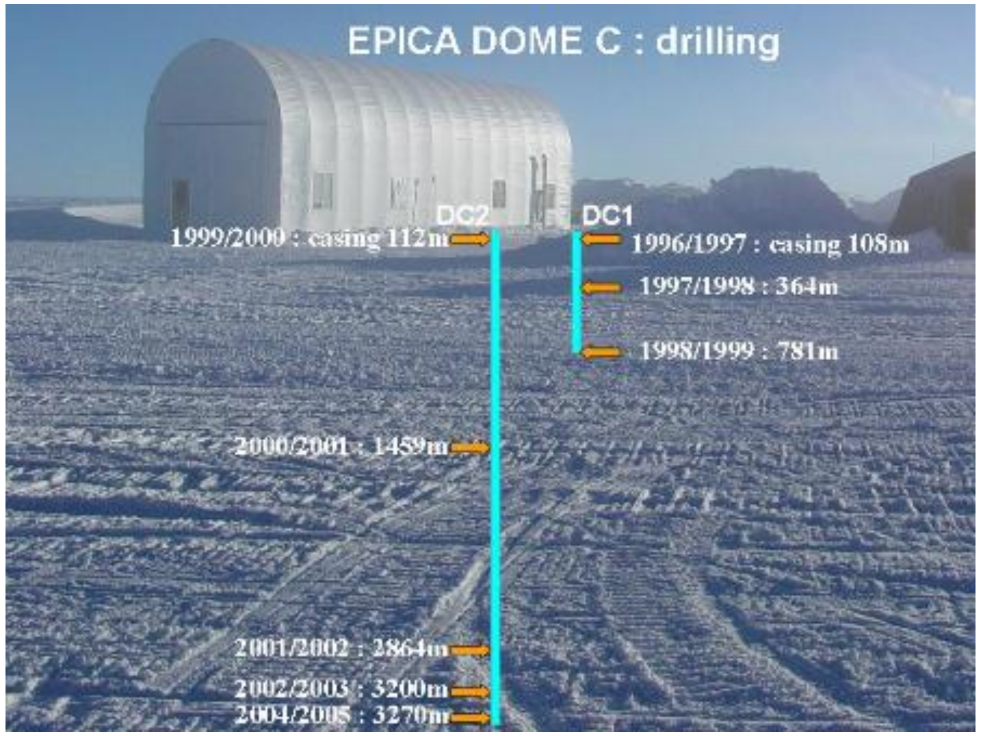
Ice core taken out of drill, Byrd, Antarctica (L. Thompson)

https://en.wikipedia.org/wiki/File:Icecore_4.jpg

Observed glacial cycle characteristics



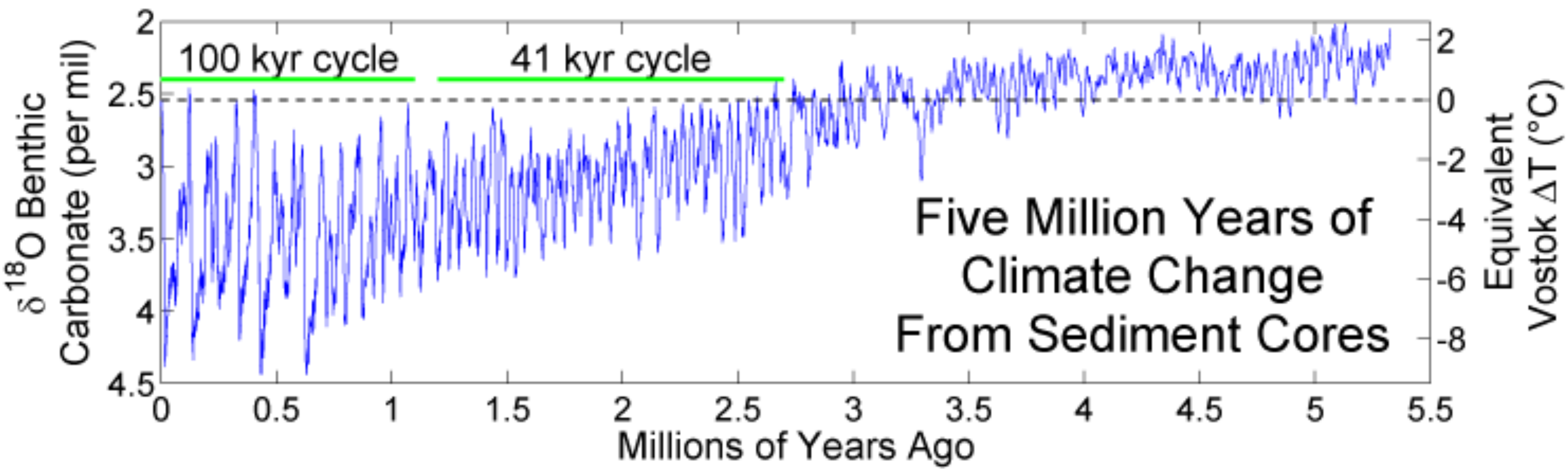
Epica ice core



Eight glacial cycles from an Antarctic ice core

EPICA community members, 2004

Observed glacial cycle characteristics



Deep sea core, past 5 M yrs

Lisiecki and Raymo (2005) LR04 Benthic Stack

https://en.wikipedia.org/wiki/Marine_isotope_stages

Observed glacial cycle characteristics

A Pliocene-Pleistocene stack of 57 globally distributed benthic D^{18}O records

Lisiecki, Lorraine E., and Maureen E. Raymo. "A Pliocene-Pleistocene stack of 57 globally distributed benthic δ18O records." Paleoceanography (2005)

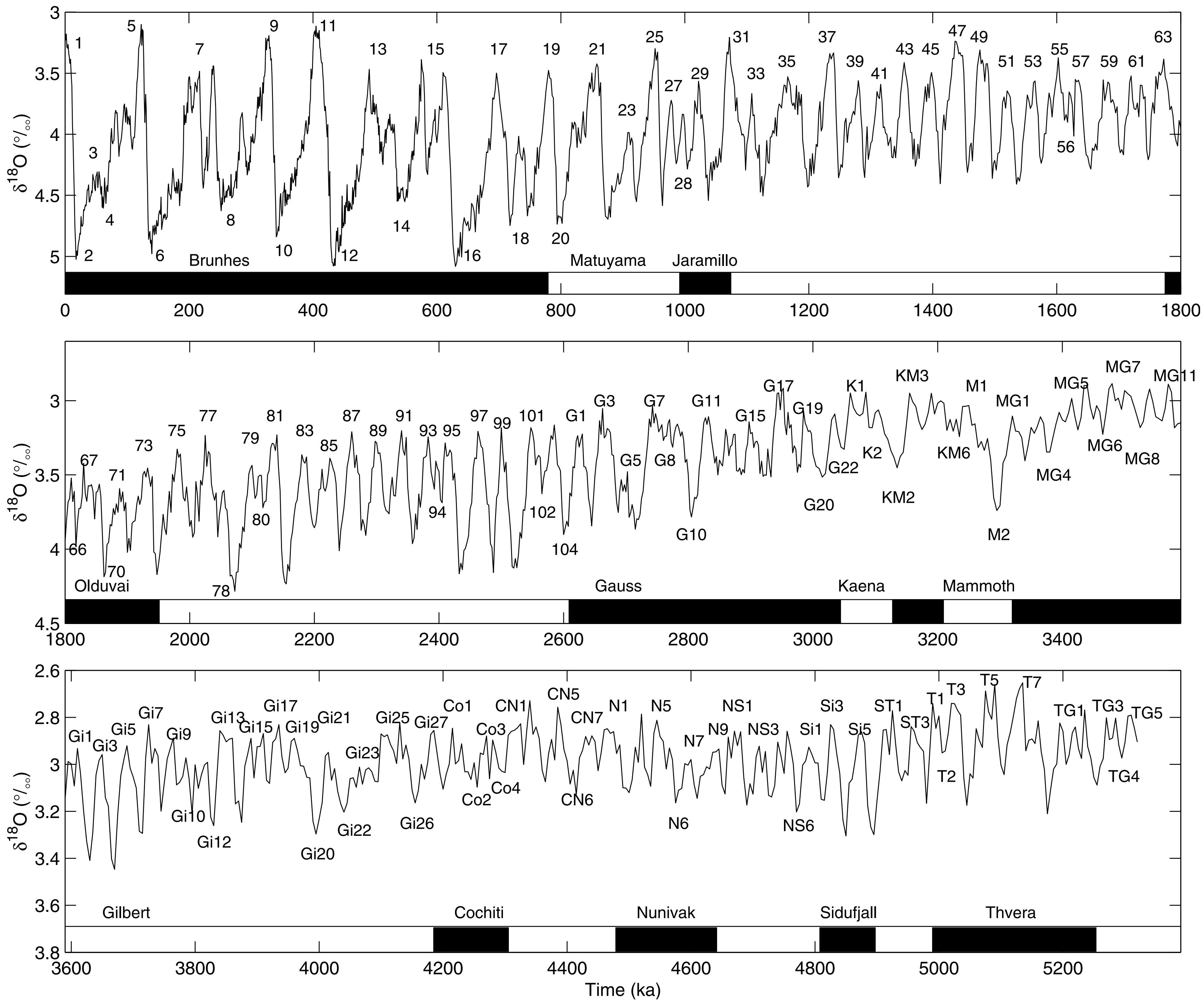
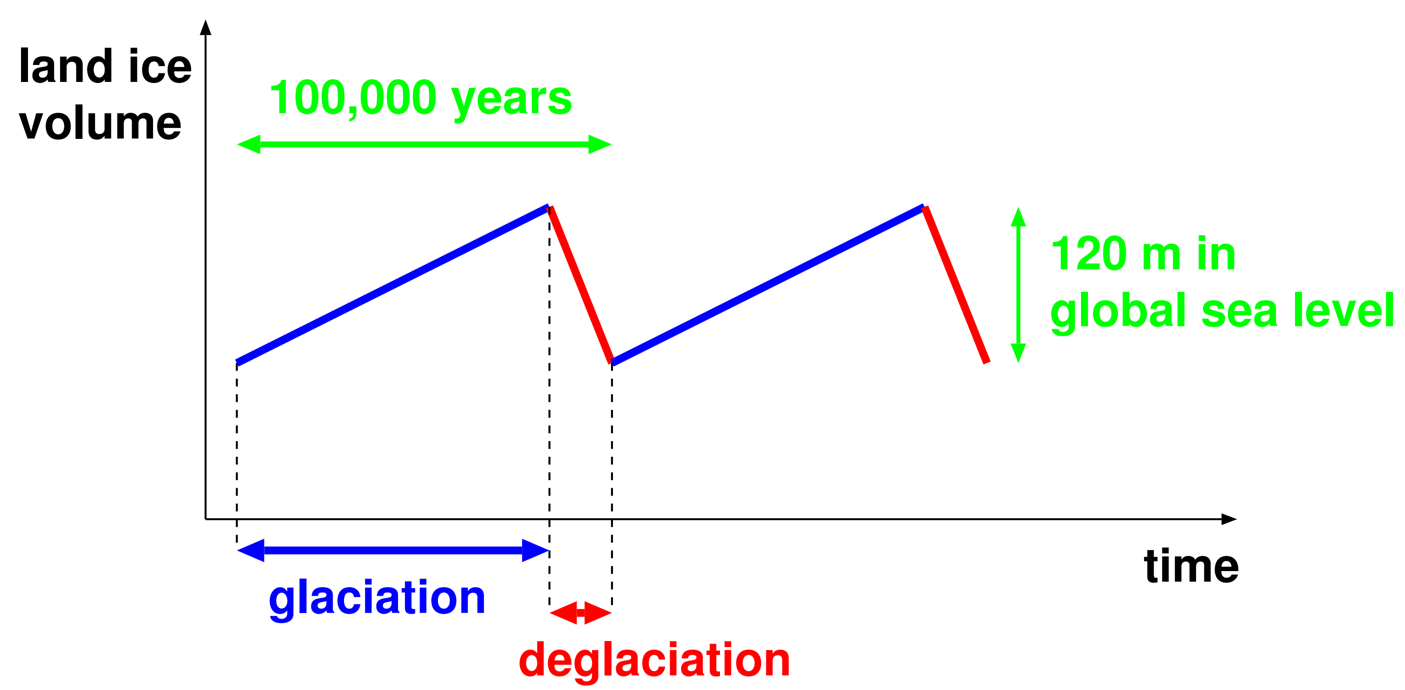


Figure 4. Td^{18}O stack constructed by the graphic correlation of 57 globally distributed benthic d^{18}O records. The scale of the vertical axis changes across panels.

Characteristics of glacial cycles to be explained by a successful theory...

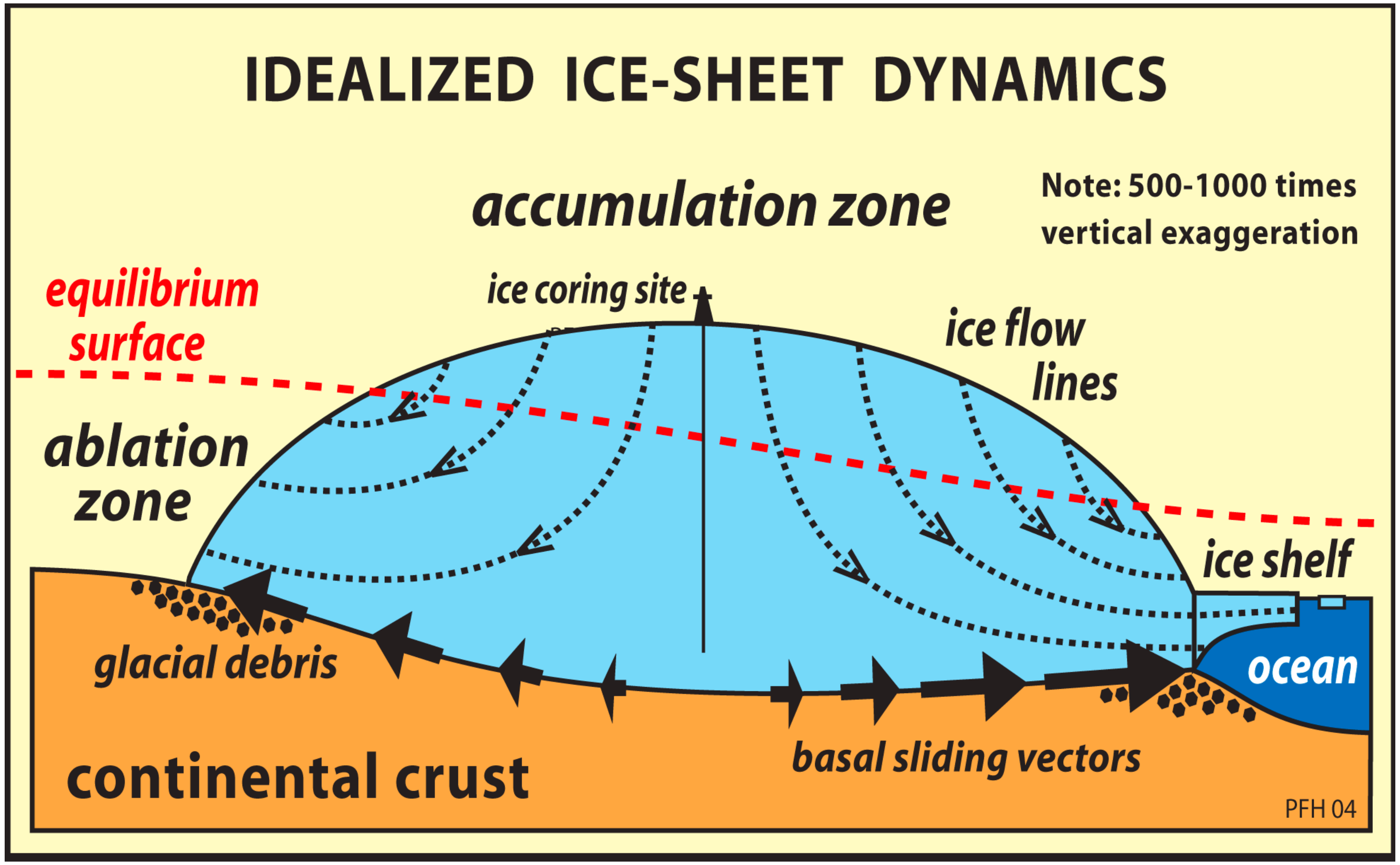
- 100 kyr time scale
- Saw-tooth structure: long glaciations (~90,000 yr), short deglaciations (10,000 yr)
- Transition from 41 kyr to 100 kyr glacial cycles ~800 kyr ago
- Atmospheric CO_2 variations during glacial cycles
- Global scale: both northern & southern hemispheres



Glacial cycle mechanism ingredients

1. energy balance and albedo feedback
2. accumulation, ablation (mass balance) as a function of ice sheet height, equilibrium line
3. Milankovitch forcing
4. ice flow and Glenn’s law
5. parabolic ice sheet profile
6. ice streams, calving
7. dust loading and enhanced ablation
8. temperature-precipitation feedback
9. shallow ice approximation
10. isostatic adjustment
11. geothermal heating

equilibrium line, accumulation zone, ablation zone



http://www.snowballearth.org/slides/Ch10-7.gif

Milankovitch forcing

Proxy record: 41 kyr and 100 kyr oscillations

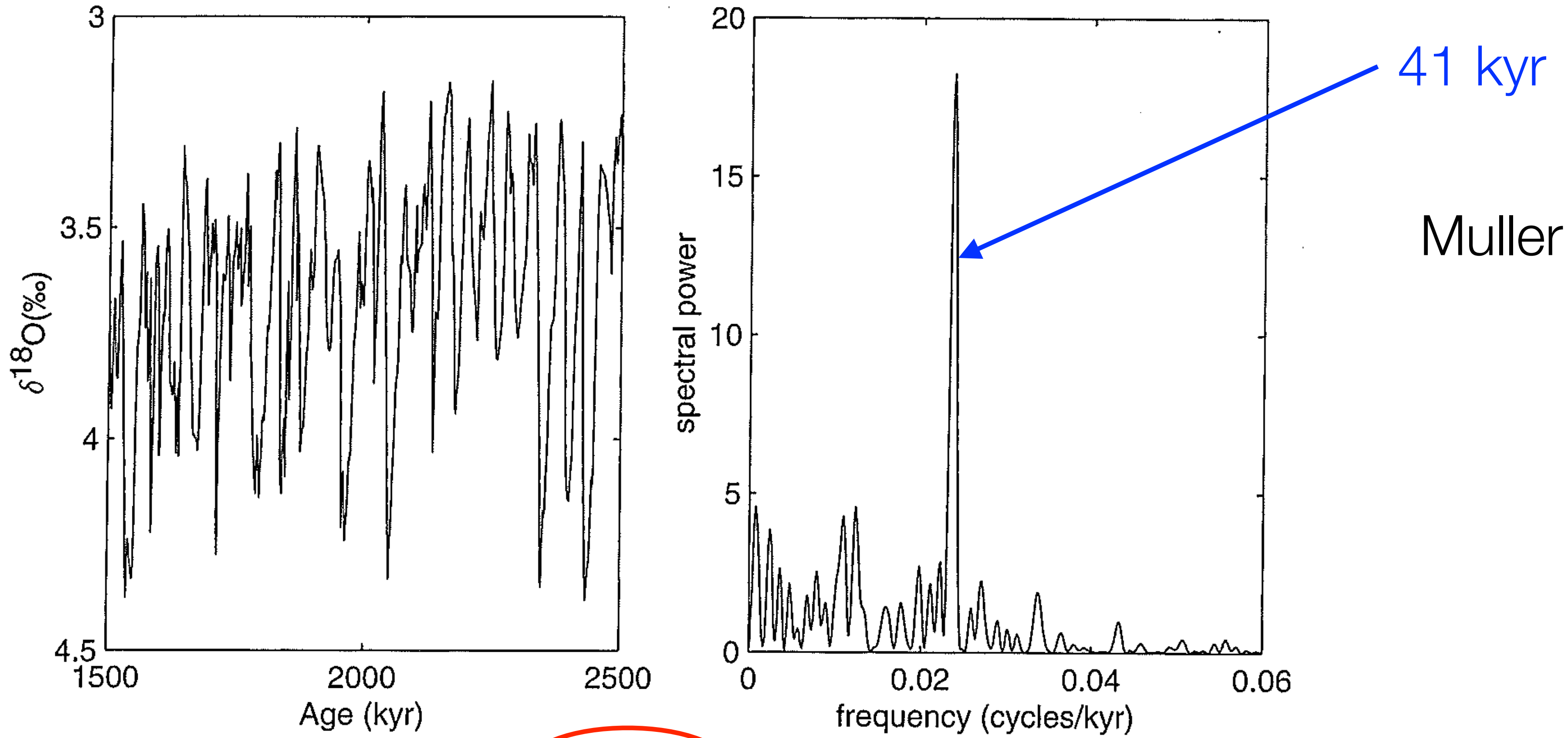


Fig. 2.1. Site 607 from 1.5 to 2.5 kyr: $\delta^{18}O$ data and spectrum.

Muller & MacDonald 2002

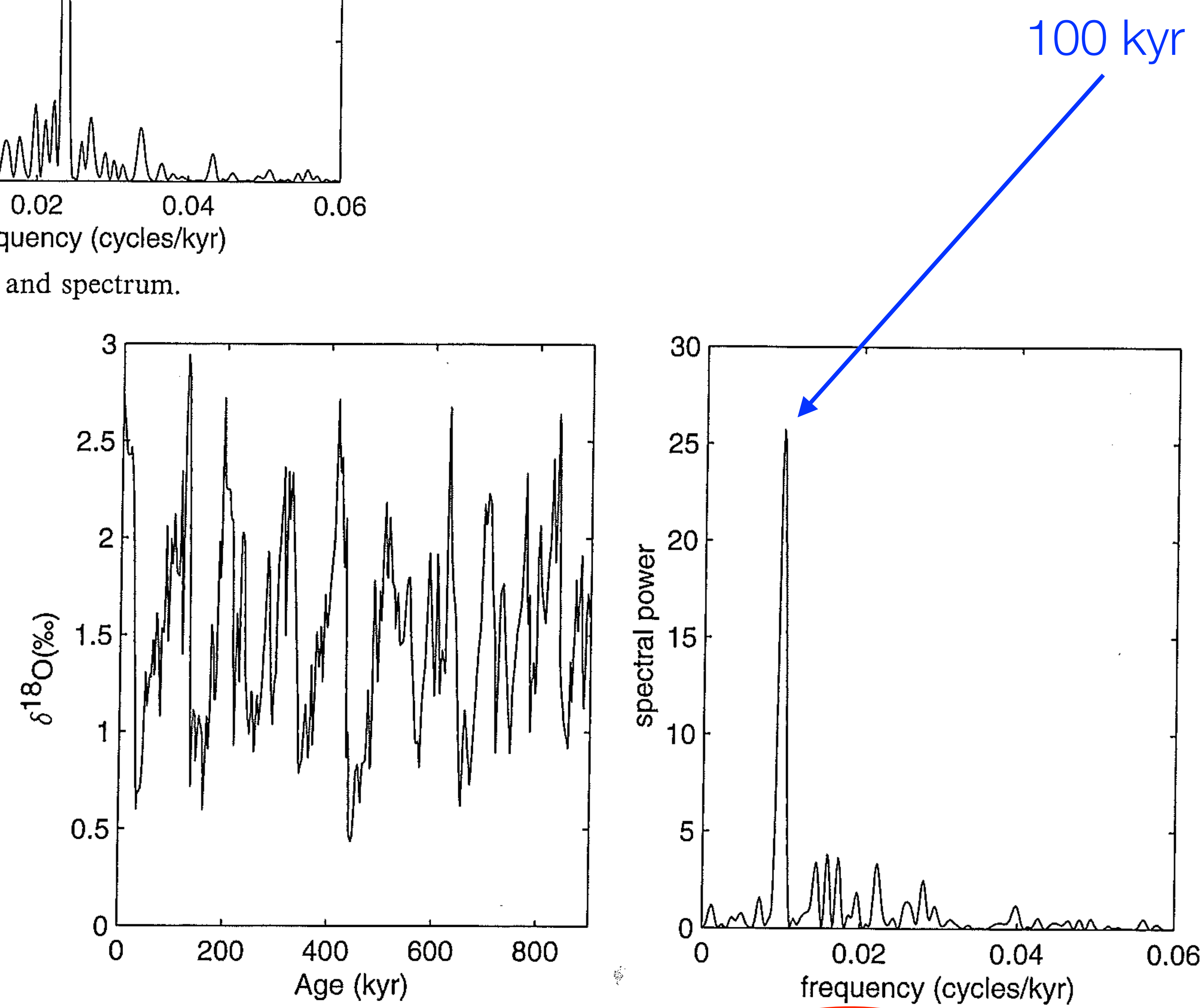
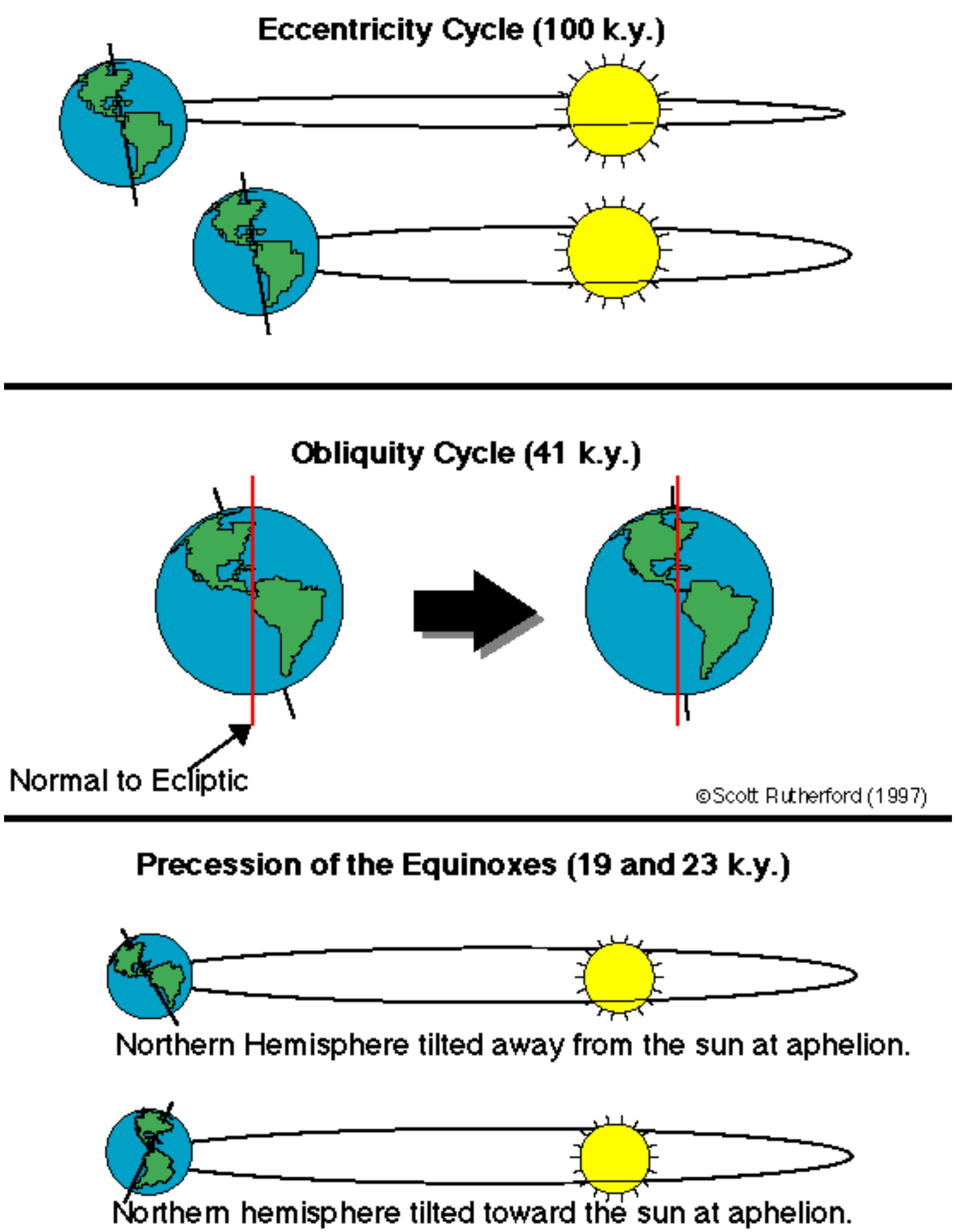


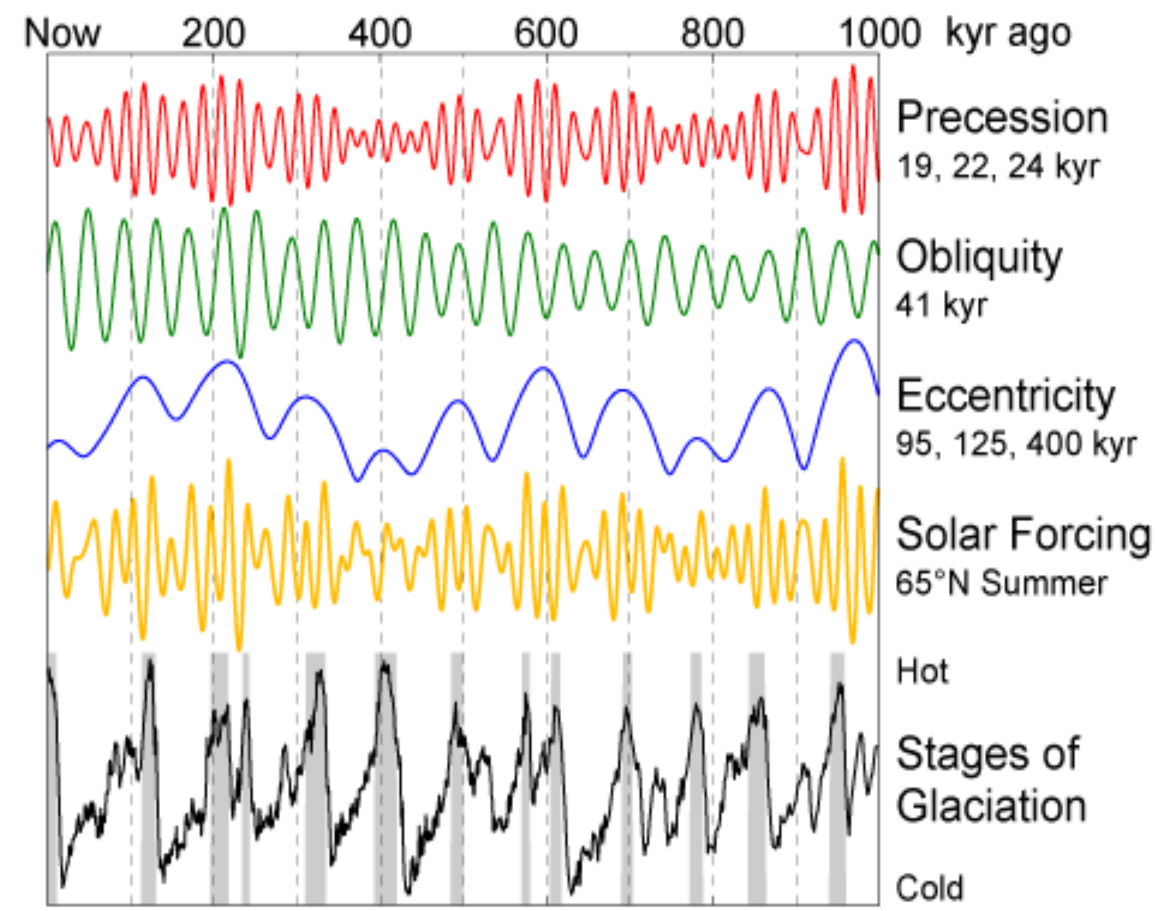
Fig. 2.2. Site 659 $\delta^{18}O$ and spectrum 0 to 900 kyr.

The spectral peak of proxy record (say of ice volume) moved from 41kyr to 100 ayr at about 800 kyr BP

Milankovitch forcing



http://deschutes.gso.uri.edu/~rutherfo/milankovitch.html

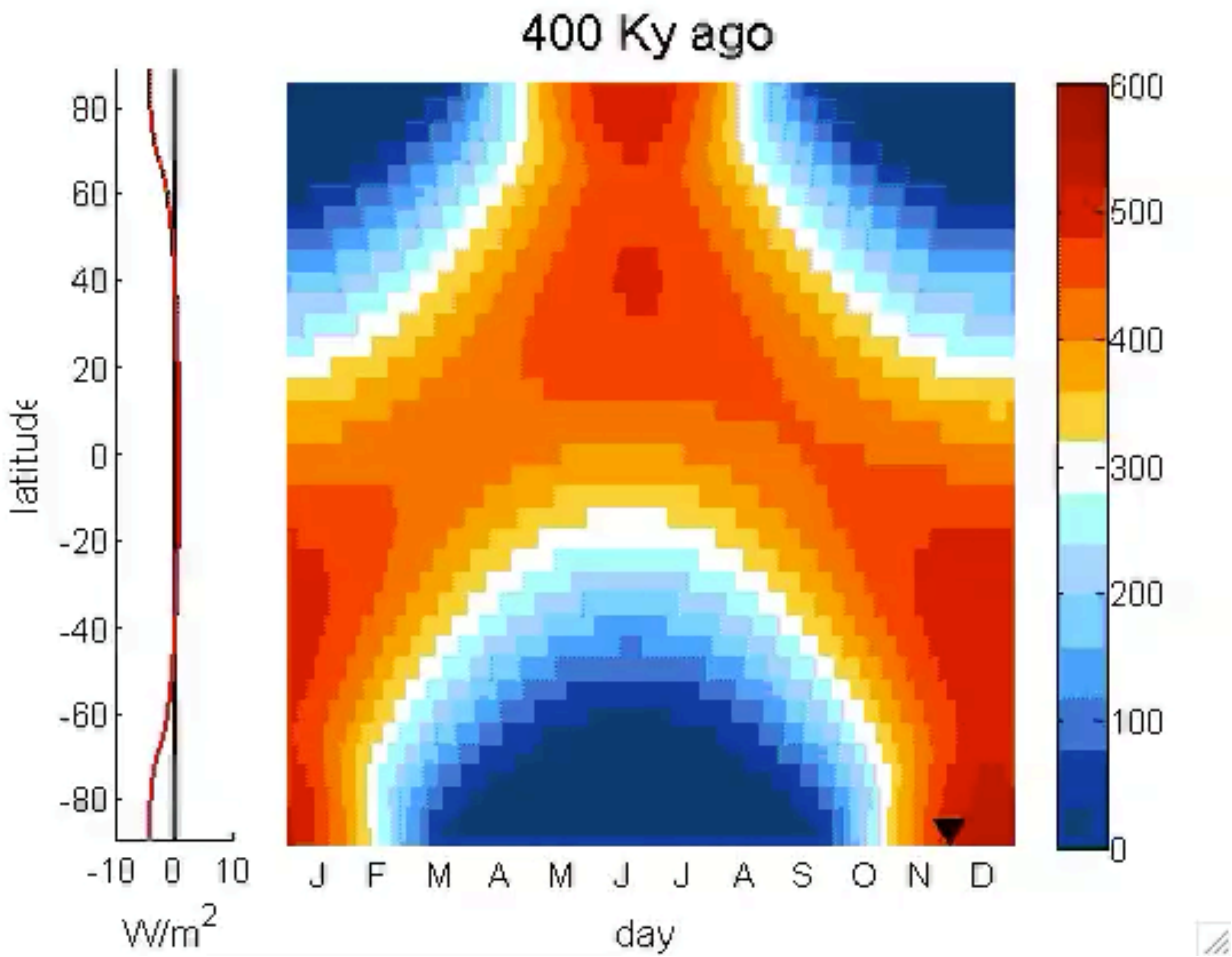


Milankovitch cycles over the past 1,000,000 years.
Source: Global Warming Art

There seems to be a connection between glaciations & summer insolation at 65N.

Hays et al 1976: insolation is the "pacemaker" of glacial cycles. How does this work...?

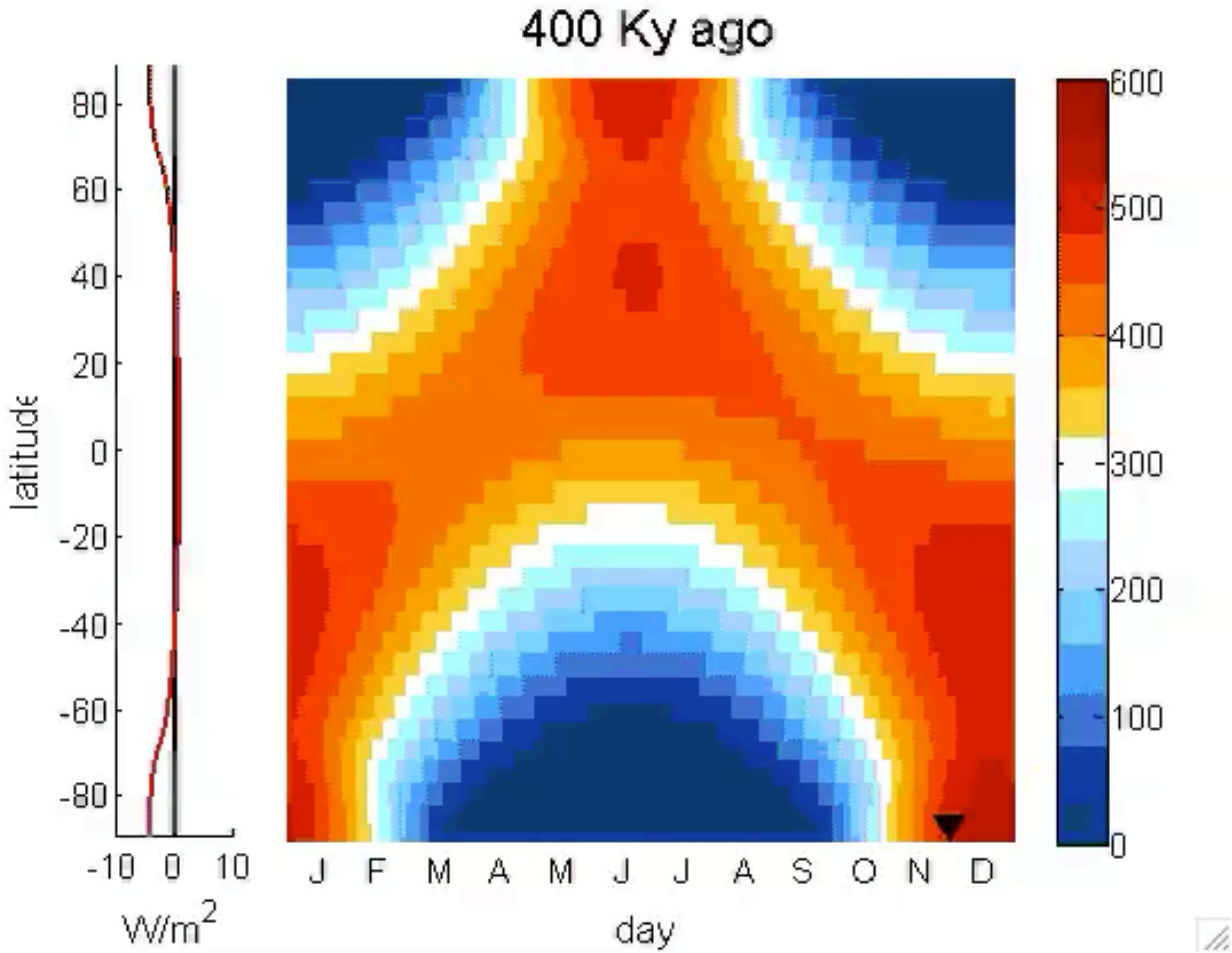
Milankovitch forcing



https://www.people.fas.harvard.edu/~phuybers/Inso/index.html

Peter Huybers

Milankovitch forcing

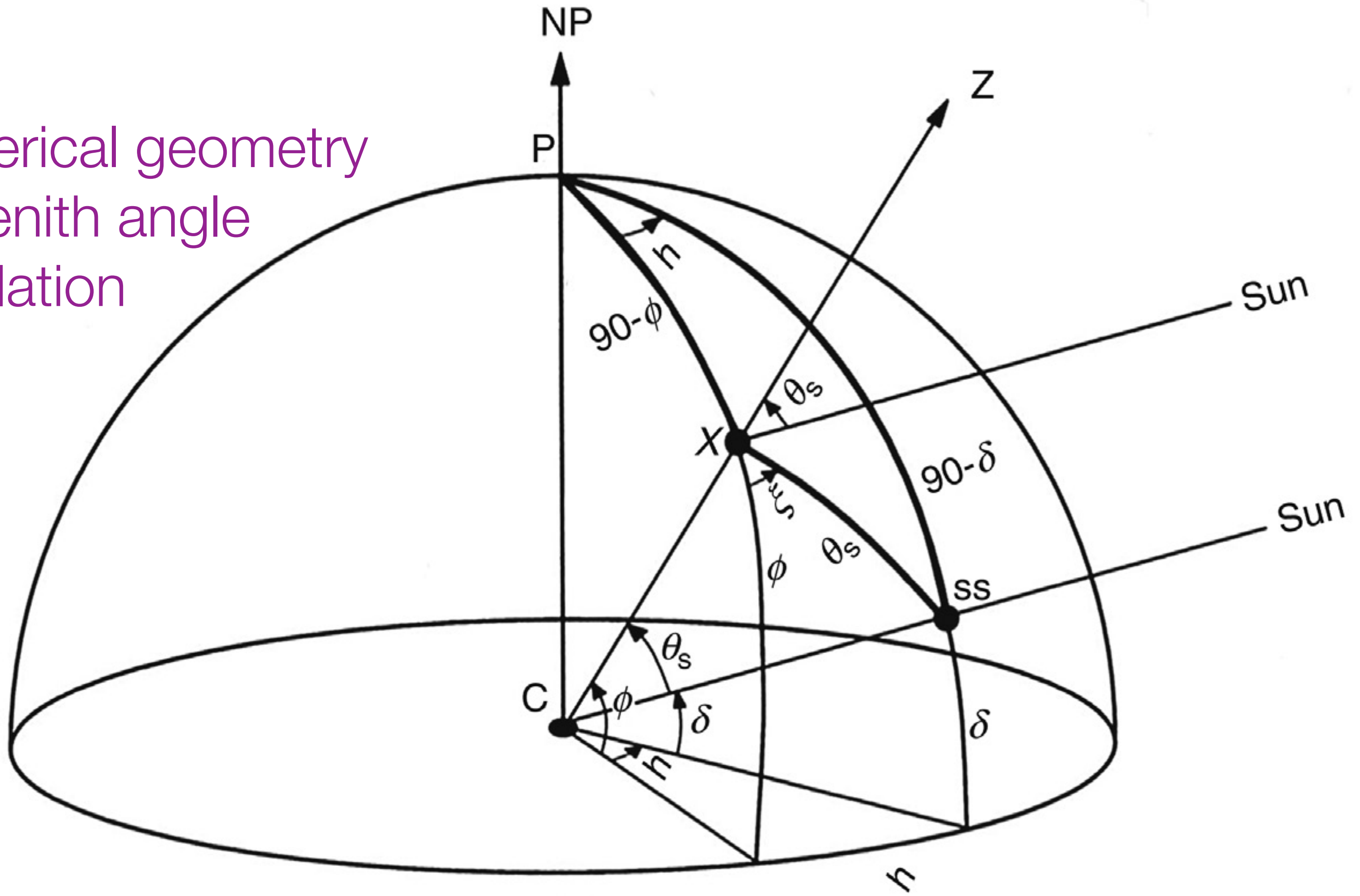


https://www.people.fas.harvard.edu/~phuybers/Inso/index.html

Peter Huybers

Geometry for calculating insolation: Hartmann Appendix A

Figure A.1 Spherical geometry for solar zenith angle calculation



$$W = \frac{S_0}{\pi}\frac{[1+e\cos(\lambda-\tilde{\omega})]^2}{(1-e^2)^2}(h_0\sin\phi\sin\delta+\cos\phi\cos\delta\sin h_0)$$

The averaged daily insolation, W, as a function of the longitude λ of Earth's orbit around the sun, measured from the vernal equinox, March 21st (June 21st is $\lambda = 90^{\circ}$), obliquity ε, precession angle $\tilde{\boldsymbol{\omega}}$, eccentricity e; S_0 is the solar constant (1360 W/m^2), δ is the declination angle of the Sun (given by $\sin\delta = \sin\varepsilon\sin\lambda$), $e = c/a = s(a^2 - b^2)^{1/2}/a$ is the eccentricity, a and b are the semi-major and semi-minor axes, ϕ is the latitude, and h_0 is the hour angle at sunrise and sunset, given by $\cos h_0 = -\tan\phi\tan\delta$.

Milankovitch forcing

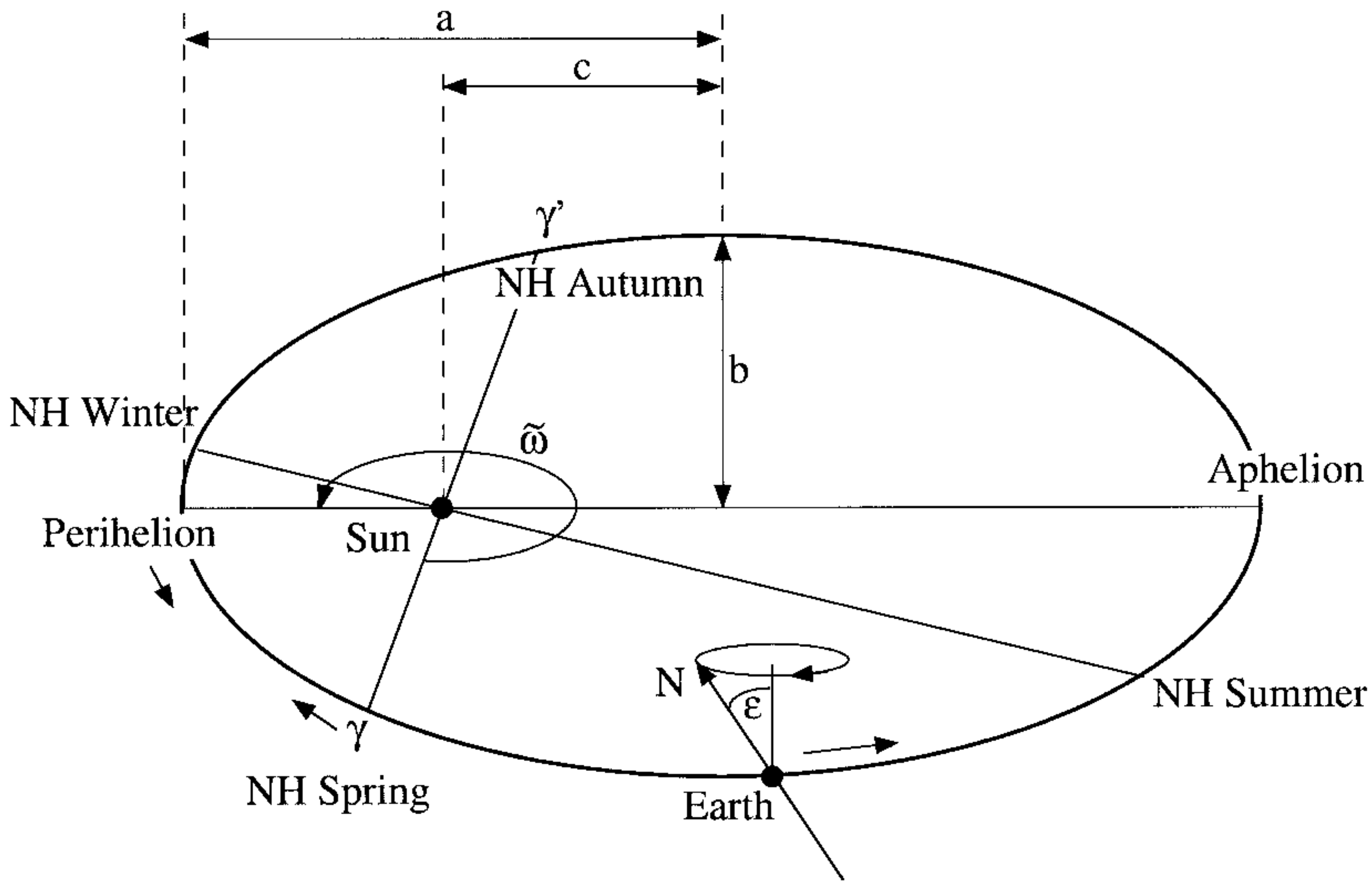


Figure 2. The orbital parameters of the Earth. Eccentricity e is defined as $e = c/a$, where a is the semimajor axis and c is the distance between the focus and the center of the ellipse. The semiminor axis b is then given by Pythagoras's theorem ($a^2 = b^2 + c^2$, which gives $b = a\sqrt{1 - e^2}$). The current eccentricity value is $e = 0.0167$, which means that the Earth's orbit is very close to a circle. The tilt of the Earth's axis with respect to the orbital plane is the obliquity ε (current value is $\varepsilon = 23.44°$). This tilt implies that the Earth equatorial plane intersects with its orbital plane, the intersection defining the $\gamma\gamma'$ line and the position of equinoxes and solstices. In the current configuration the Earth is closest to the Sun (perihelion) around January 3, just a few weeks after the Northern Hemisphere winter. This position, relative to the vernal equinox γ, is measured by the $\tilde{\omega}$ angle.

Paillard 2001

Milankovitch forcing: eccentricity

current Earth eccentricity=0.016722

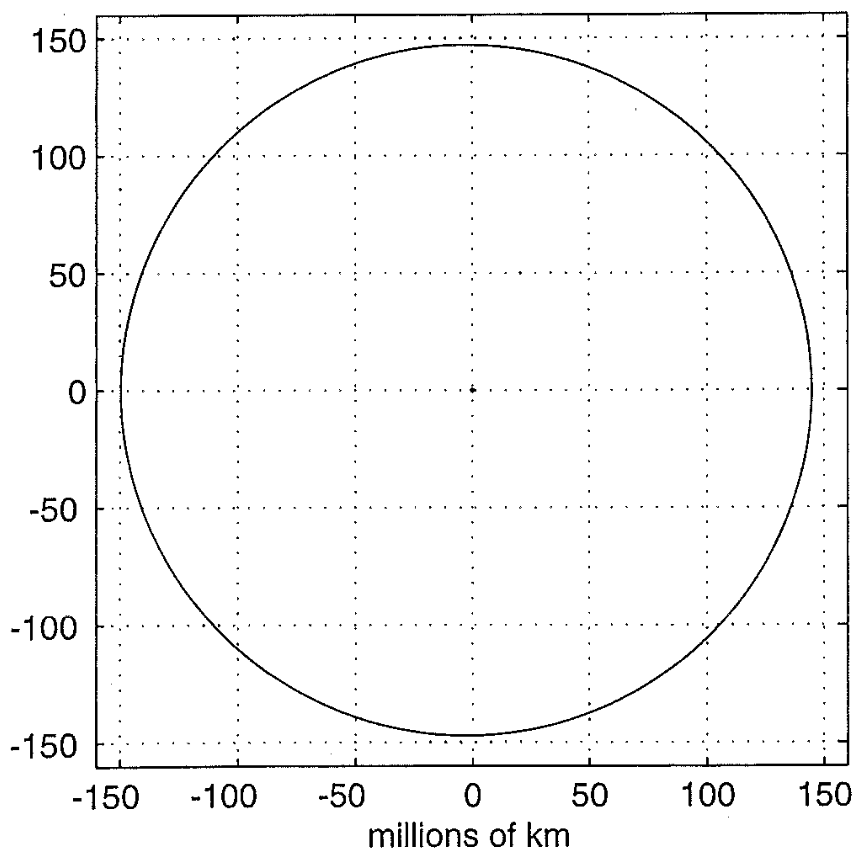


Fig. 2.5. Earth's orbit, to scale.

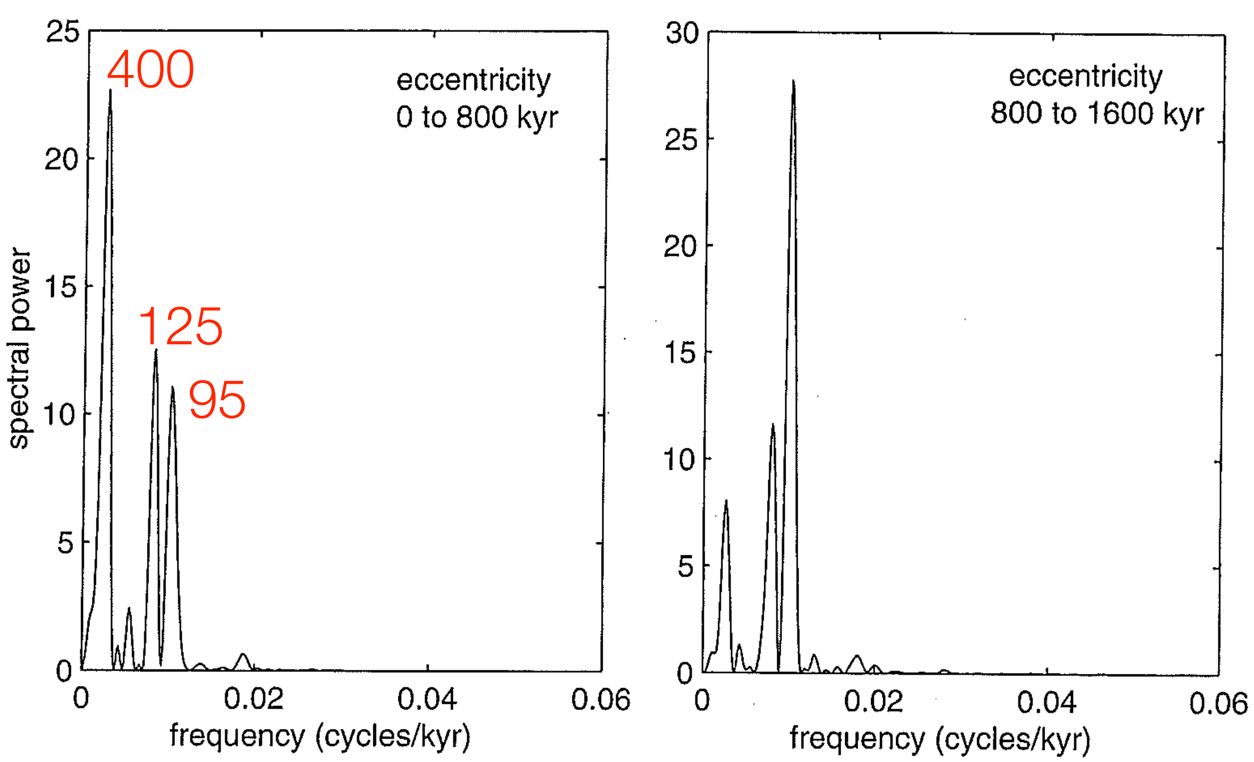


Fig. 2.9. Spectrum of eccentricity.

Muller & MacDonald 2002

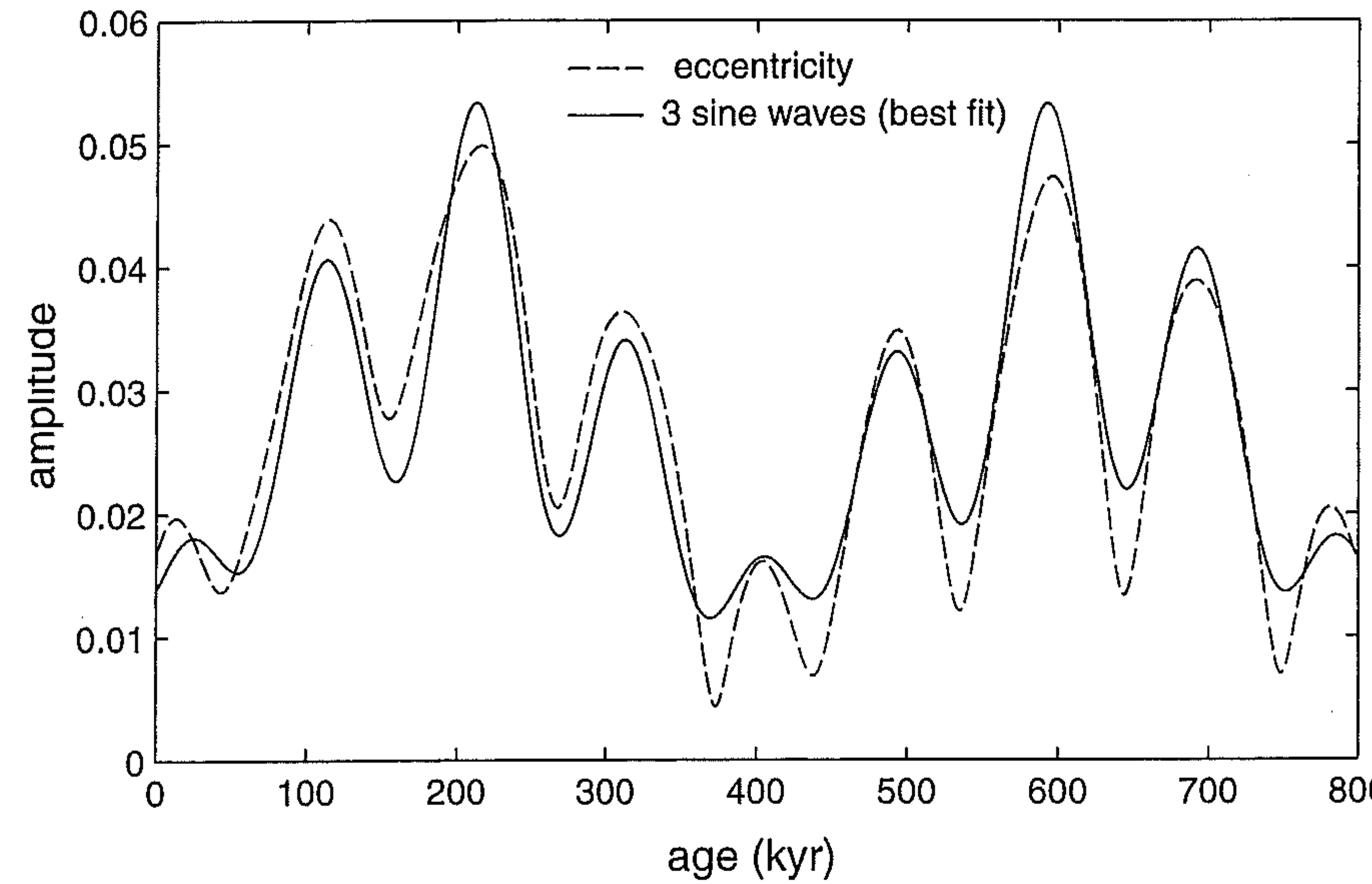
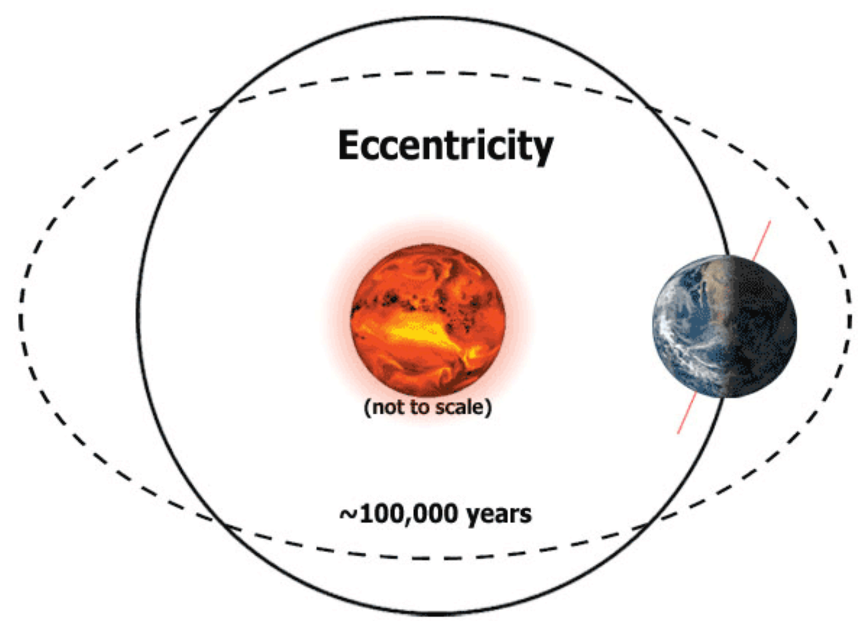
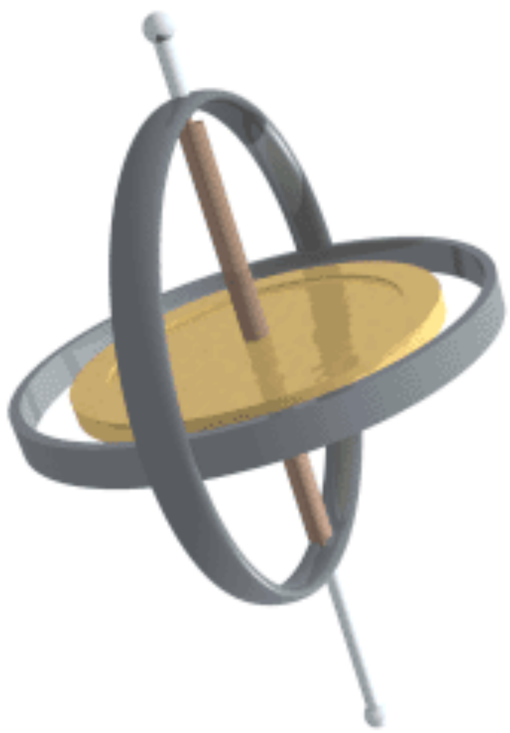


Fig. 2.7. Sum of sine waves with periods 95, 125 and 400 kyr.

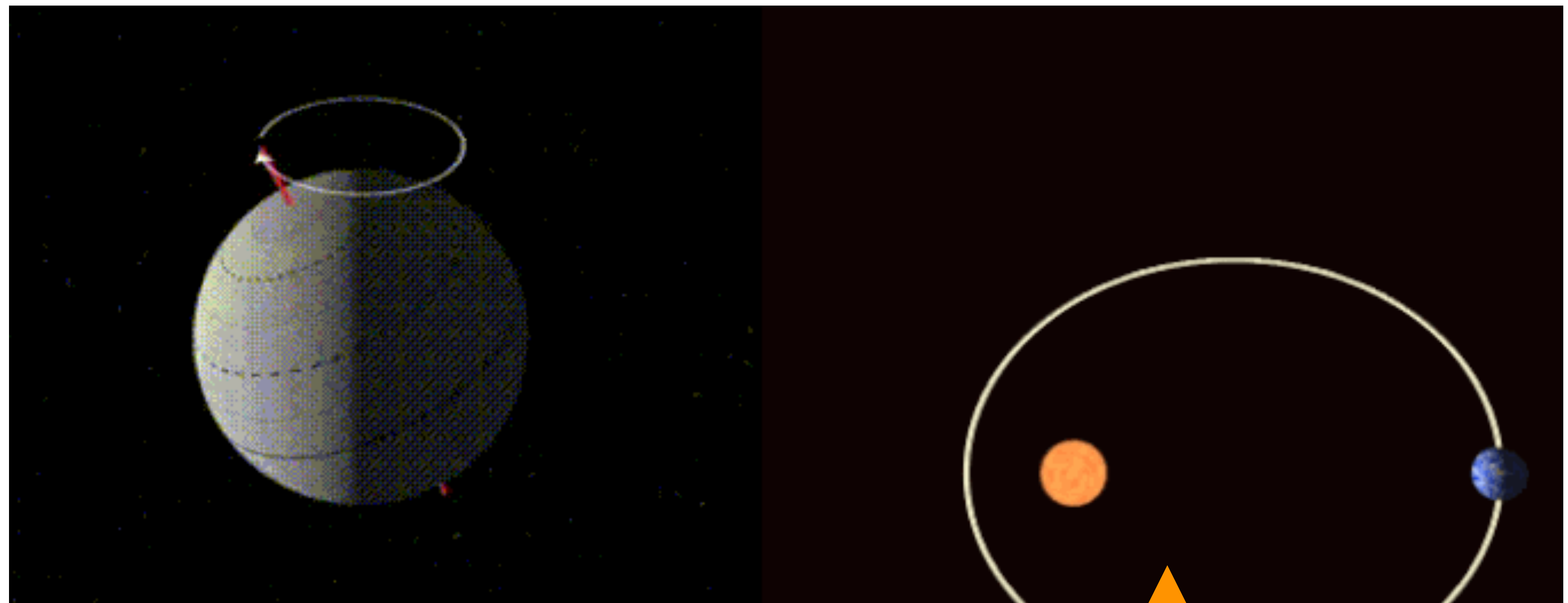


https://www.ua-magazine.com/2022/04/16/astronomy-may-have-influenced-ancient-human-species-and-gave-rise-the-homo-sapiens/

Milankovitch forcing: precession of orbit and of axis of rotation



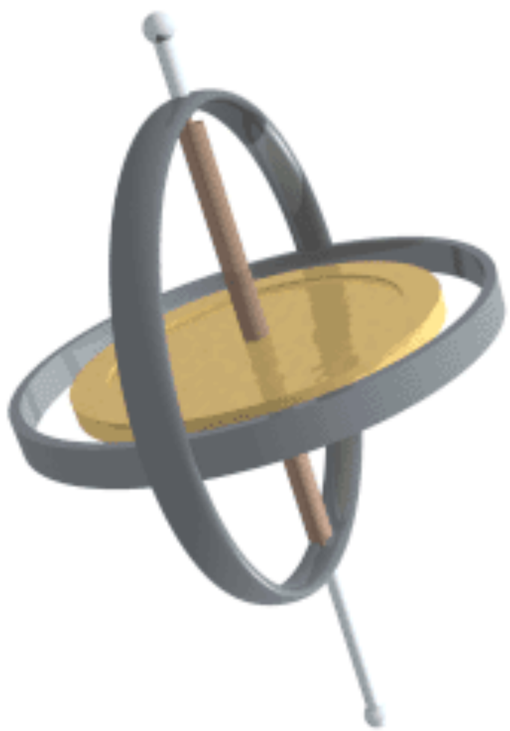
https://commons.wikimedia.org/wiki/File:Gyroscope_precession.gif



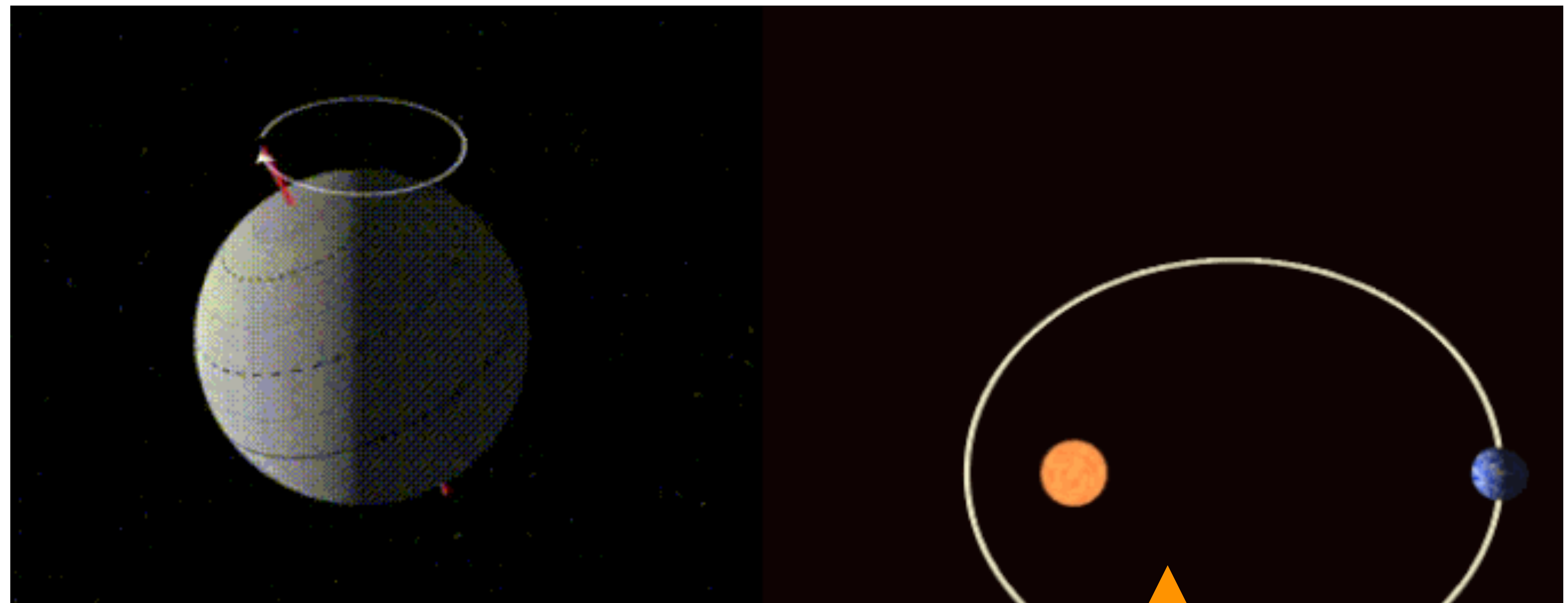
Rotational precession [left] and orbital precession [right]. [Robert Simmon / NASA; WillowW / Wikimedia Commons] downloaded from https://www.technologyshout.com/there-is-a-new-hypothesis-about-how-uranus-tilts-on-its-side-24/

Note speed of orbiting at different phases of cycle, due to Kepler's laws

Milankovitch forcing: precession of orbit and of axis of rotation



https://commons.wikimedia.org/wiki/File:Gyroscope_precession.gif



Rotational precession [left] and orbital precession [right]. [Robert Simmon / NASA; WillowW / Wikimedia Commons] downloaded from https://www.technologyshout.com/there-is-a-new-hypothesis-about-how-uranus-tilts-on-its-side-24/

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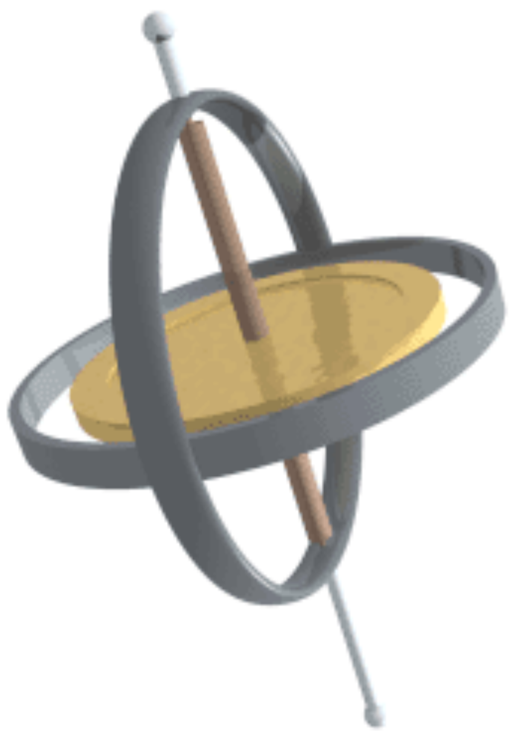
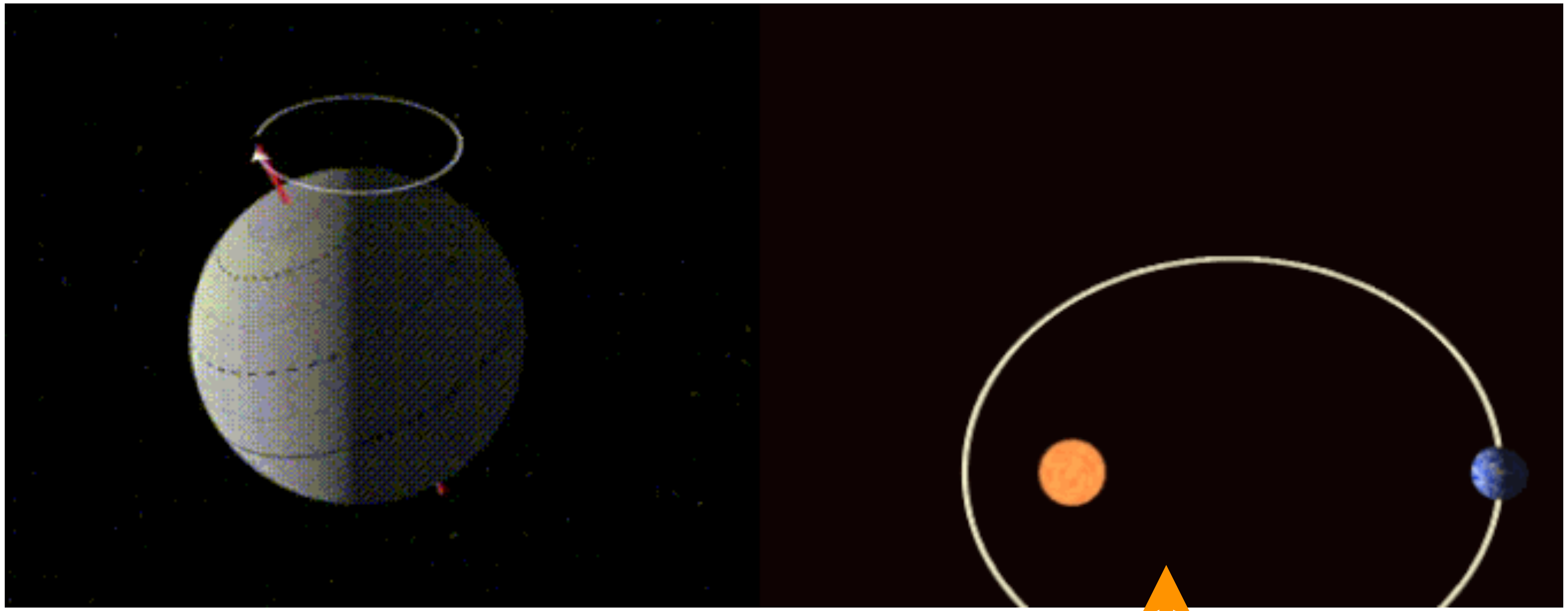


https://commons.wikimedia.org/wiki/File:Gyroscope_precession.gif



Rotational precession [left] and orbital precession [right]. [Robert Simmon / NASA; WillowW / Wikimedia Commons] downloaded from https://www.technologyshout.com/there-is-a-new-hypothesis-about-how-uranus-tilts-on-its-side-24/

Note speed of orbiting at different phases of cycle, due to Kepler's laws

Milankovitch forcing: precession

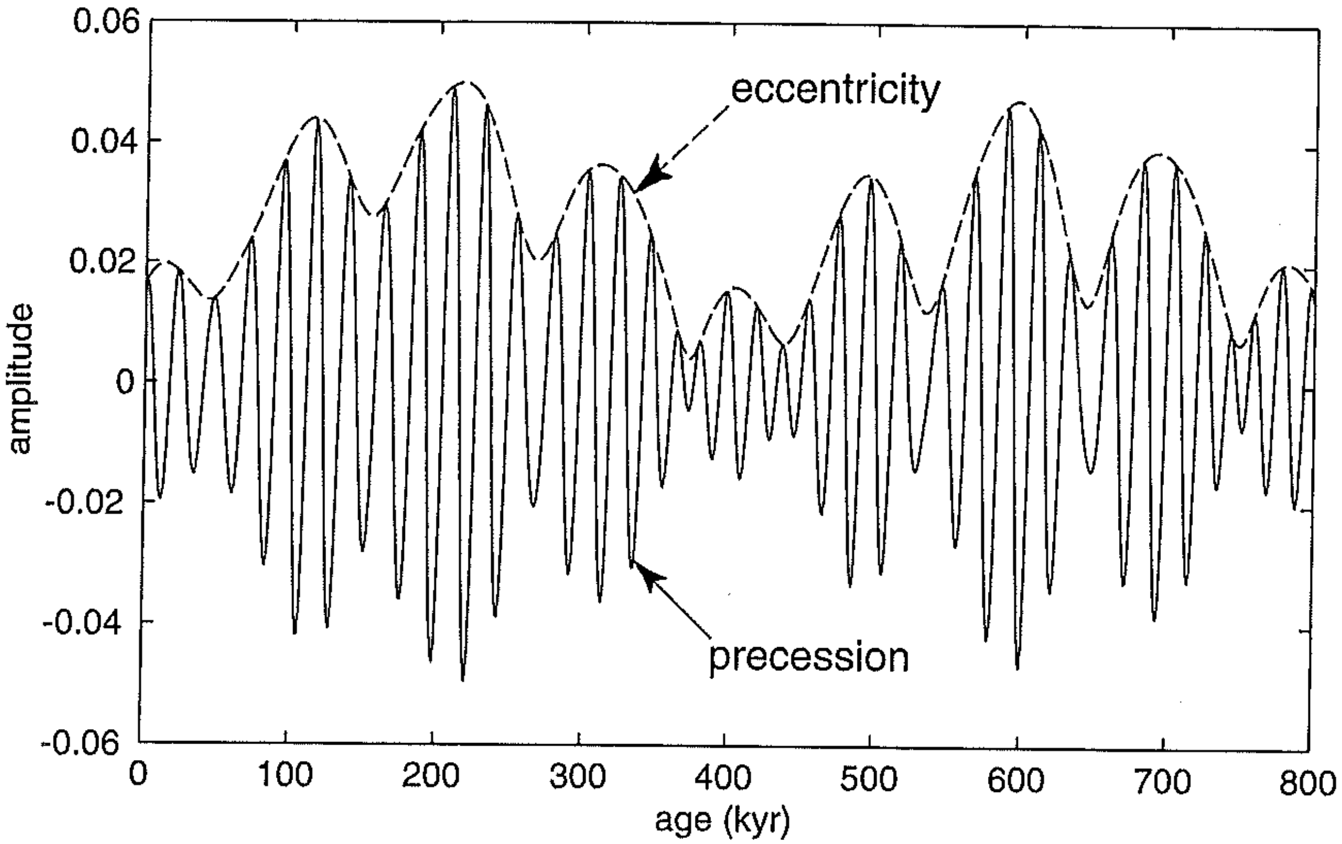


Fig. 2.10. Precession parameter p and eccentricity.

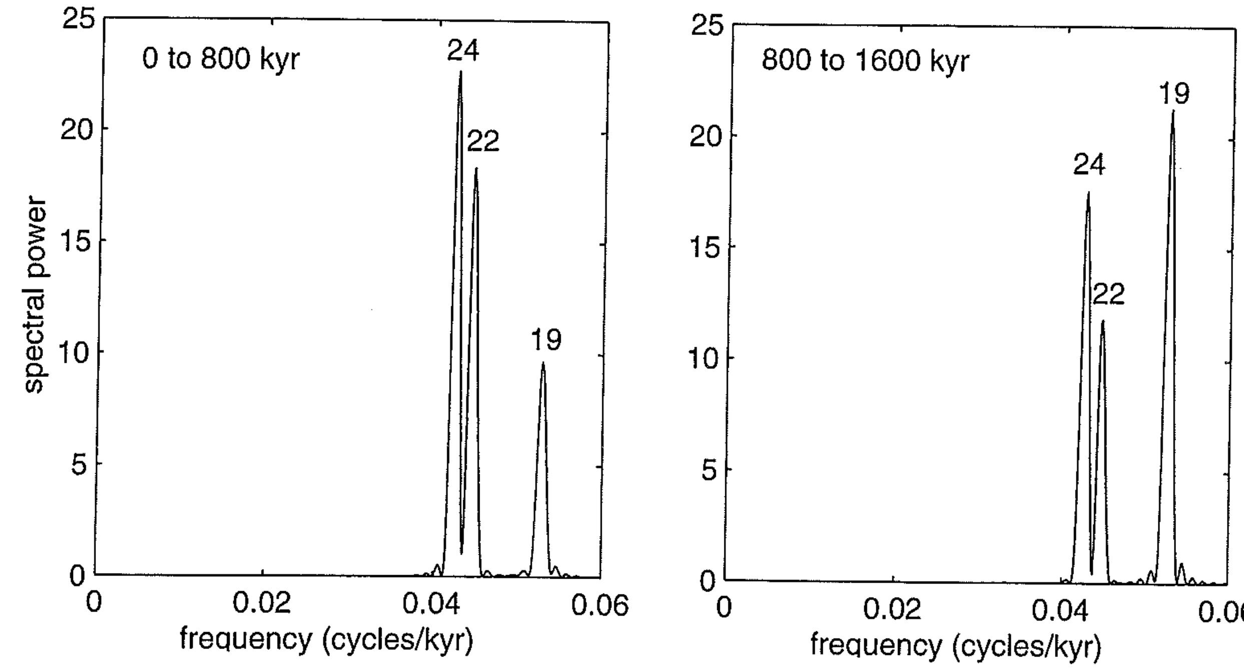
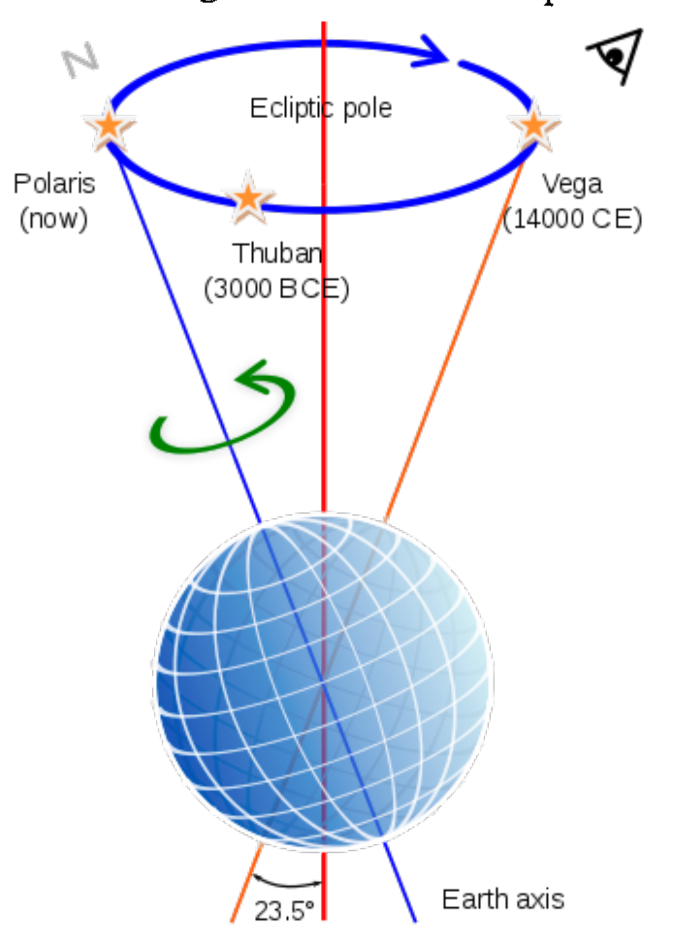


Fig. 2.11. Spectrum of precession.

$$p = e \sin \omega_M$$

climate precession parameter



https://commons.wikimedia.org/wiki/File:Precession-sphere-EN.svg

precession: moon & sun pulling on equatorial bulge of Earth

Precession effect is anti-symmetric with respect to seasons & hemispheres; annual average of precession vanishes at each latitude; precession has no effect when eccentricity is zero (circular orbit); Paillard, 2001: ”energy received at a given latitude & between two given orbital positions does not depend on the climatic precession. However, the time necessary for the Earth to move between these two positions (e.g., length of summer season) changes w/climatic precession. The insolation — amount of energy received per unit time — therefore changes w/climatic precession, through length of seasons.”

Milankovitch forcing: precession+eccentricity

In-class workshop

back of the envelope calculation to show that in spite of the small eccentricity, the combined effect of eccentricity and precession on summer insolation is very significant

- Assume a (moderately large) Earth orbit eccentricity of 0.03

Milankovitch forcing: precession+eccentricity

In-class workshop

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- Calculate the maximum top of the atmosphere summer insolation at 65N at each of the two configurations.

Milankovitch forcing: precession+eccentricity

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- How big is the difference in W/m^2 between the two configurations? As a percent fraction of the weaker insolation?

Milankovitch forcing: precession+eccentricity

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[solution: presenter notes]

Milankovitch forcing: obliquity

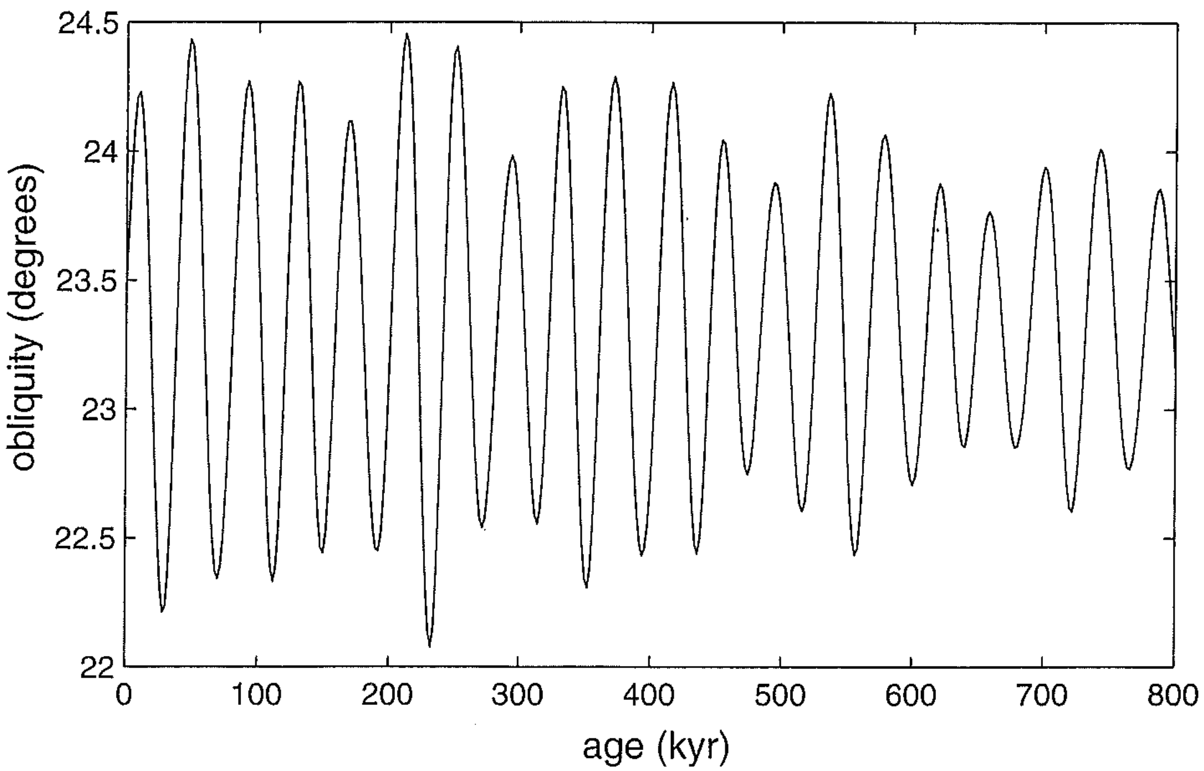


Fig. 2.12. Obliquity.

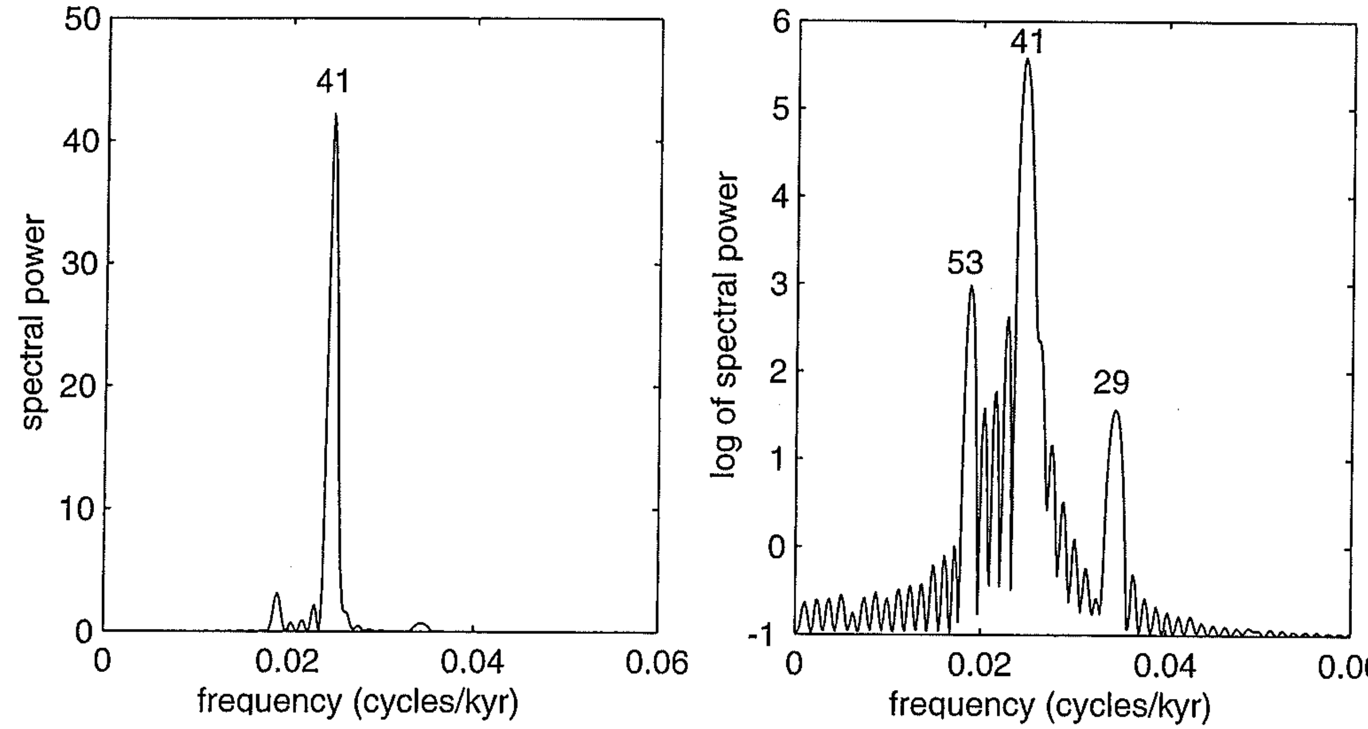
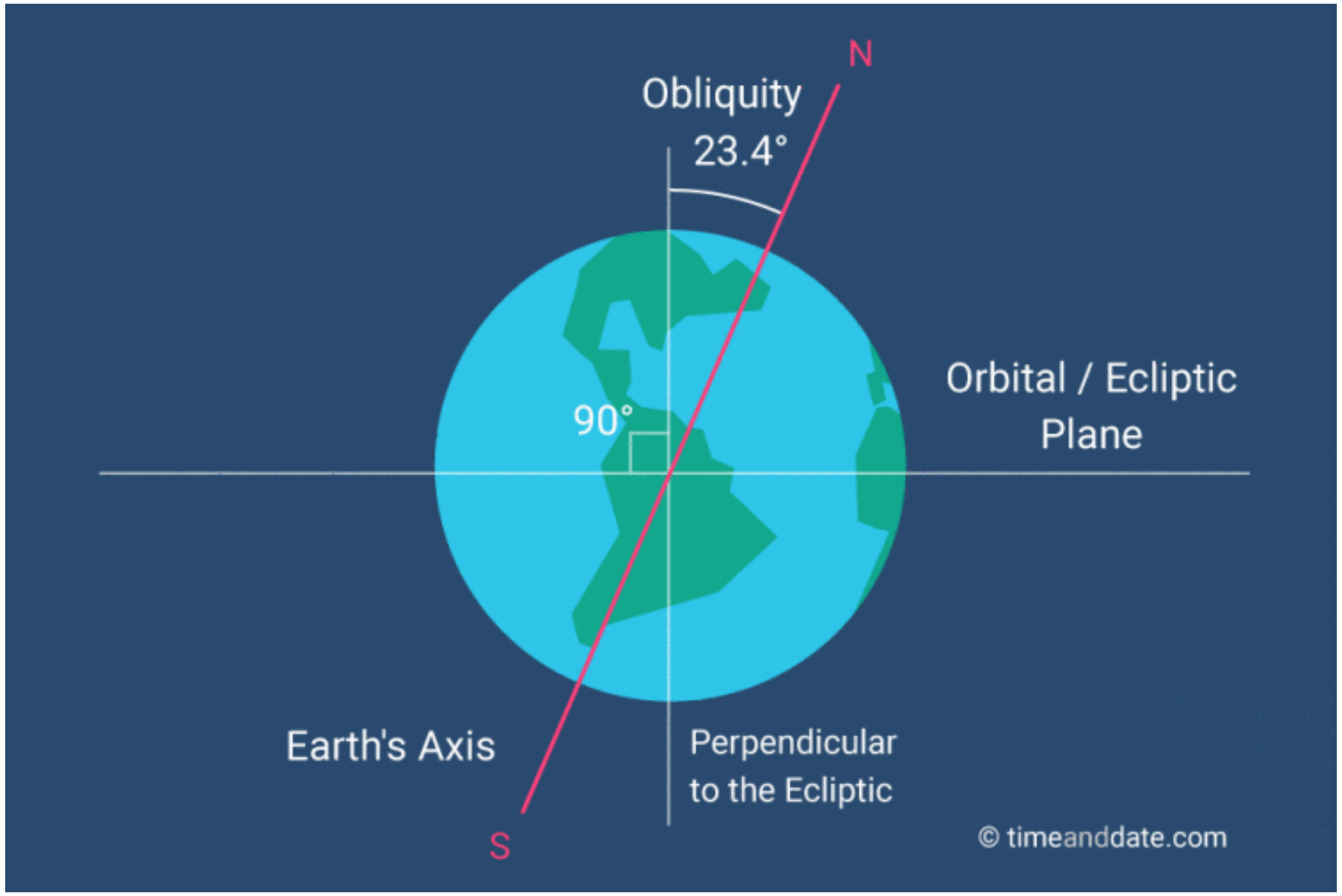


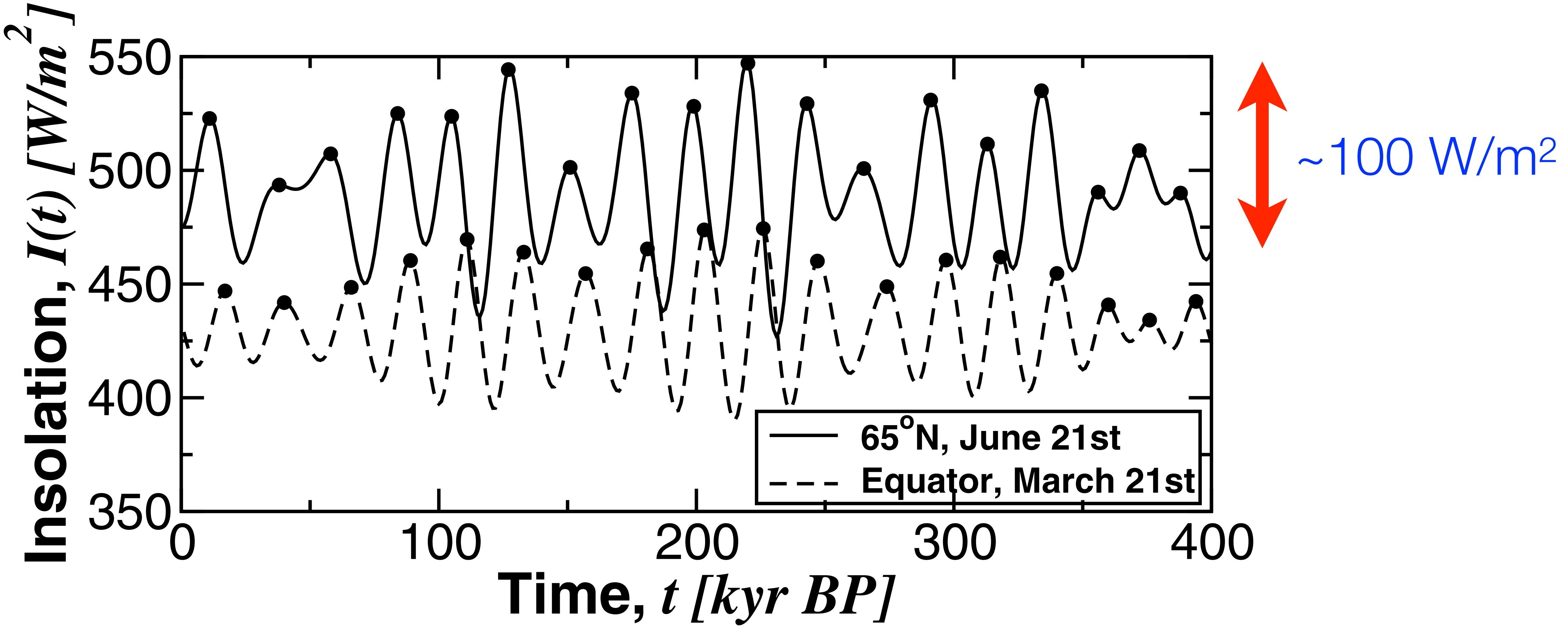
Fig. 2.13. Spectrum of obliquity, linear and log plots.

Muller & MacDonald 2002

Obliquity does affect the annual mean insolation at a given latitude, but not the global average. ➨ Antisymmetric effect on the N/S hemispheres. Larger obliquity leads to more radiation at the poles in summers, but still none at winter, so more generally high latitude annual insolation depends on obliquity.



Milankovitch forcing: combining precession, obliquity, eccentricity



June insolation different latitudes

Figure 1. Insolation time series at 65°N for 21 June (longitude measured from the vernal equinox is l = 90°) and at the equator for 21 March (the vernal equinox, l = 0°) [Laskar, 1990]. Note that the 21 March insolation (warm season in the equatorial Pacific) leads the insolation of 21 June (Northern Hemisphere (NH) summer solstice) at 65°N by 5 kyr.

ASHKENAZY AND TZIPERMAN 2006

Milankovitch forcing: combining precession, obliquity, eccentricity

July insolation power at different latitudes

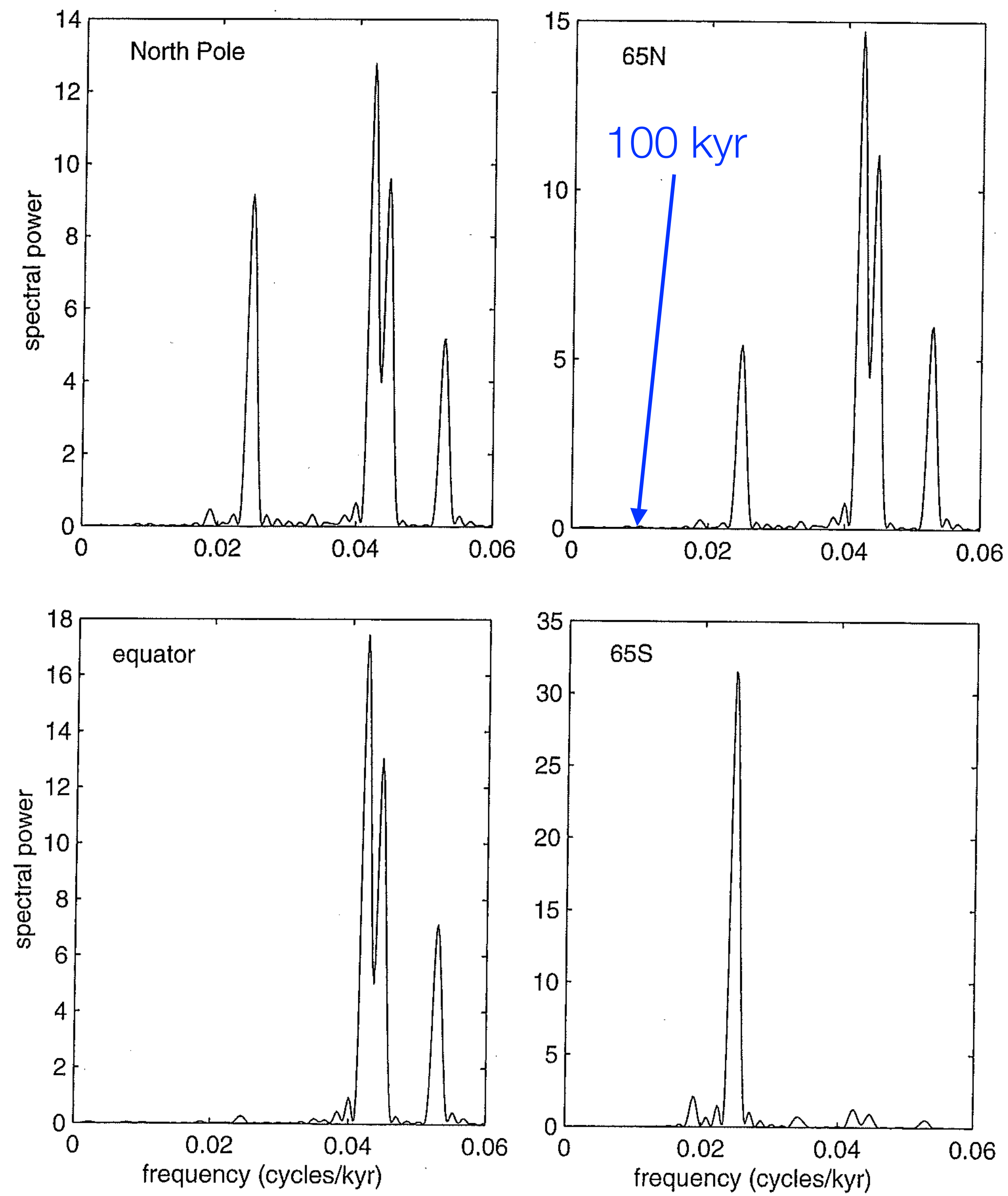


Fig. 2.14. Spectra of July insolation, at latitudes 90N, 65N, 0 and 65S.

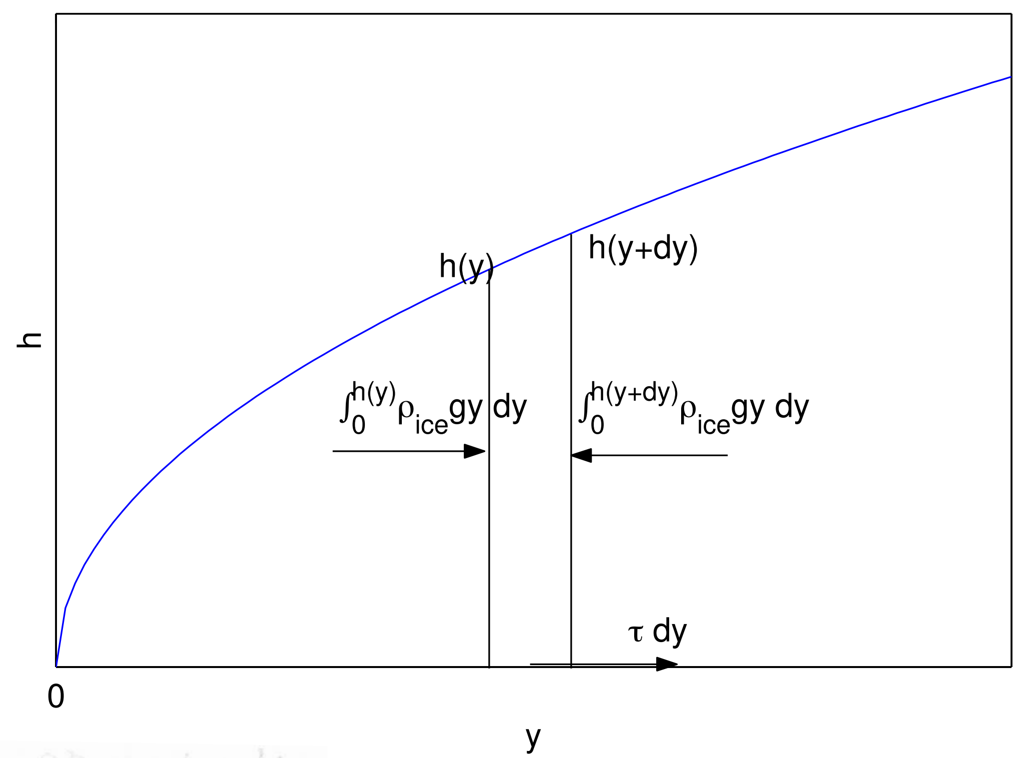
Muller & MacDonald 2002

notes: parabolic ice sheet

(simple version only, 1-WH-lect_08_2001.pdf p 100

use next slide)

Parabolic ice sheet profile



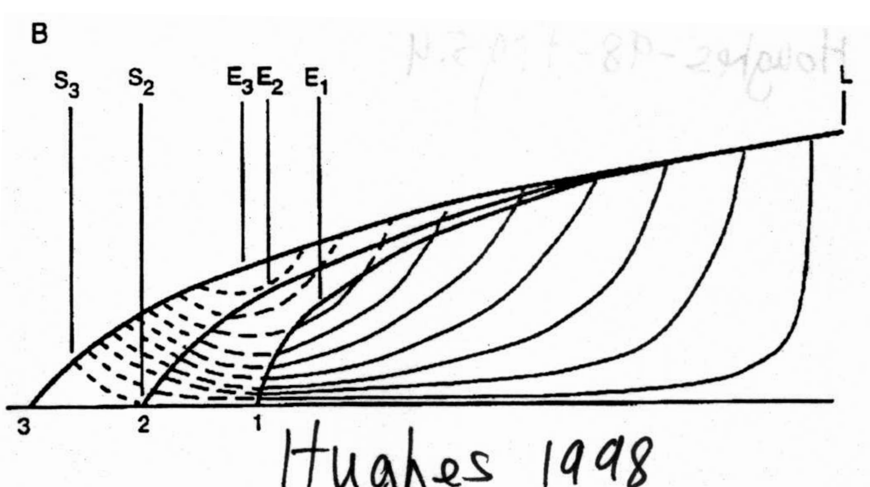


Figure 5.4: Parabolic profiles for a plastic ice sheet on a horizontal bed. (*A*) Profile 1: τ_o = 105 kPa for an advancing ice sheet. Profile 2: τ_o = 66 kPa for an equilibrium ice sheet. Profile 3: τ_o = 42 kPa for a retreating ice sheet. (*B*) Ice trajectories for ice sheets that are advancing (solid curves), in equilibrium (solid and broken curves), and retreating (solid, broken, and dashed curves). Equilibrium points are E_1 for advancing, E_2 for equilibrium, and E_3 for retreating ice sheets. Stagnation points are S_2 for equilibrium and S_3 for retreating ice sheets.

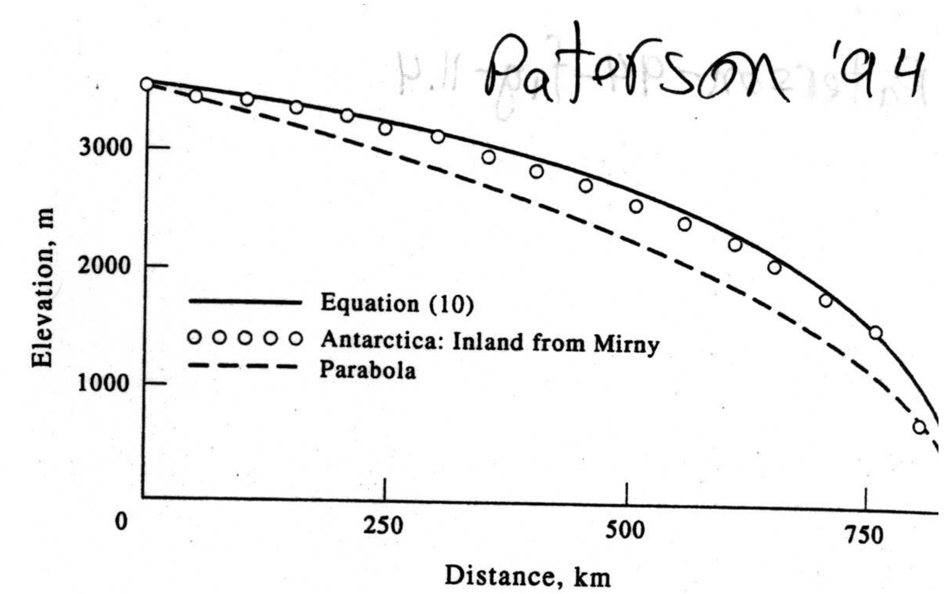
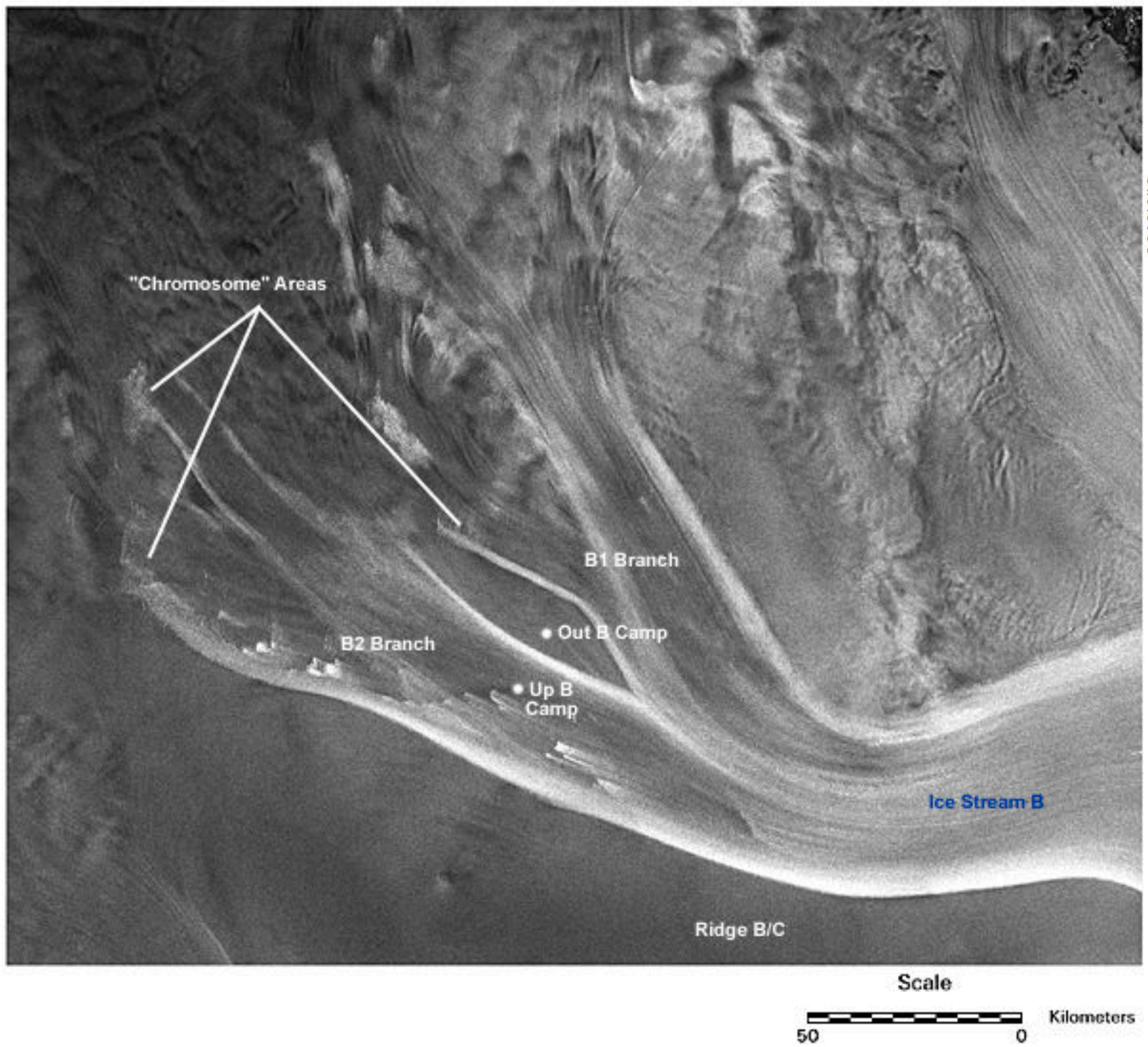


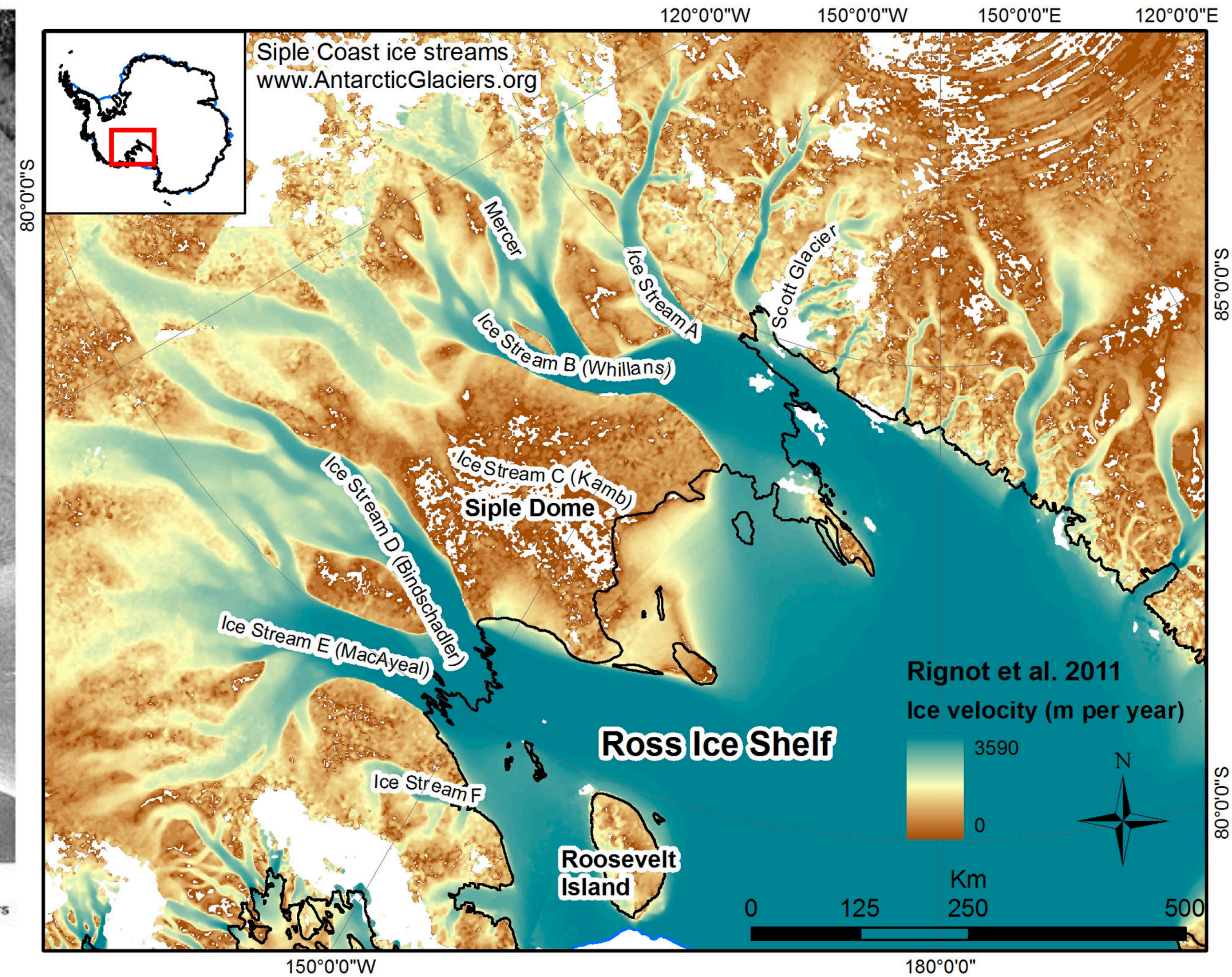
FIG. 11.4. Profile of Antarctic Ice Sheet inland from Mirny compared with theoretical profiles. Data from Vialov (1958).

Ice streams, calving, ablation



ice stream B, Antarctica

https://nsidc.org/support/faq/what-features-can-i-detect-ramp-amm-1-sar-image-mosaic-antarctica



Rignot et al 2011

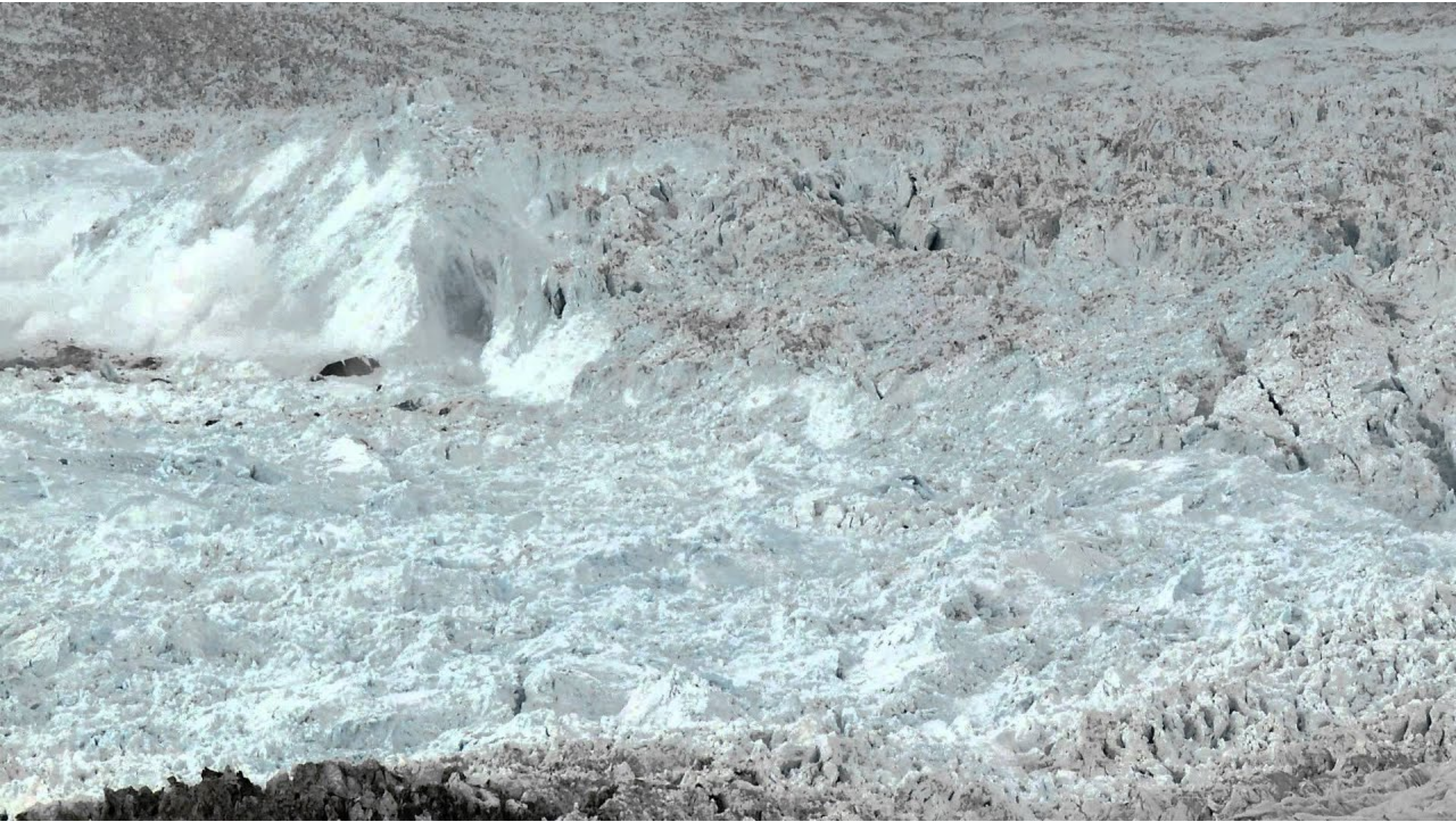
Ablation, ice flow
Antarctic ice streams

https://www.jpl.nasa.gov/video/details.php?id=1015

Ablation, ice flow
Antarctic ice streams

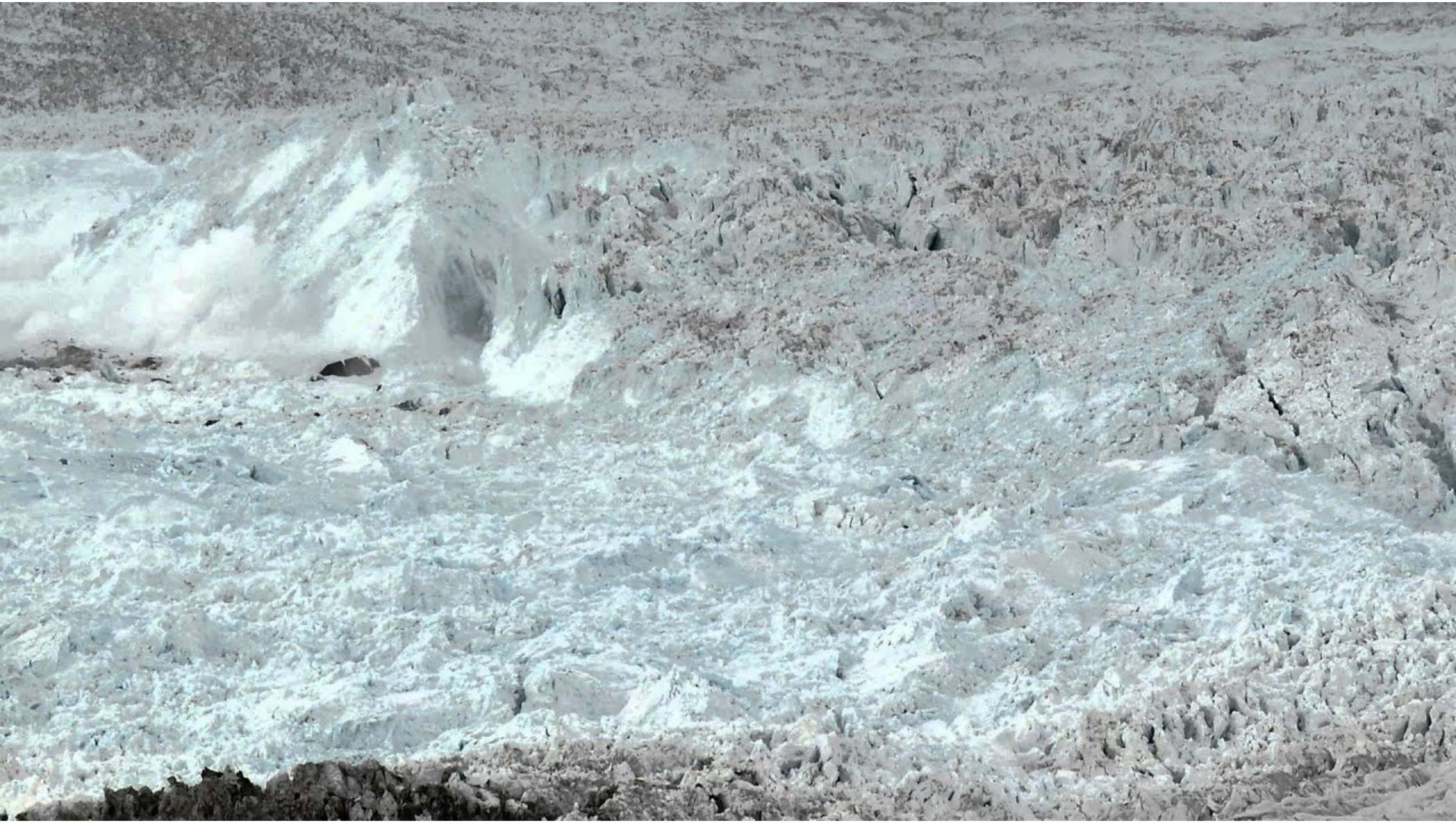
https://www.jpl.nasa.gov/video/details.php?id=1015

Greenland Calving event, “Chasing ice” film



https://www.youtube.com/watch?v=hC3VTgIPoGU

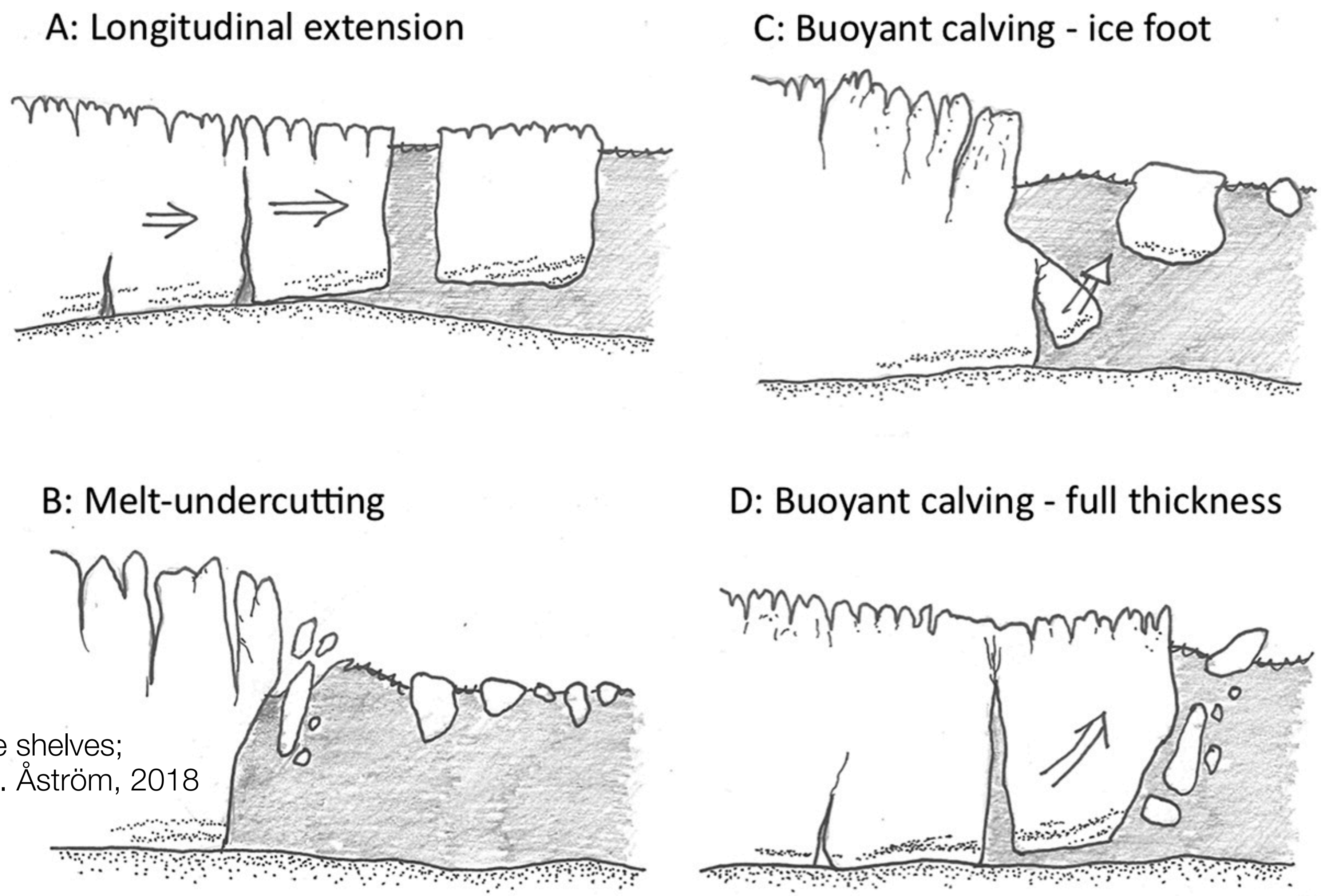
Greenland Calving event, “Chasing ice” film



https://www.youtube.com/watch?v=hC3VTgIPoGU

Ablation

Calving: role of buoyancy forces



Calving glaciers and ice shelves;
Douglas I. Benn, Jan A. Åström, 2018

Figure 2. A selection of key calving styles: **(a)** rifting due to longitudinal extension, **(b)** collapse of overhang following undercutting by subaqueous melt, **(c)** buoyant calving: release of a protruding 'ice foot' below the waterline and **(d)** buoyant calving: uplift of a super-buoyant glacier tongue.

Temperature precipitation feedback

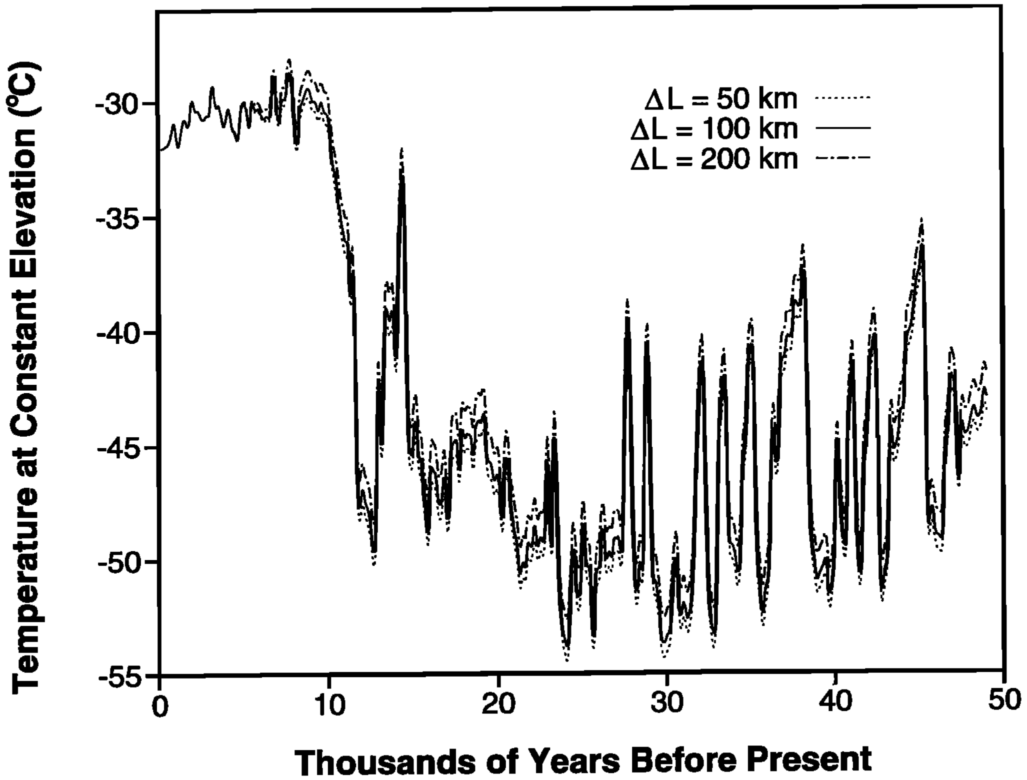


Figure 3. Temperature history according to calibrated isotope curve, corrected for elevation changes. The data have been smoothed with a 250-year triangular filter so that the effect of different elevation corrections, corresponding to different marginal retreat distances, can be seen.

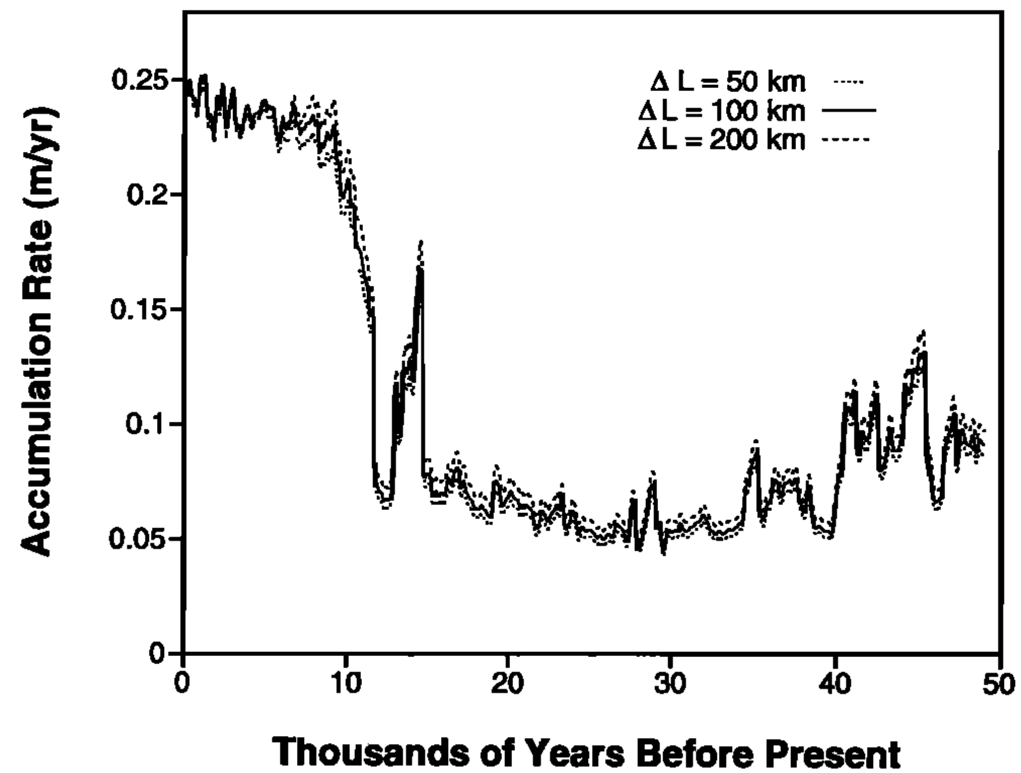


Figure 5. Accumulation rate histories for different marginal retreat distances.

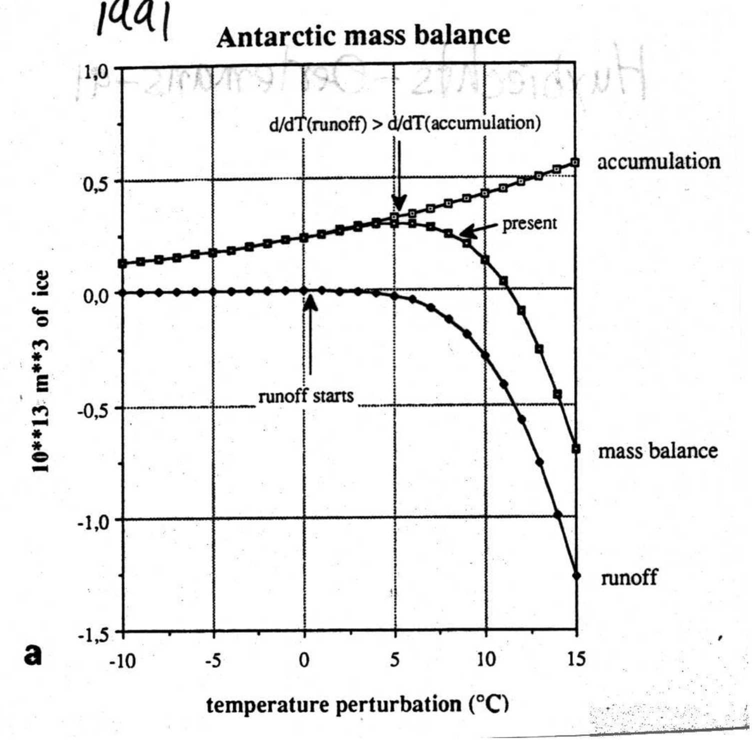
Will greenhouse warming lead to Northern Hemisphere ice-sheet growth?

Gifford H. Miller* & Anne de Vernal† 1992

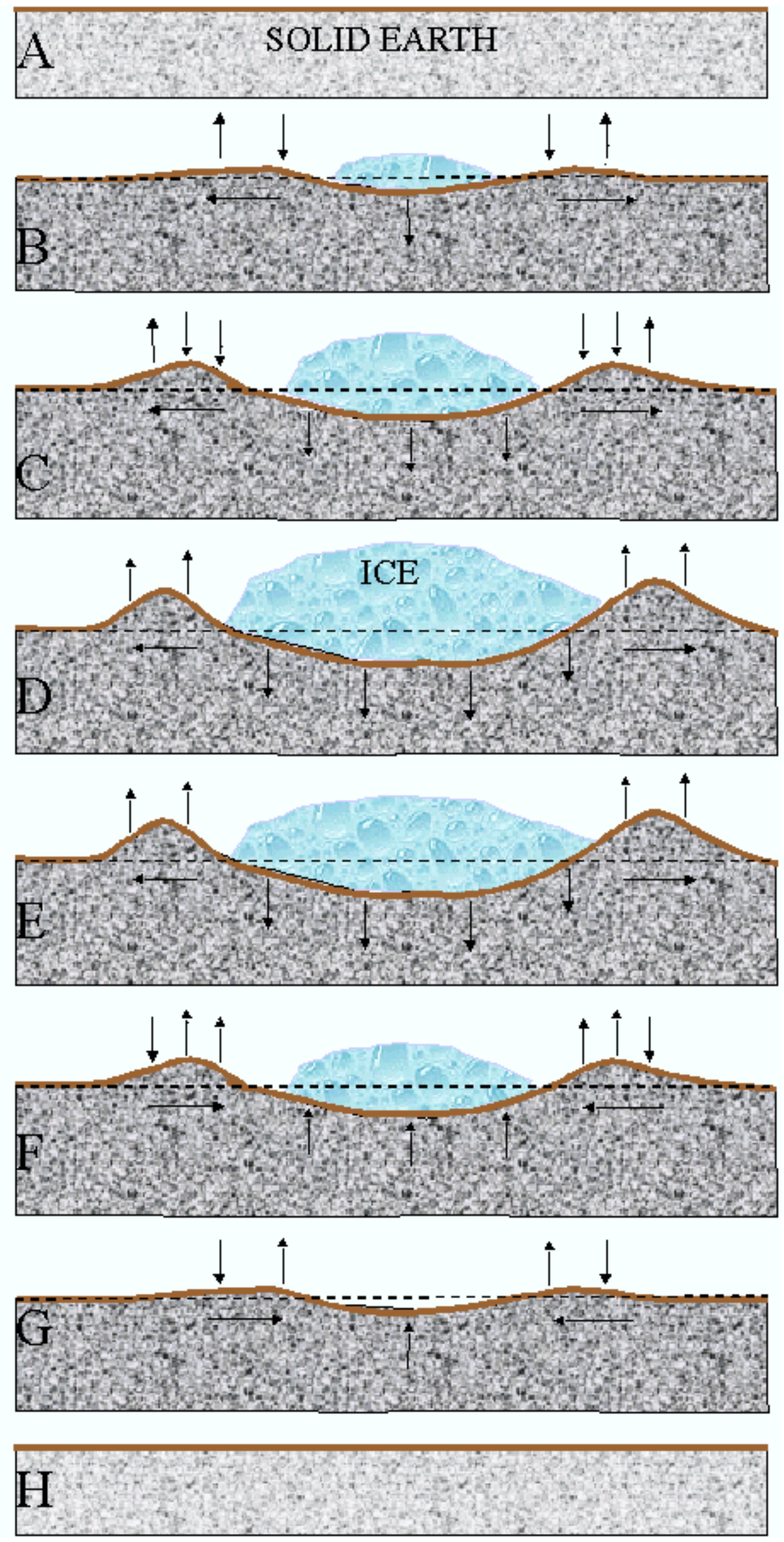
* Center for Geochronological Research, INSTAAR, University of Colorado, Boulder, Colorado 80309-0450, USA
† GEOTOP, Université du Québec à Montréal, CP 8888, Succursale "A", Montréal, Québec, Canada, H3C 3P8

Huybrechts and Oerlemans:

1991



isostatic adjustment



Early glacial theories

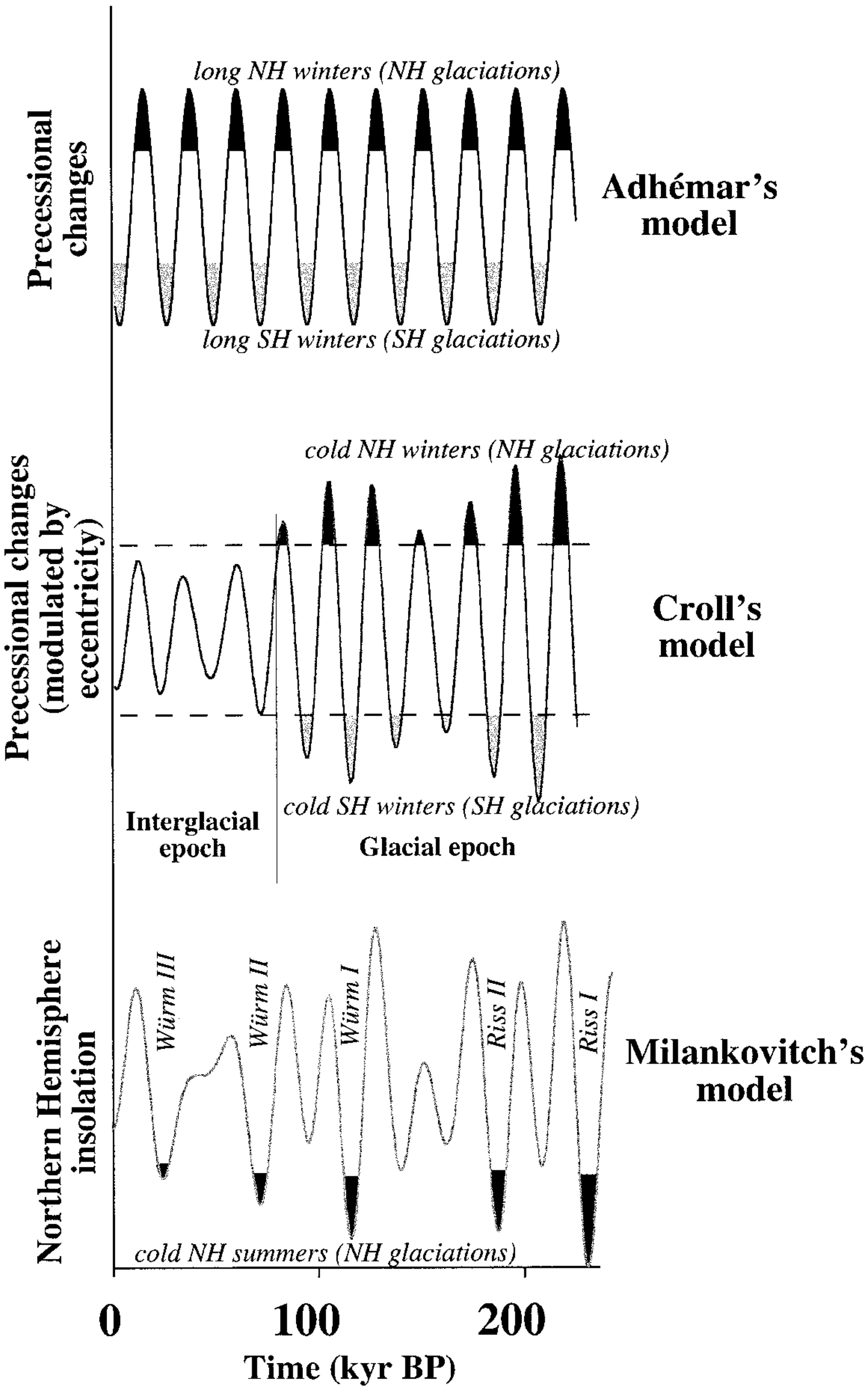


Figure 1. The ice ages according to **Adhemar [1842]**, **Croll [1875]**, and **Milankovitch [1941]**. Adhemar was aware only of the precession of equinoxes, and he related the glacial ages to the lengths of the seasons. Croll benefited from the advances in astronomy and was aware of changes in the other astronomical parameters, though he could not compute the obliquity changes. In his view, the interglacial epoch is associated with small eccentricity and therefore with small precessional changes. **Milankovitch** was the first to integrate the effect of all astronomical parameters and to compute explicitly the insolation at the top of the atmosphere. He **understood that summer, not winter, was the critical season**. His insolation minima were associated with the major alpine glacier advances recorded by geological evidence.

Paillard 2001

Early glacial theories

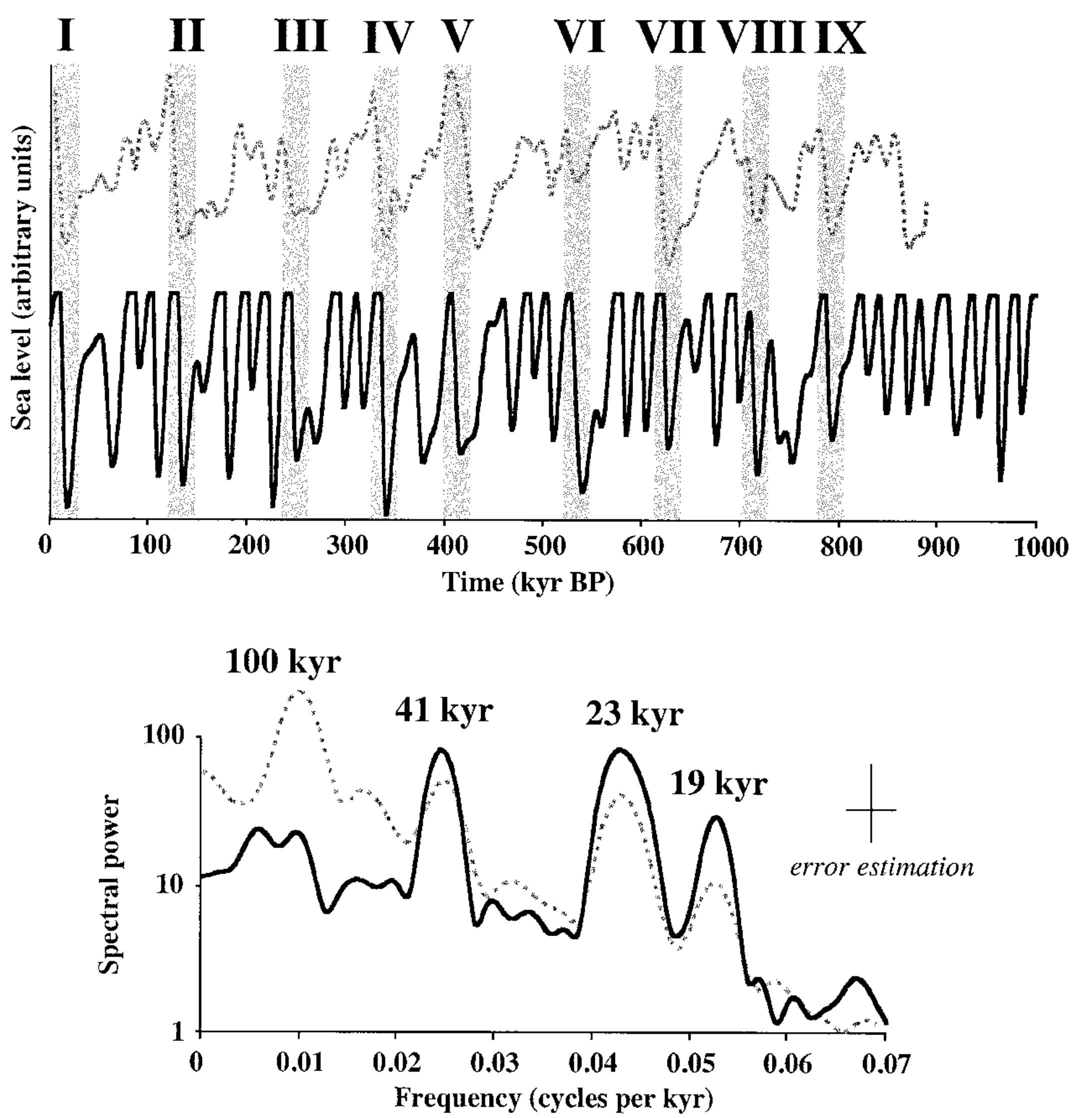
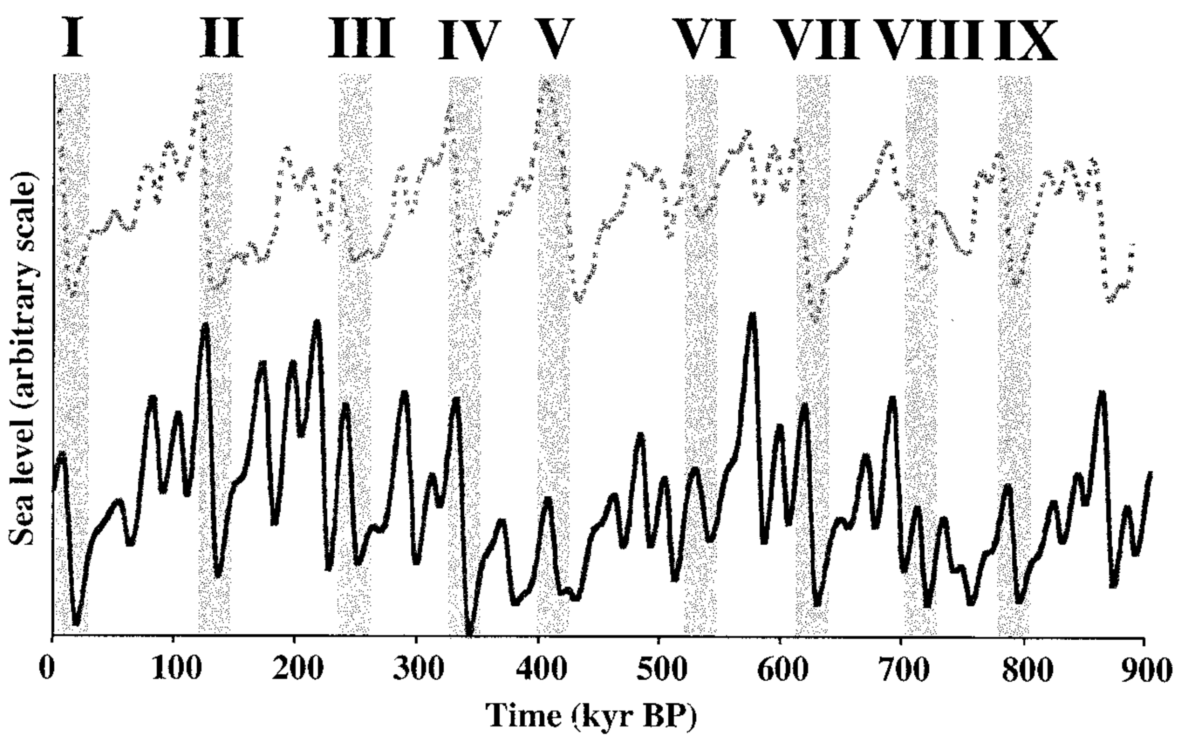


Figure 9. Results from the **Calder [1974] model**. The threshold i_0 is equal to 502 W m^2, and the ratio k_A/k_M is chosen equal to 0.22. The forcing ***i*** is the summer solstice insolation at 65N [Laskar, 1990]. The result is very sensitive to these choices. The agreement with the record is quite poor, but this crude model still predicts the major transitions at the right time, a feature that many, more sophisticated models do not reproduce well. An isotopic record is given here for comparison [Bassinot et al., 1994b].

$$\frac{dV}{dt} = -k(i - i_0),$$

Paillard 2001

Early glacial theories



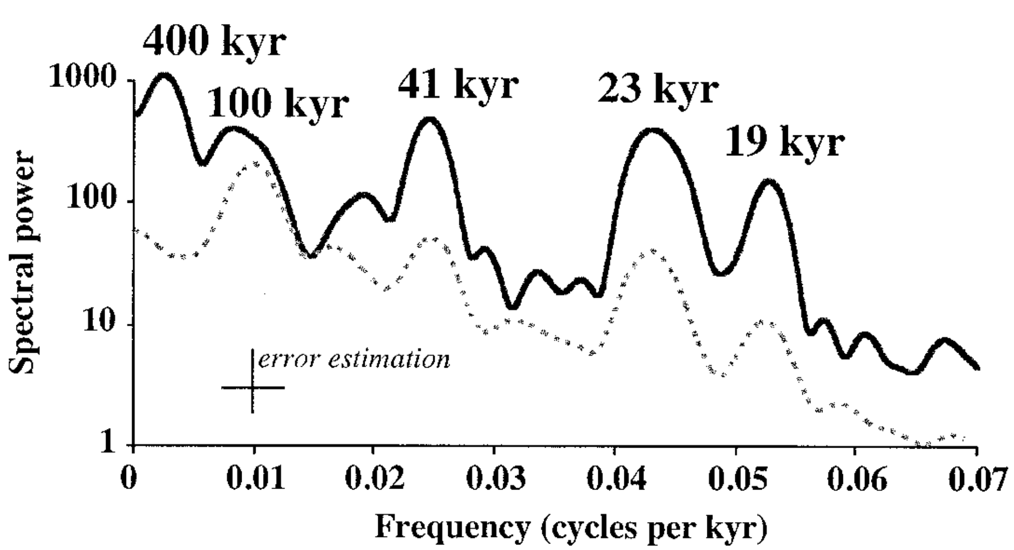
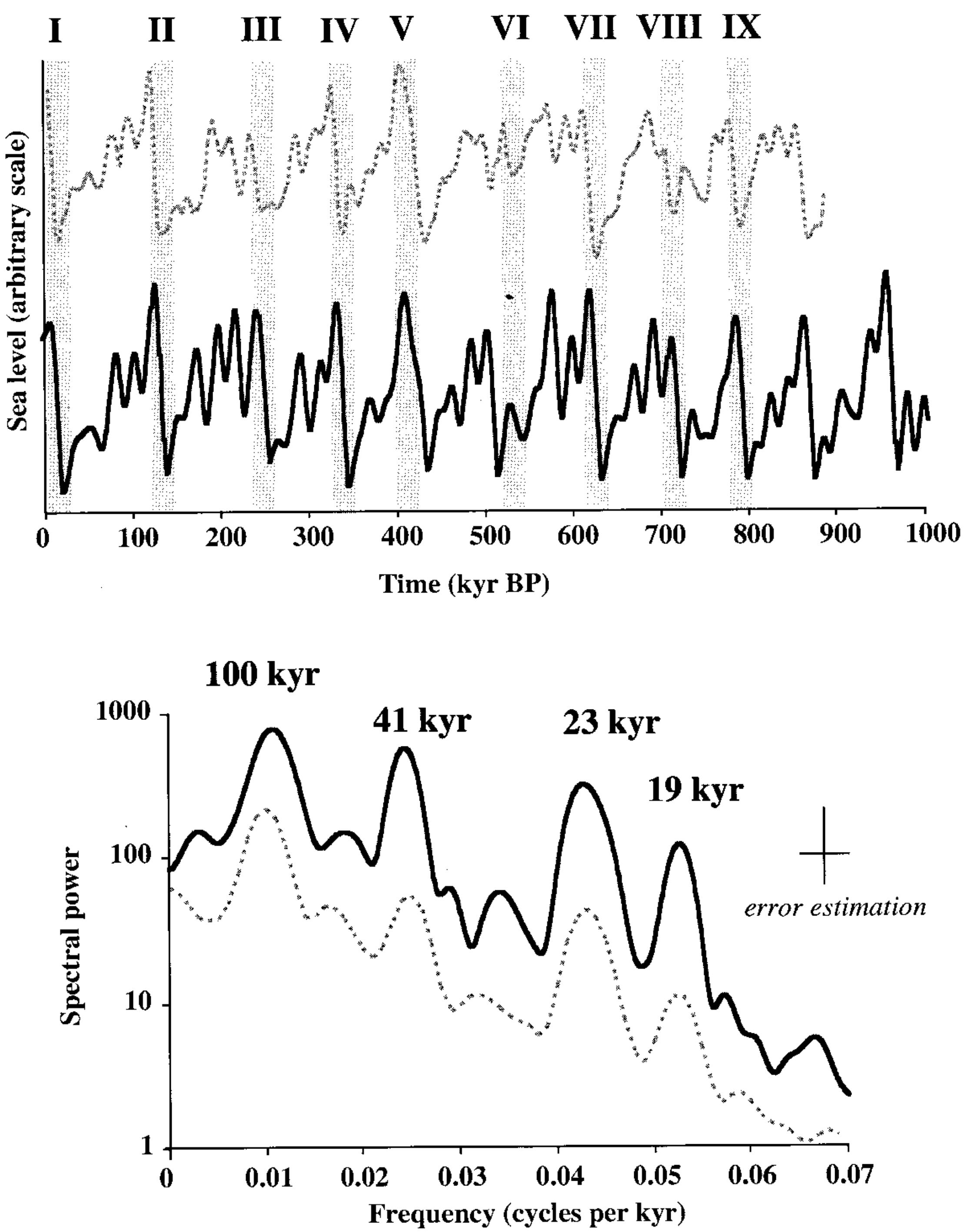


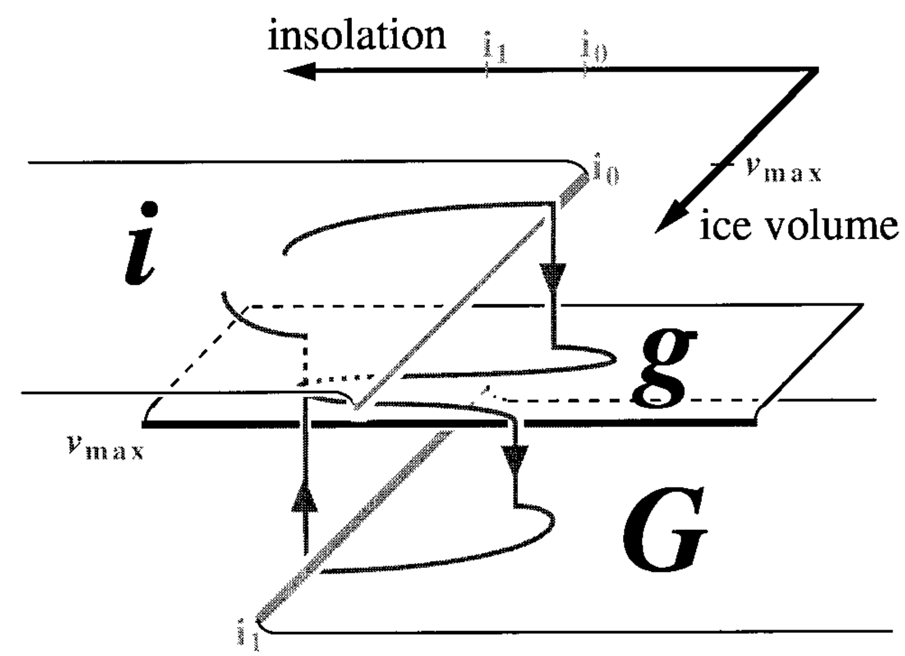
Figure 10. Same as Figure 9, but for the Imbrie and Imbrie [1980] model. The forcing ***i*** is the summer solstice insolation at 65N. The time constants are τ_M=42 kyr and τ_A=10 kyr.

$$\frac{dV}{dt} = \frac{(i - V)}{\tau},$$

Paillard 2001

An Imbrie-like glacial theory





$$\frac{dV}{dt} = -\frac{(V_R - V)}{\tau_R} - \frac{F}{\tau_F},$$

Figure 13. Same as Figure 9, but for Paillard's [1998] model. Threshold values are $\boldsymbol{i}_0$=0.75 and $\boldsymbol{i}_1$=0. Time constants are τ_i=10 kyr, τ_G=τ_g=50 kyr, and τ_F=25 kyr. A total of about 14 model parameters.

Paillard 2001

100 kyr cycle as a nonlinear response to Milankovitch

Le-Treut and Ghil (1983): the 100 cycle as a difference tone of insolation's 19k and 23k frequencies, due to nonlinear glacial dynamics.

$$\frac{1}{109} kyr^{-1} = f_1 - f_2 = \frac{1}{19} kyr^{-1} - \frac{1}{23} kyr^{-1}$$

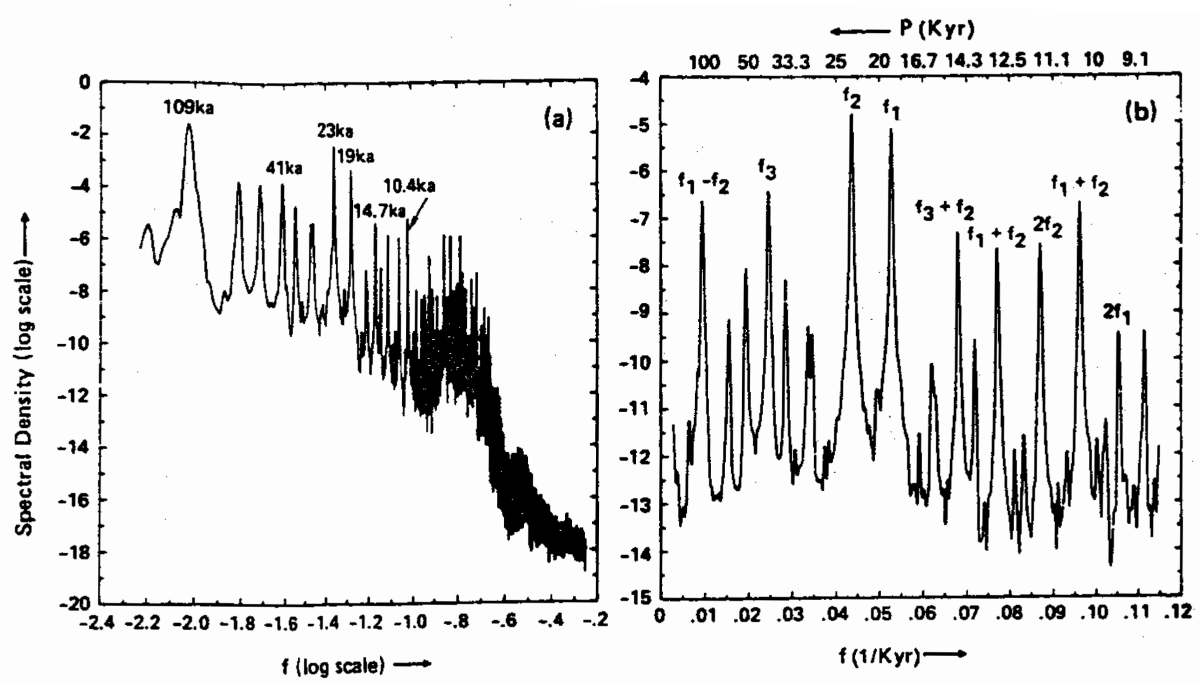


Fig. 10. Power spectrum of (a) simulated marine $\delta^{18}0$ and (b) ice-core $\delta^{18}0$ records. The periods associated with the labeled peaks in panel (b) are, from left to right, 109 kyr, 41 kyr, 23 kyr, 19 kyr, 14.7 kyr, 13 kyr, 11.5 kyr, 10.4 kyr and 9.5 ky (from [90]).

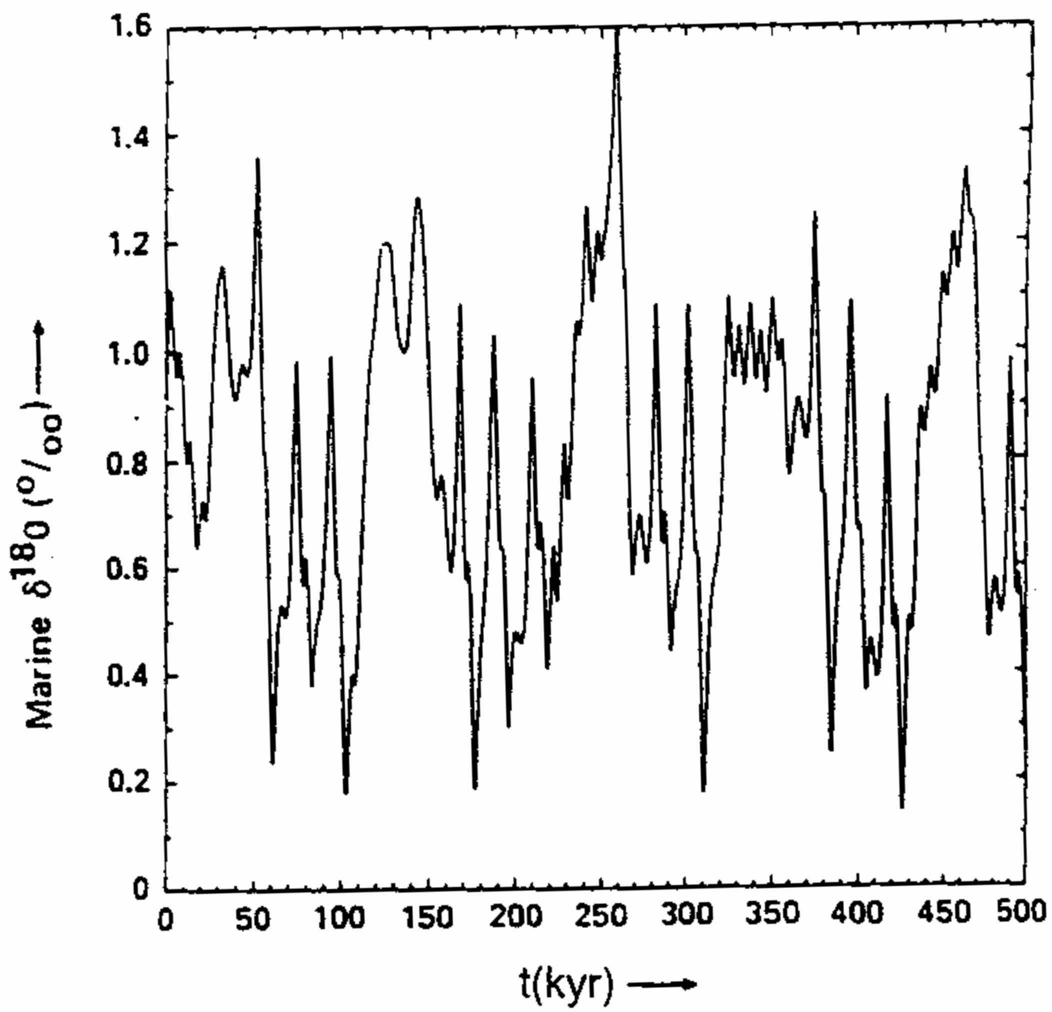


Fig. 11. Model-simulated marine $\delta^{18}0$ record; its power spectrum appears in Fig. 10a (from [90]).

Then Rial (1999):

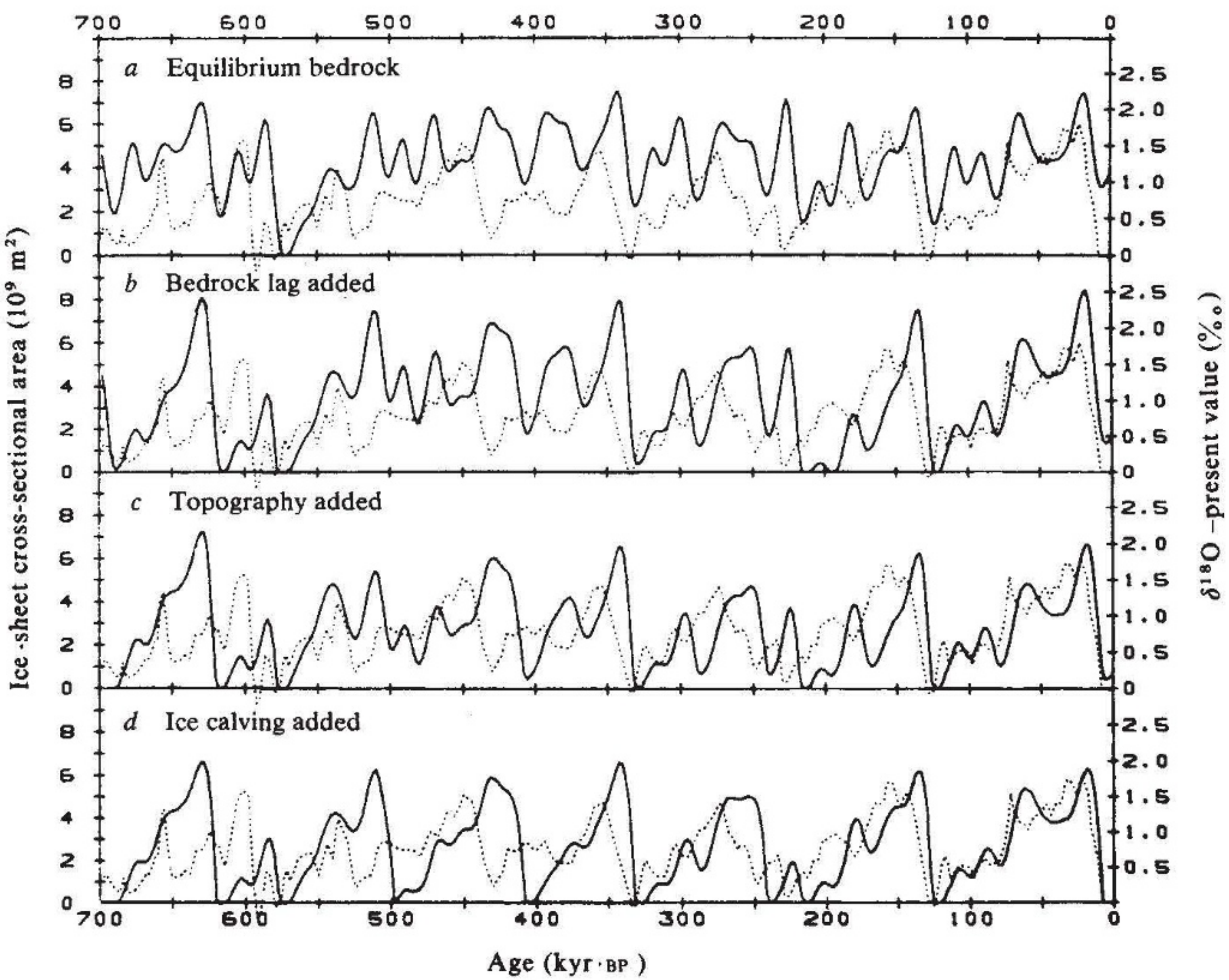
$$\frac{1}{107} kyr^{-1} = \frac{1}{95} kyr^{-1} - \frac{1}{826} kyr^{-1}$$

100 kyr cycle from orbitally-forced ice sheet models

$$\frac{\partial h}{\partial t} = A \frac{\partial}{\partial x}\left[h^{\alpha} \left| \frac{\partial (h+h')}{\partial x} \right|^{\beta} \frac{\partial (h+h')}{\partial x} \right] + G(h+h', x, \text{orbit})$$

Shallow ice approximation with a prescribed surface mass balance

$$G = \begin{cases} a(h+h'-E) - b(h+h'-E)^2 & \text{if } h+h'-E \leq 1{,}500 \text{ m} \\ 0.56 & \text{if } h+h'-E > 1{,}500 \text{ m} \end{cases} \text{ m yr}^{-1}$$



A simple ice sheet model yields realistic 100 kyr glacial cycles

David Pollard, 1982

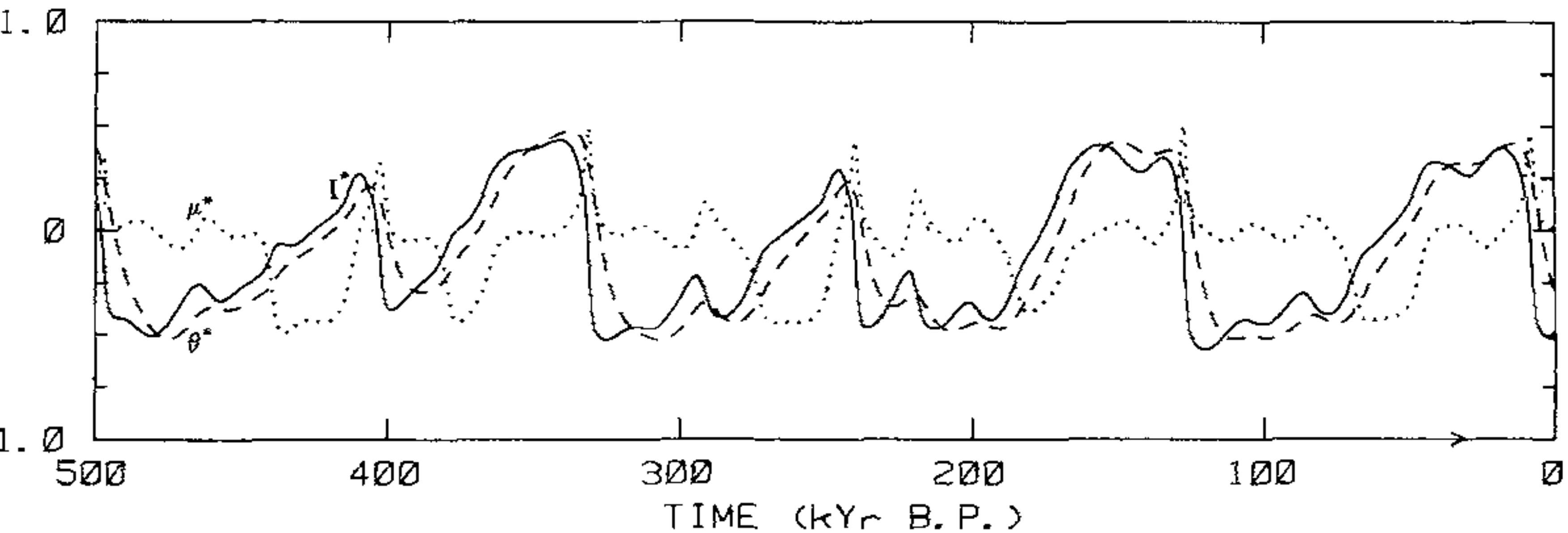
100 kyr cycle from toy “Earth system” models

Barry Saltzman: Carbon dioxide and the delta18O record of late-Quaternary climatic change: a global model (Clim Dyn, 1:77–85, 1987)

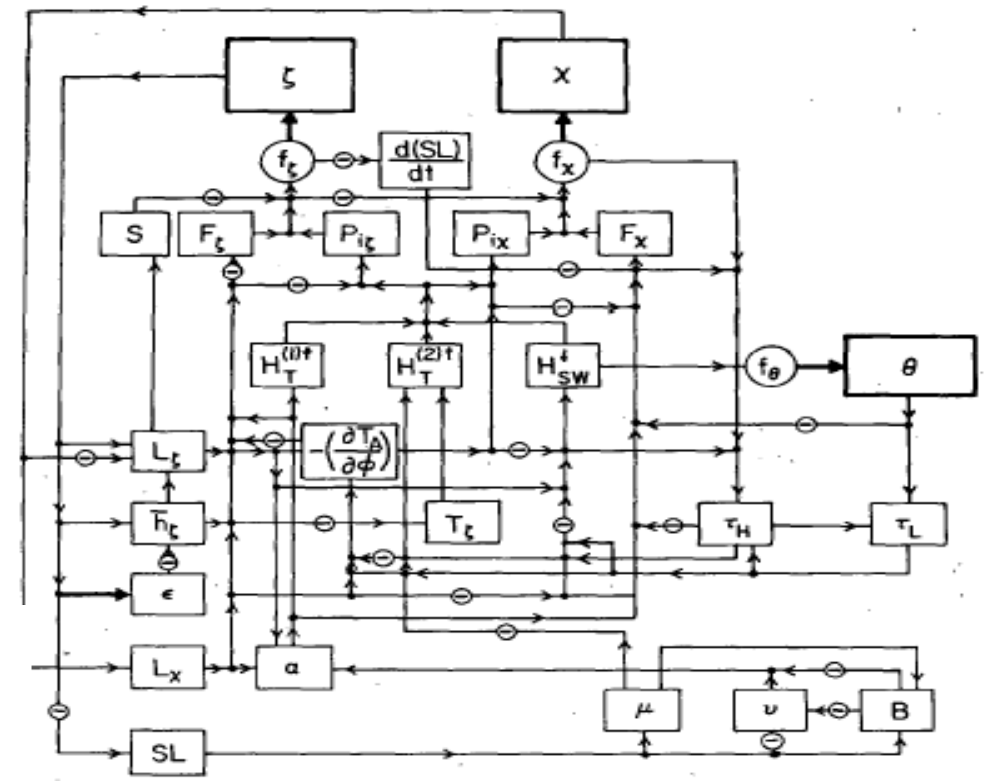
$$\frac{dX}{dt^*} = -\alpha_1 Y - \alpha_2 Z - \alpha_3 Y^2 \quad (17)$$

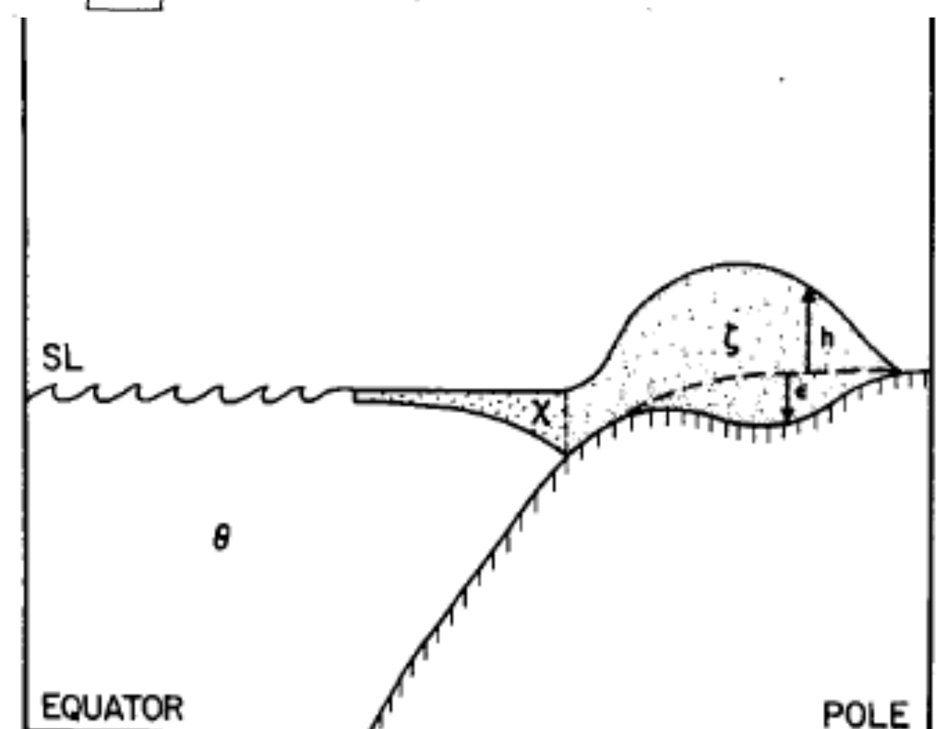
$$\frac{dY}{dt^*} = -\beta_0 X + \beta_1 Y + \beta_2 Z - (X^2 + 0.004 Y^2) Y + F_Y \quad (18)$$

$$\frac{dZ}{dt^*} = X - \gamma_2 Z \quad (19)$$



Orbitally-forced solution of the three-component dynamical system (17)-(19) in non-dimensional units, mu (solid curve), I* (dotted), and theta* (dashed)

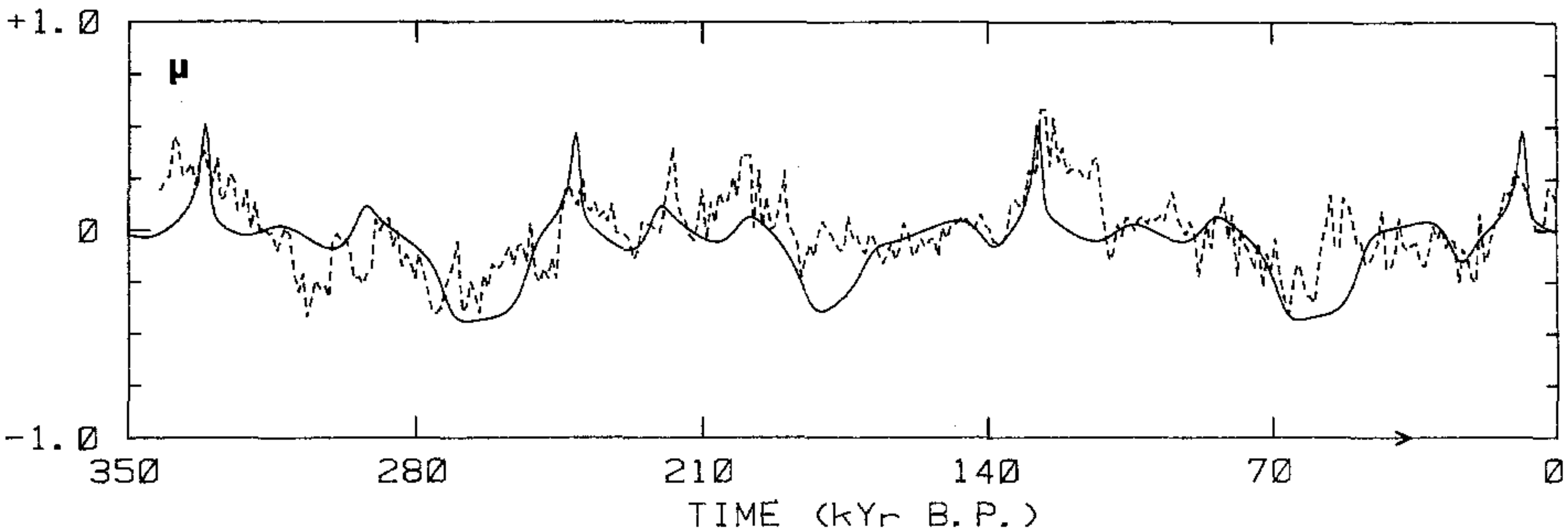


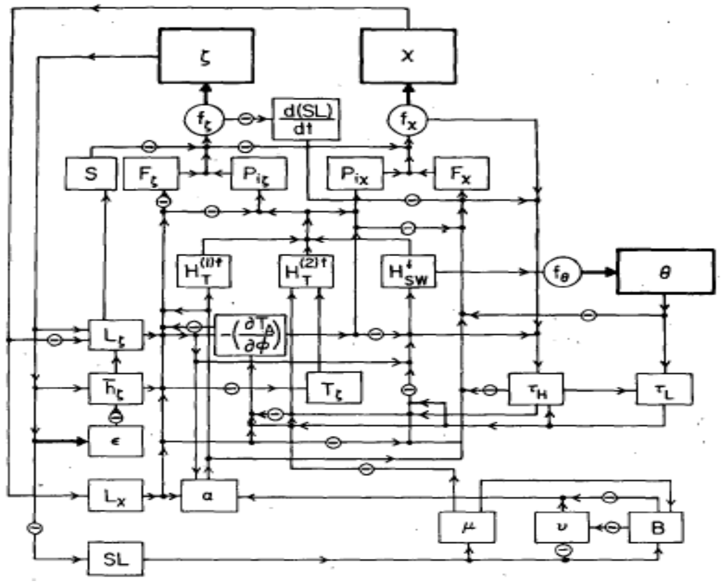


100 kyr cycle from toy "Earth system" models

Barry Saltzman: Carbon dioxide and the delta18O record of late-Quaternary climatic change: a global model (Clim Dyn, 1:77–85, 1987)

A great fit to observations can be overdone…:





Comparison of solution for atmospheric carbon dioxide concentration (solid) with the variations inferred by Shackleton and Pisias (1985) for core V19-30 (dashed).

100 kyr cycle from toy "Earth system" models

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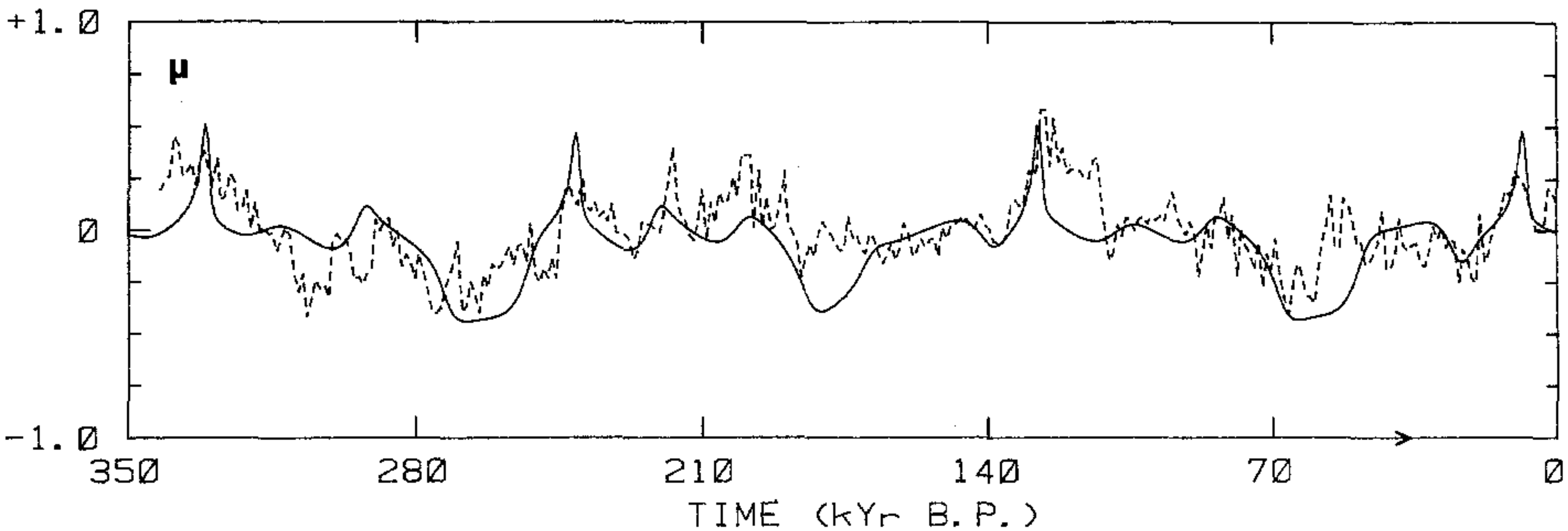
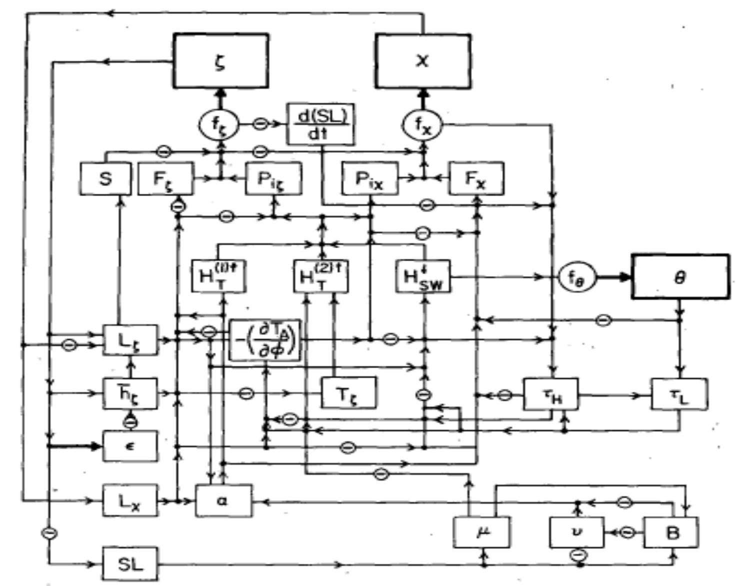
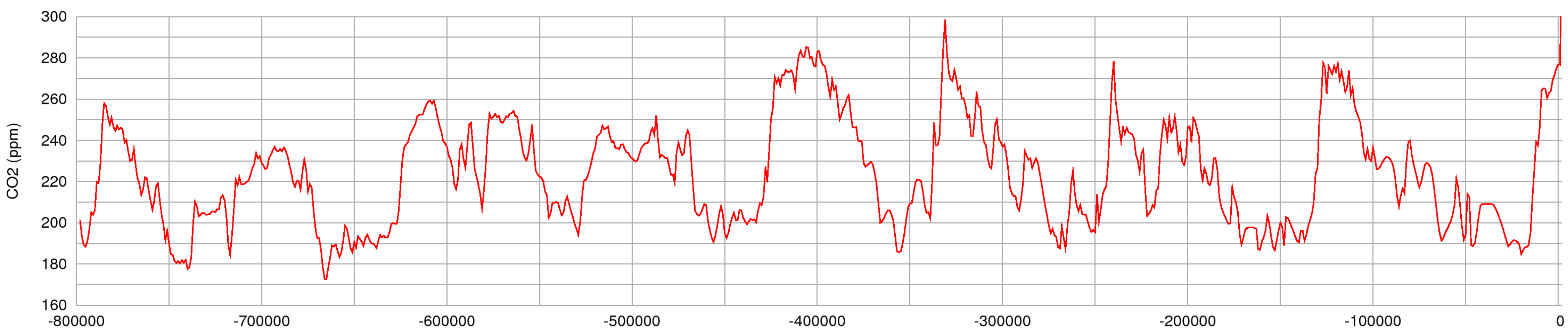


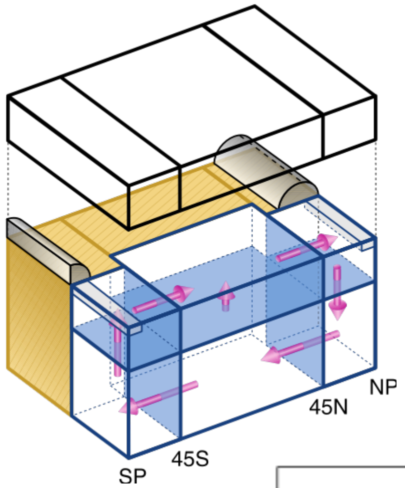


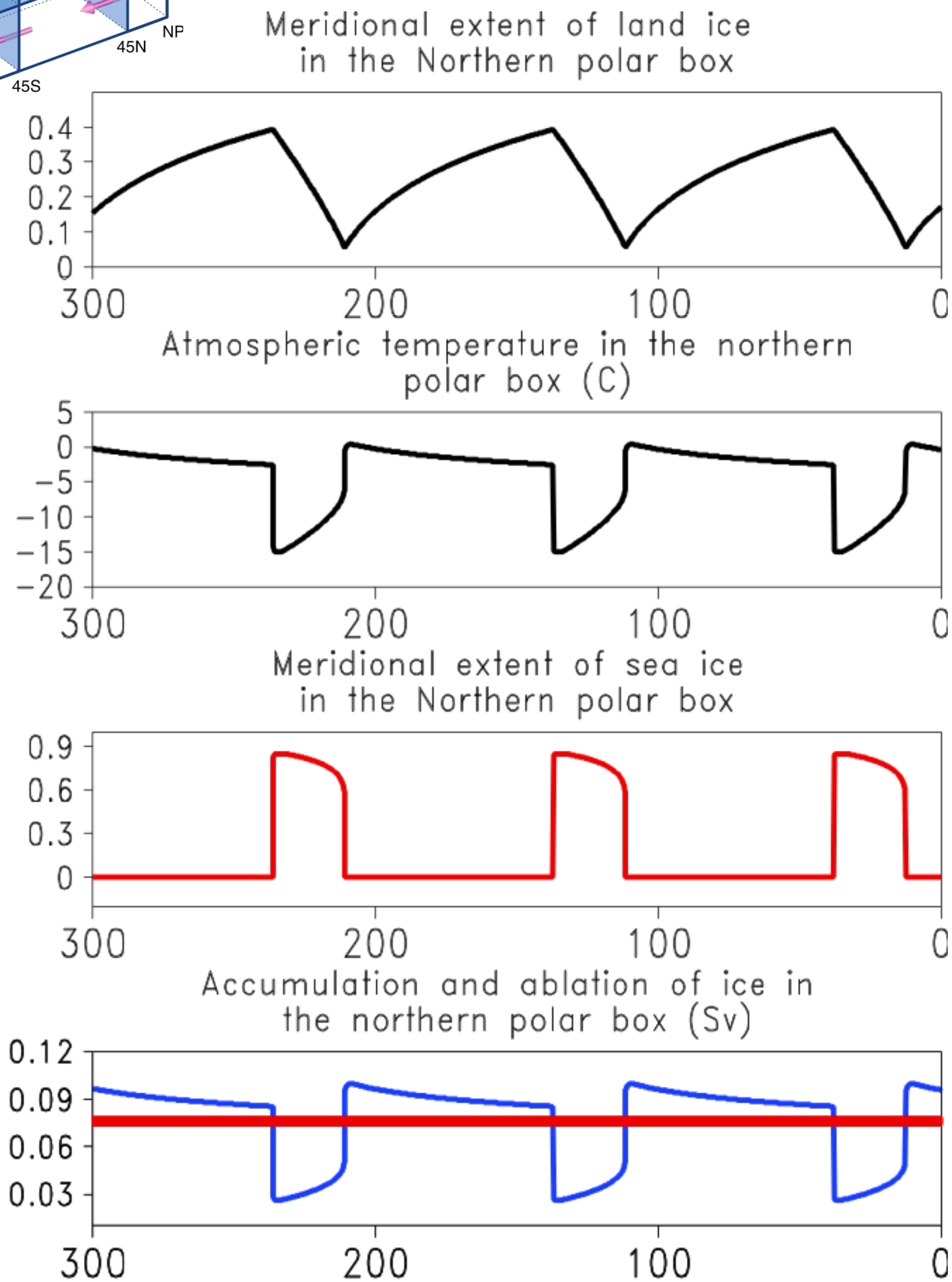
Comparison of solution for atmospheric carbon dioxide concentration (solid) with the variations inferred by Shackleton and Pisias (1985) for core V19-30 (dashed).

Unfortunately, the ice core record that came out soon after that, looks very different:



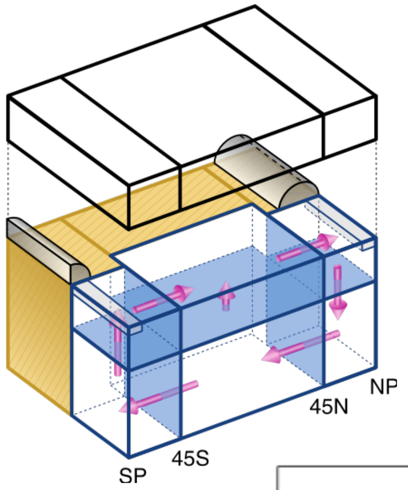
“Sea ice switch” glacial mechanism



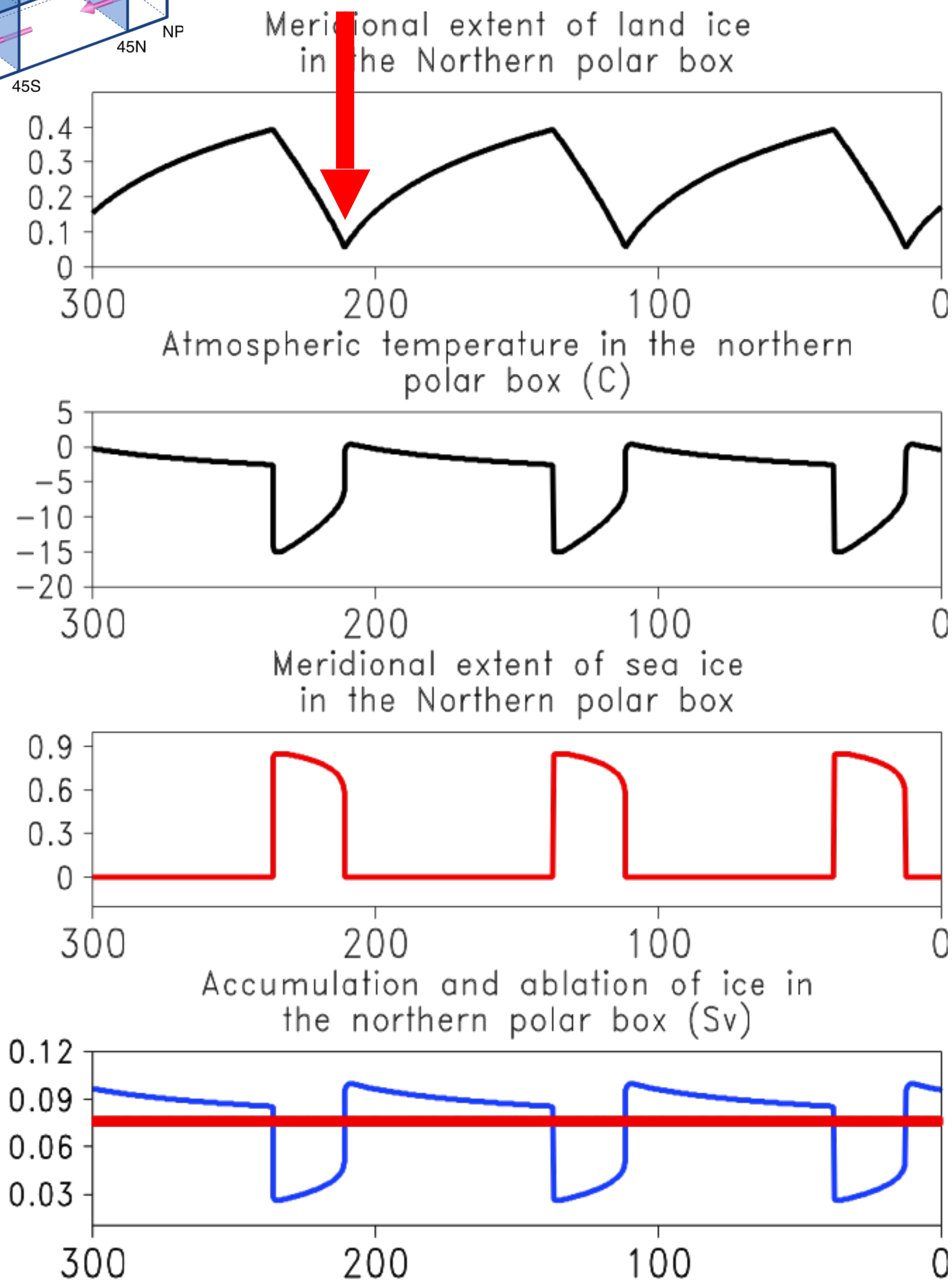


Gildor and Tziperman, ~2000

“Sea ice switch” glacial mechanism



Interglacial, no ice, warm atmosphere, large accumulation.



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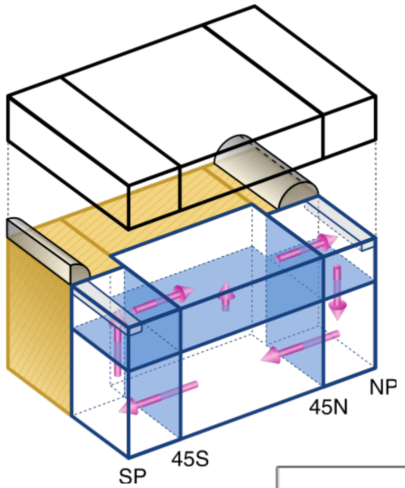
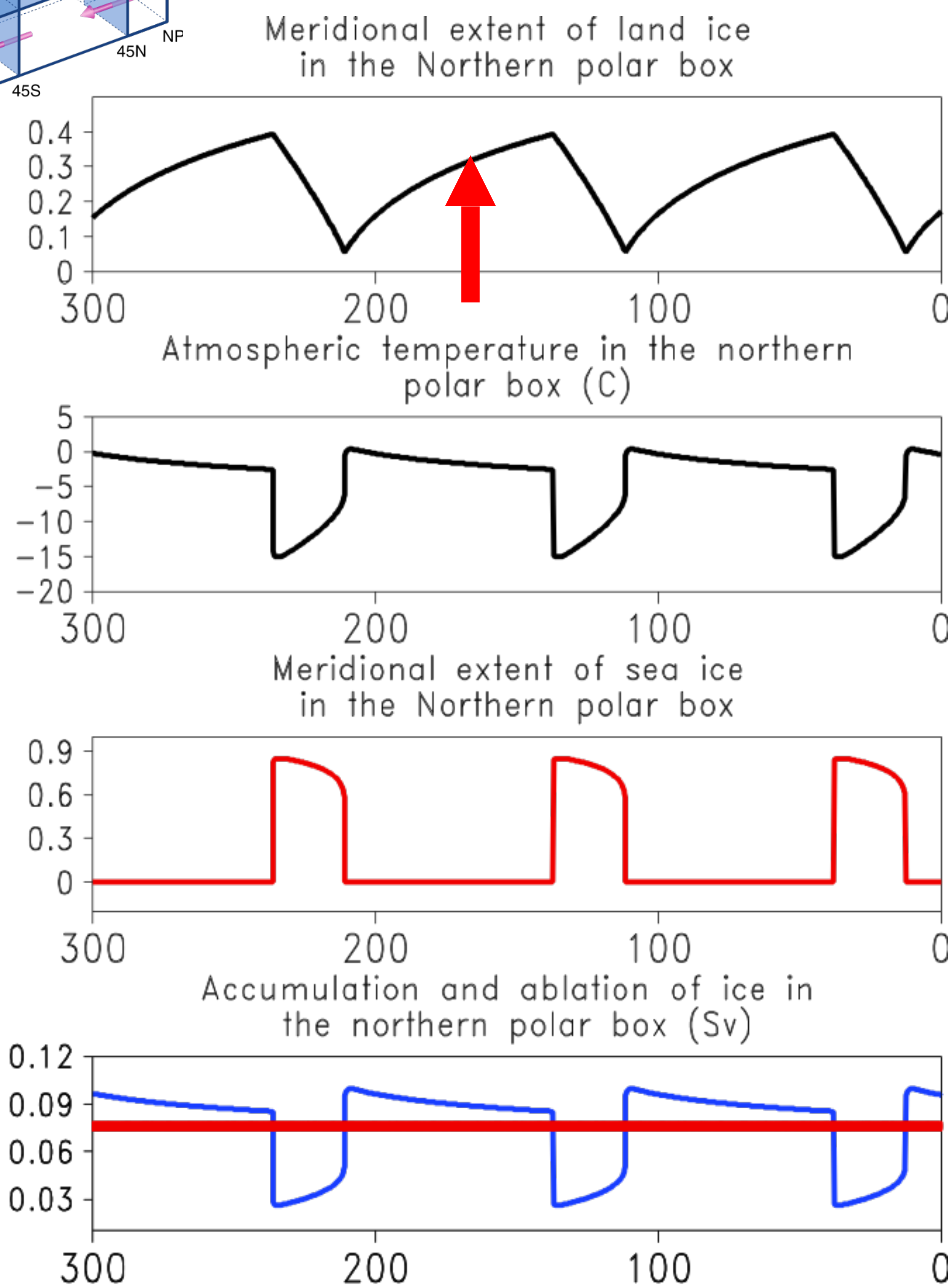


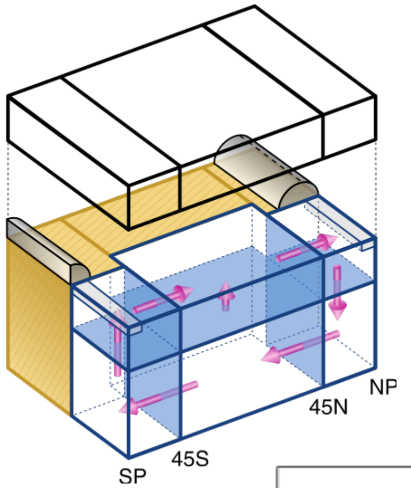


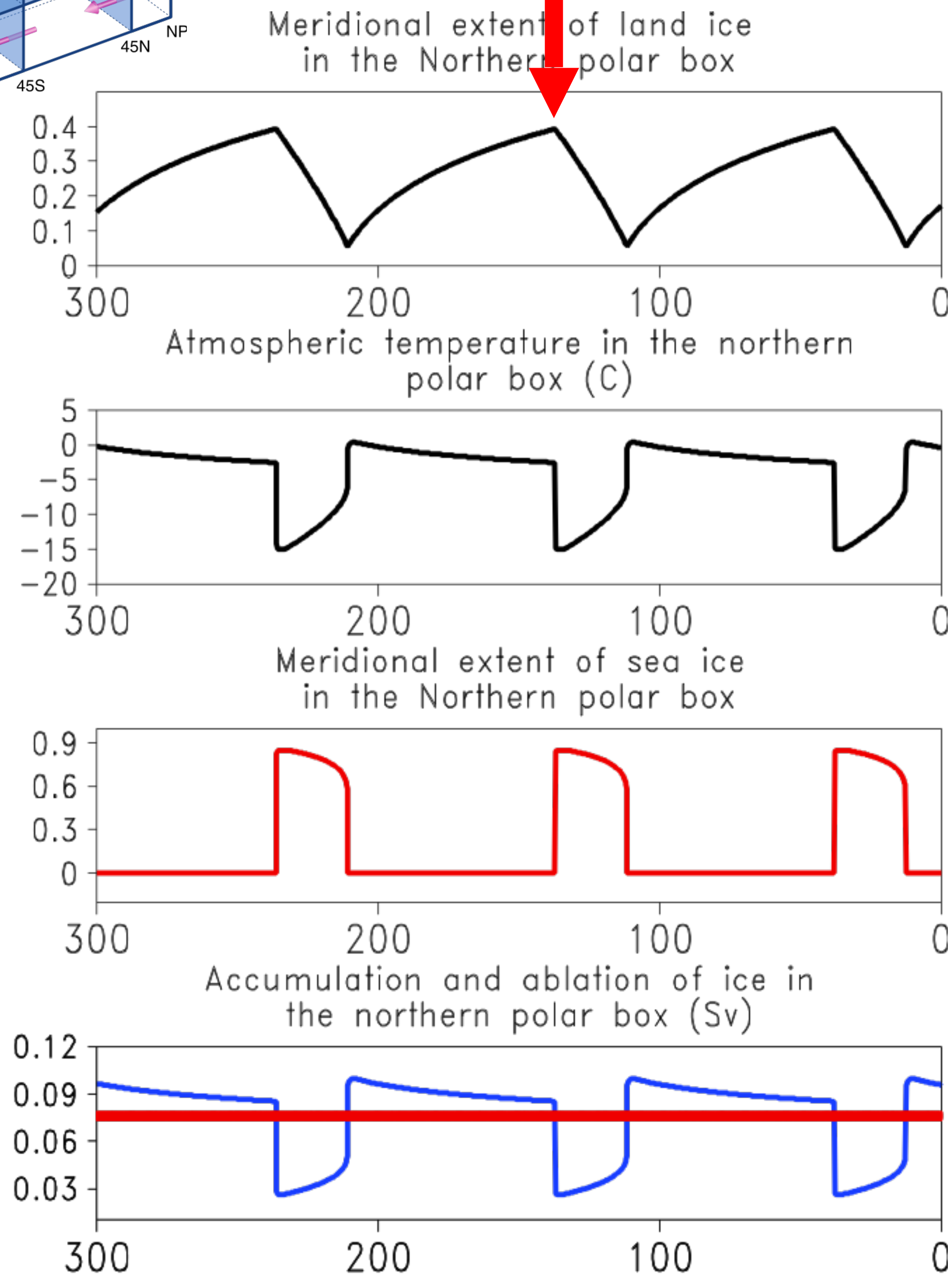
Interglacial, no ice, warm atmosphere, large accumulation.

Land ice sheets grow, temperature decreases.

Gildor and Tziperman, ~2000

“Sea ice switch” glacial mechanism





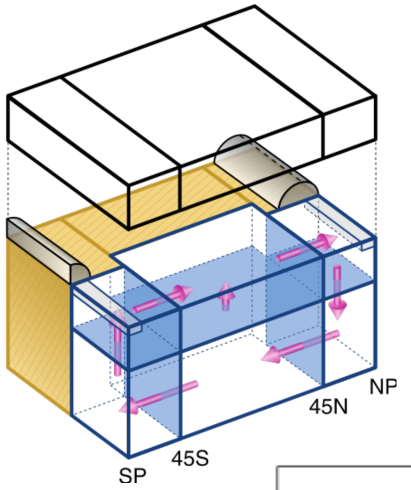
Interglacial, no ice, warm atmosphere, large accumulation.

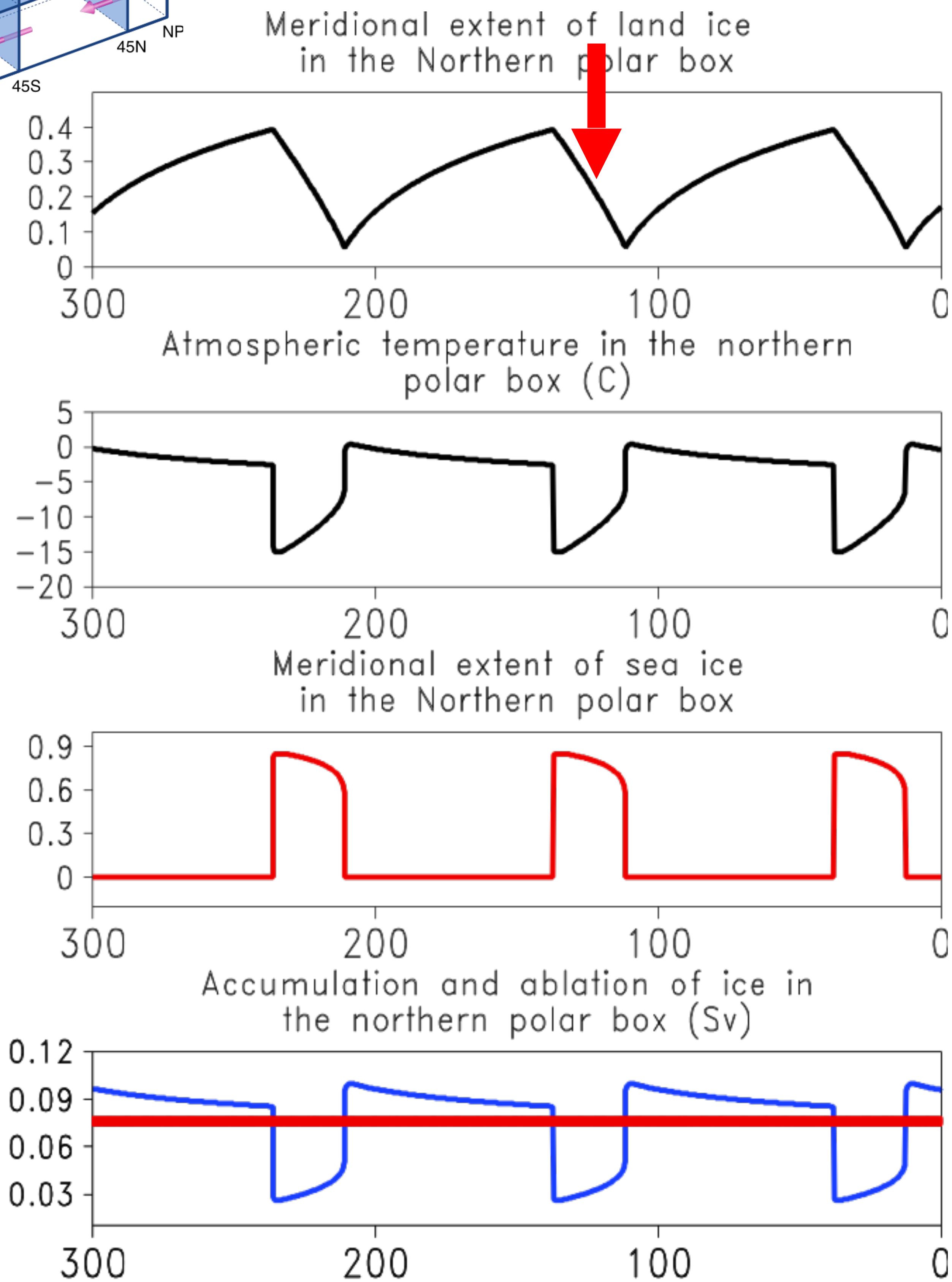
Land ice sheets grow, temperature decreases.

Cooling leads to sea ice formation, ice albedo feedback causes switch-like rapid sea ice growth.

Gildor and Tziperman, ~2000

“Sea ice switch” glacial mechanism





Interglacial, no ice, warm atmosphere, large accumulation.

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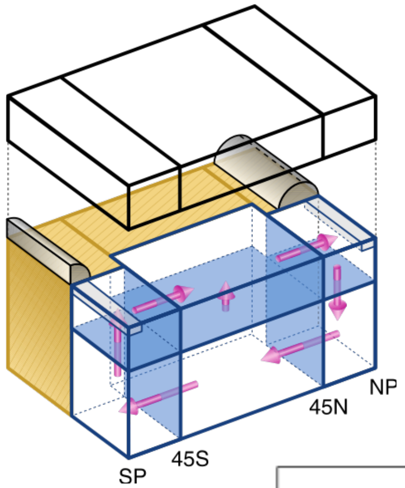
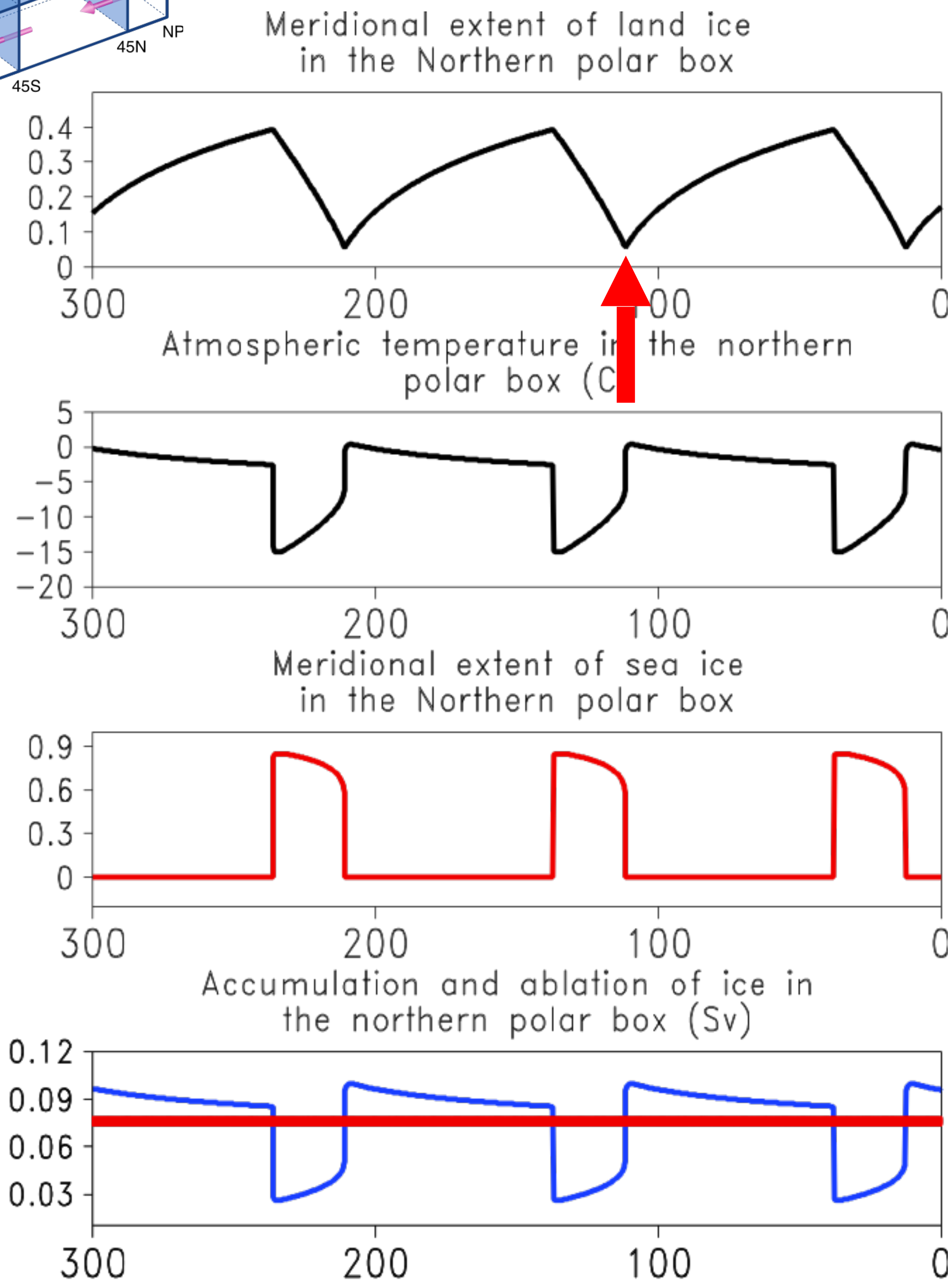
Cooling leads to sea ice formation, ice albedo feedback causes switch-like rapid sea ice growth.

Cold, dry, weak accumulation, ablation continues. Land ice retreats.

Gildor and Tziperman, ~2000

“Sea ice switch” glacial mechanism





Interglacial, no ice, warm atmosphere, large accumulation.

Land ice sheets grow, temperature decreases.

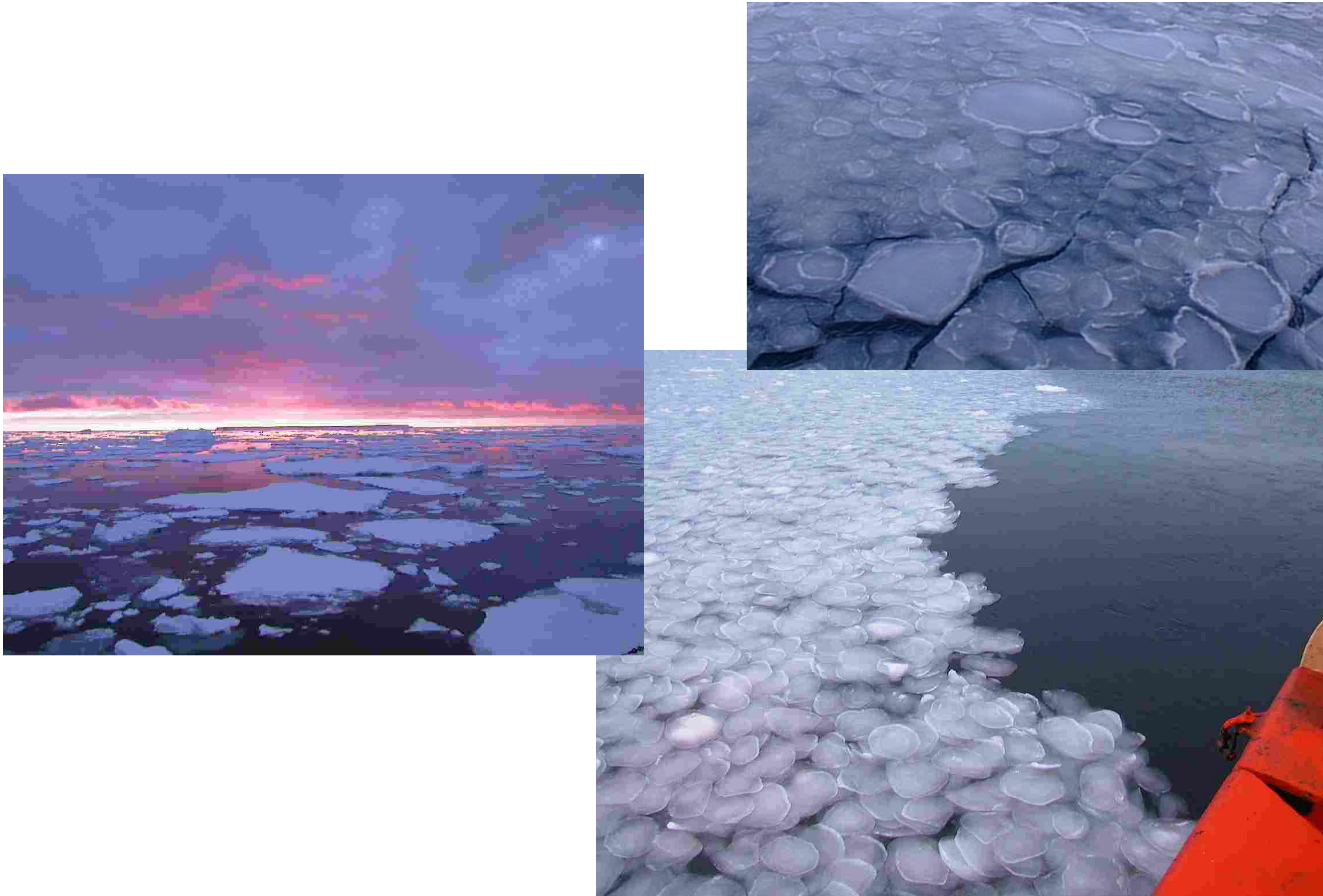
Cooling leads to sea ice formation, ice albedo feedback causes switch-like rapid sea ice growth.

Cold, dry, weak accumulation, ablation continues. Land ice retreats.

Sea ice melts, first slowly then again abruptly due to sea ice feedbacks. Back to initial state.

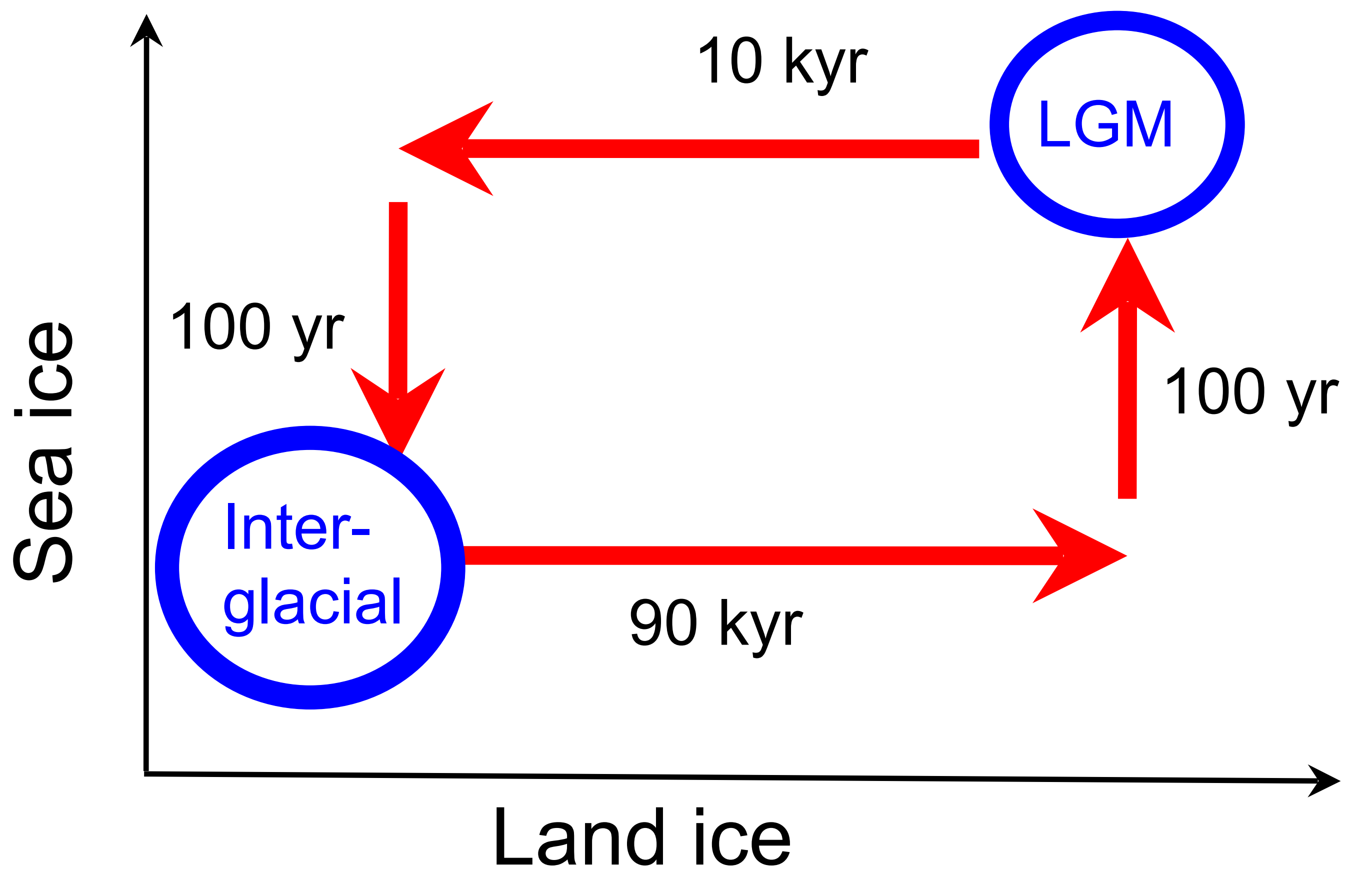
Gildor and Tziperman, ~2000

Sea ice ‘pancakes’

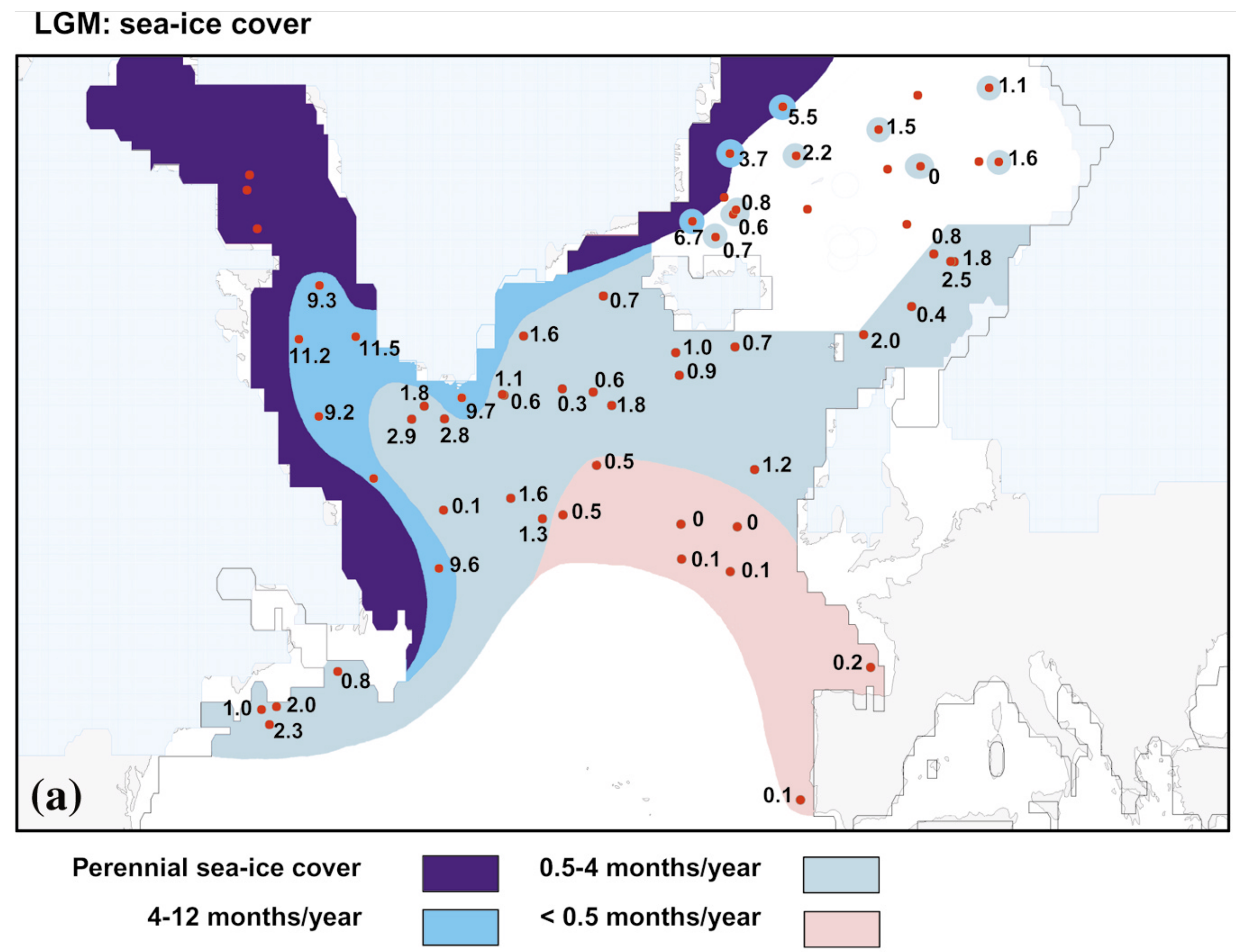


Observational evidence for sea ice switch?

Mechanism predicts a hysteresis between sea ice & land ice



This mechanism, unlike many others, is falsifiable: need to examine phase between land ice & sea ice.



Currently, only sea ice proxy at a single time slice, LGM (21kyr) exists. (Anne de Vernal et al, 2000)

Insolation-driven glacial hysteresis

(Abe-Ouchi et al 2013)

A coupled ice-sheet climate model shows a hysteresis loop with insolation.
When Insolation varies with Milankovitch forcing, this leads to glacial-like oscillations

This represents a case where there would have been no glacial oscillations without time-varying Milankovitch forcing.

Kirstin Koepnick

Insolation-driven glacial hysteresis

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A coupled ice-sheet climate model shows a hysteresis loop with insolation.
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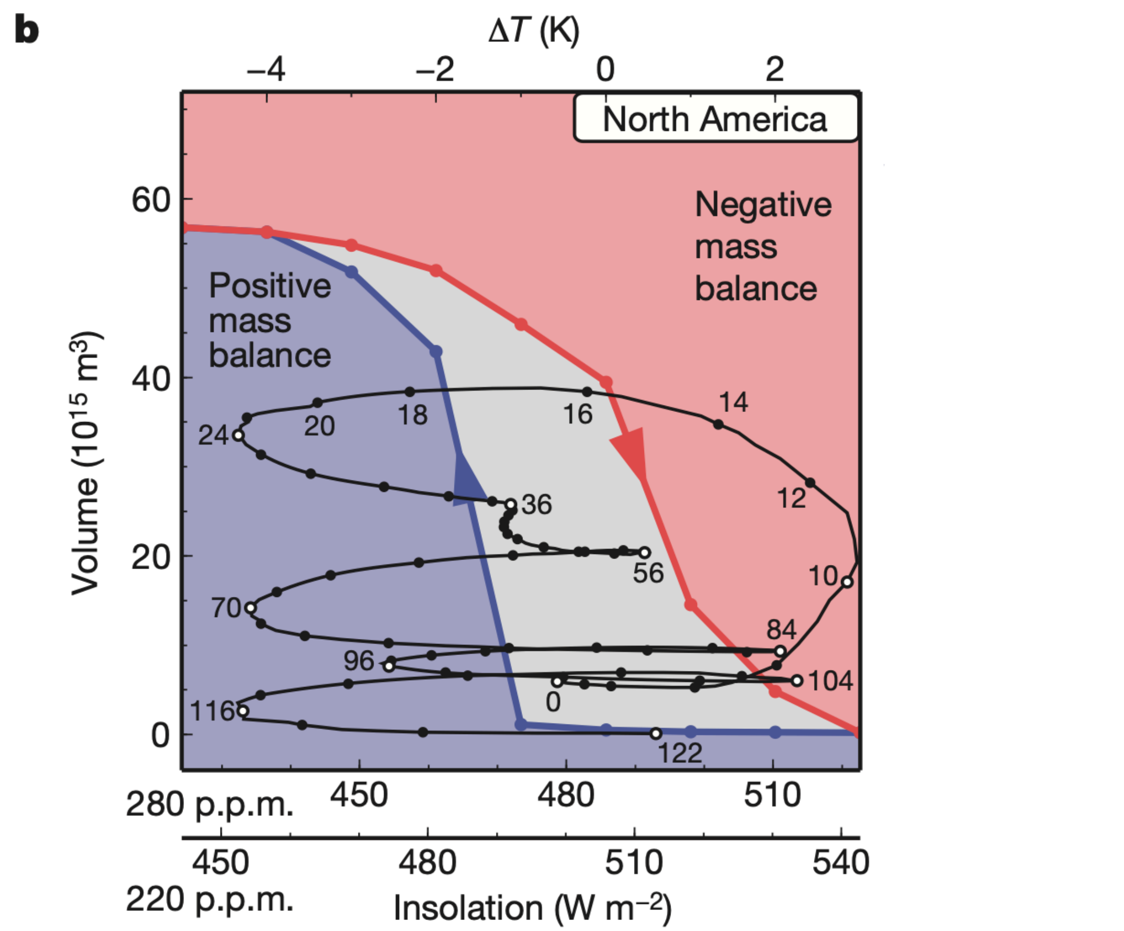
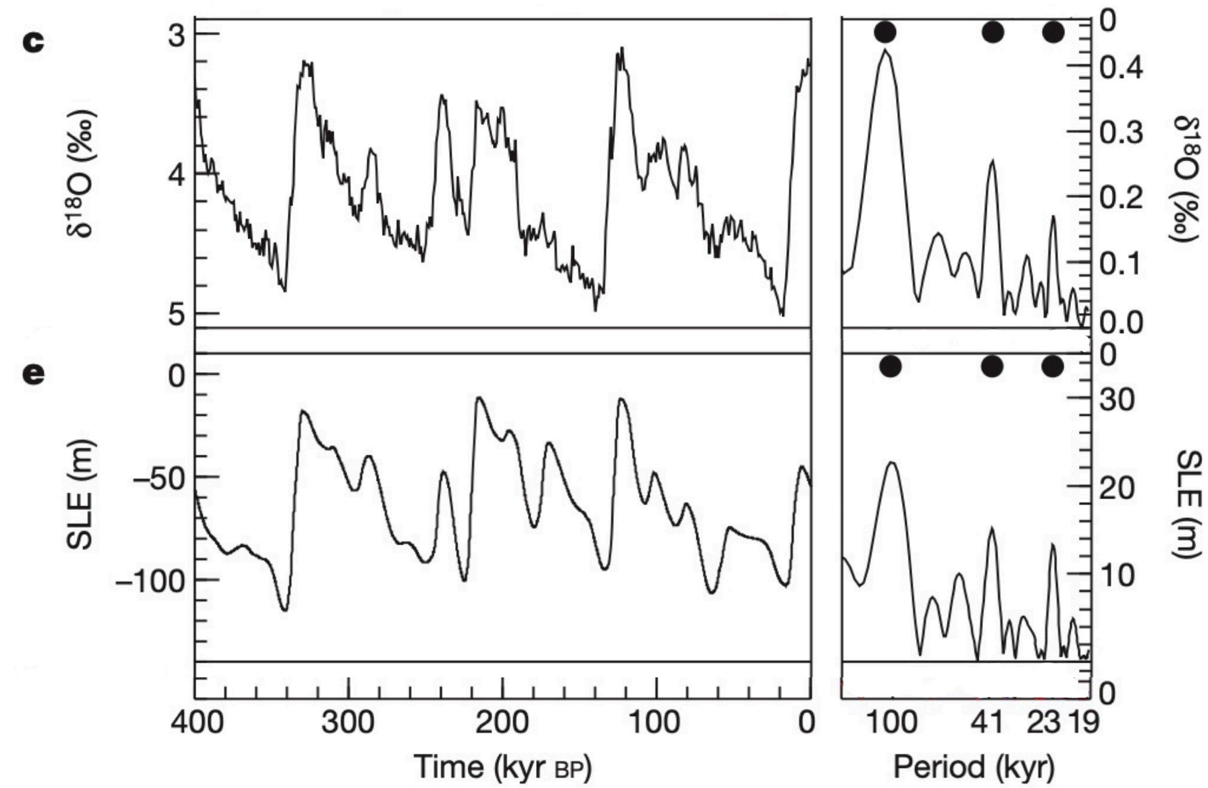


Fig 2b: Ice volume as a function of insolation. Showing hysteresis loop (upper branch in red, lower in blue), and model trajectory (black) for last 122 kyr under time-dependent Milankovitch insolation.



Ice volume time series and spectrum for observations (1c) and model run for the last 400 kyrs (1e).

This represents a case where there would have been no glacial oscillations without time-varying Milankovitch forcing.

Kirstin Koepnick

Milankovitch forcing:

Variations in the Earth's Orbit: Pacemaker of the Ice Ages

For 500,000 years, major climatic changes have followed variations in obliquity and precession.

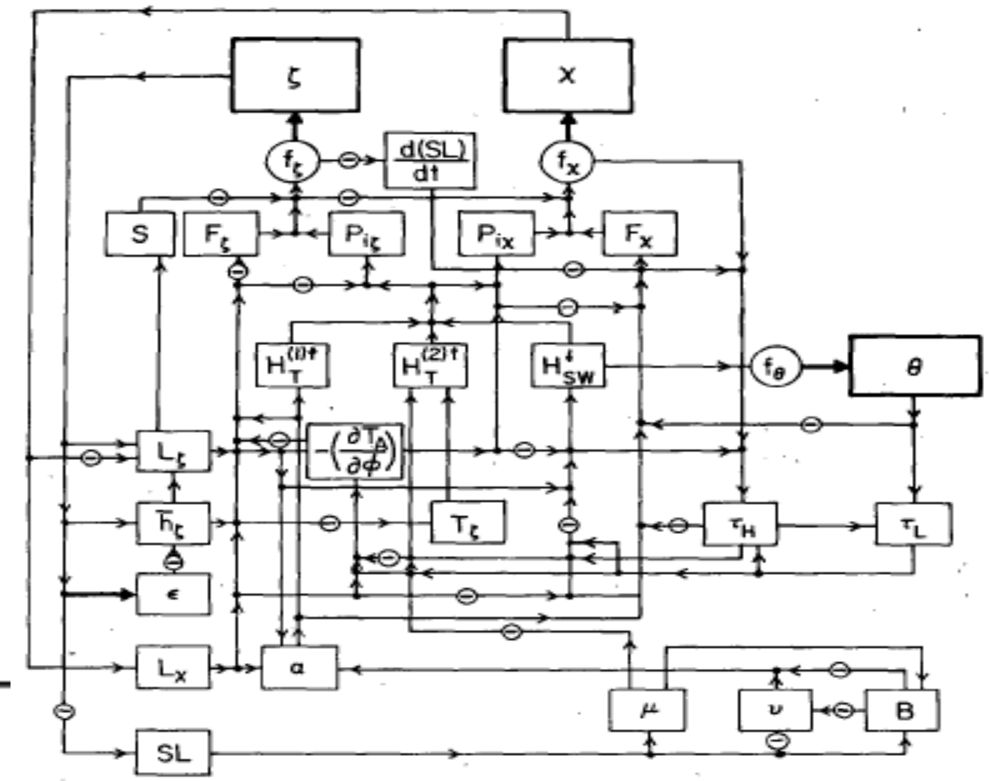
J. D. Hays, John Imbrie, N. J. Shackleton **1977**

"pacemaker" = nonlinear phase locking?

First: analysis of phase locking of fireflies

Three "successful" glacial models: 1

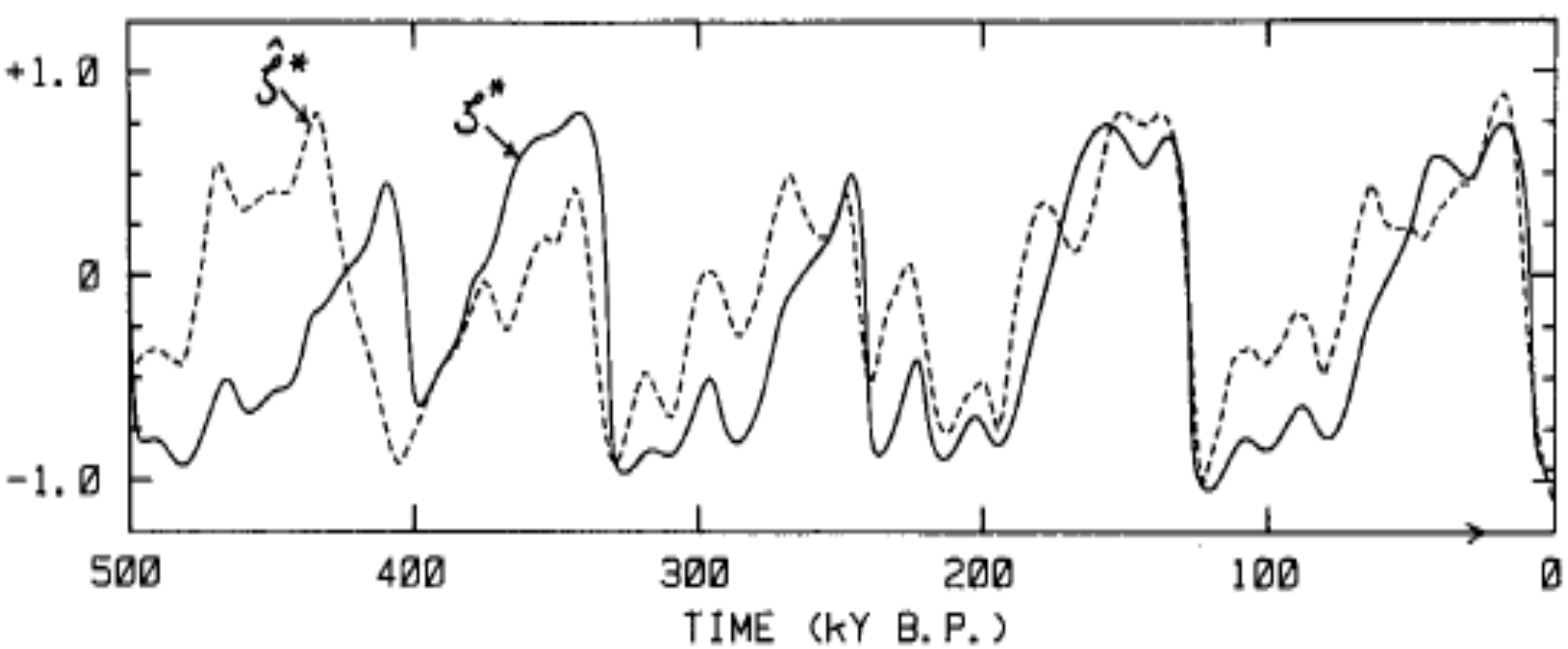
Saltzman, Hansen and Maasch, 1984: glacial cycles are due to the interaction of land ice, ice shelves and deep ocean temp.

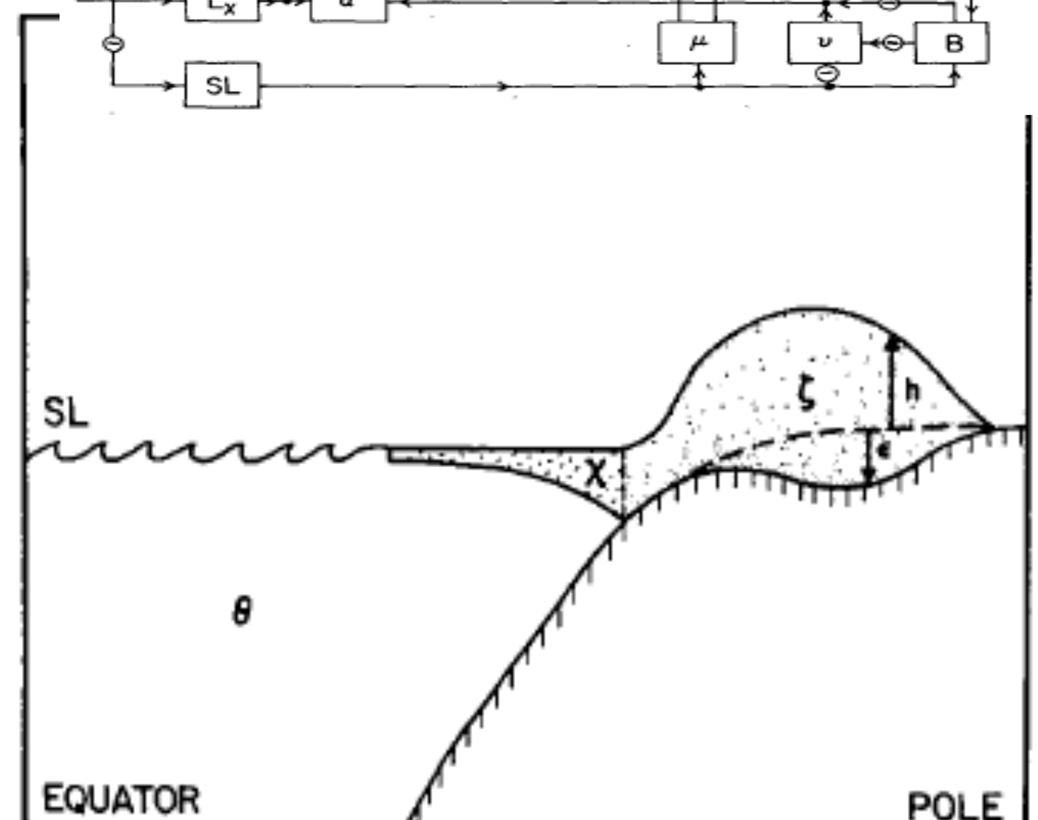


$$\frac{d\zeta'}{dt} = a_0\zeta' + a_1(1 - a_2\chi')\chi' - a_3\theta' + F_\zeta + R_\zeta, \quad (1)$$

$$\frac{d\chi'}{dt} = b_0\zeta' + b_1(1 - b_2\chi' - b_3\chi'^2 - b_4\zeta'^2)\chi' - b_5\theta' + F_\chi + R_\chi, \quad (2)$$

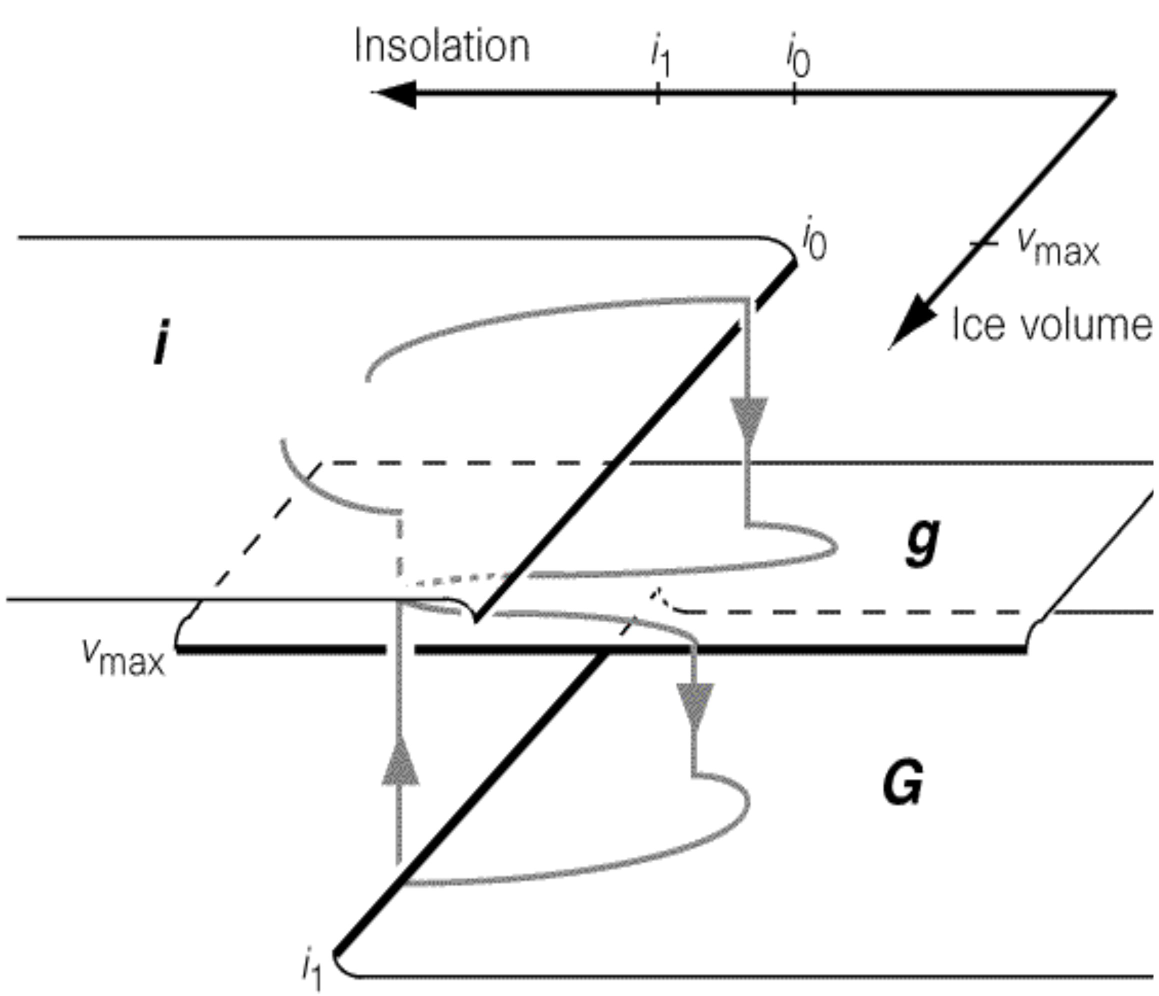
$$\frac{d\theta'}{dt} = c_0\zeta' + c_1\chi' - c_2\theta' + F_\theta + R_\theta, \quad (3)$$

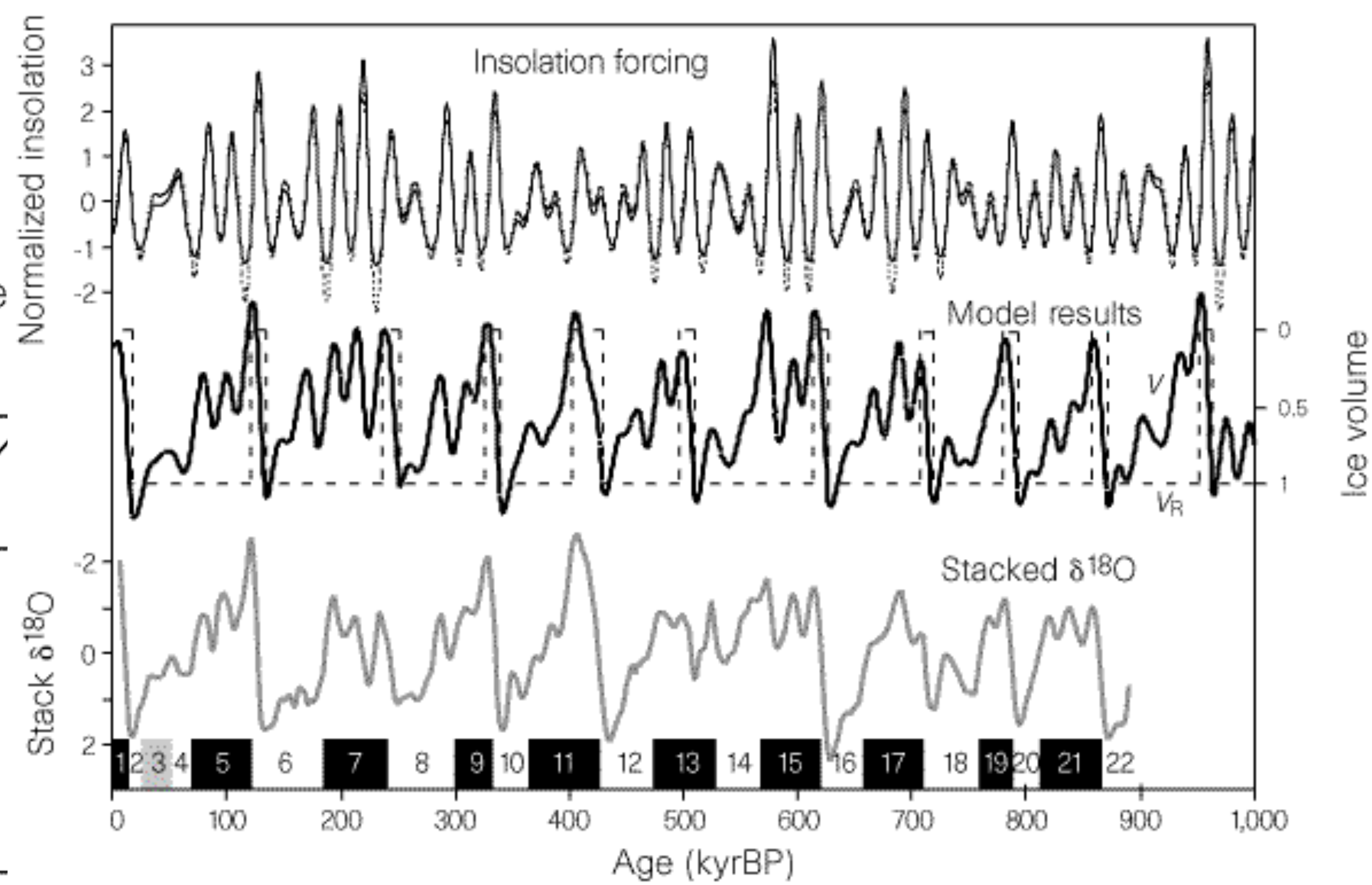




Three “successful” glacial models: 2

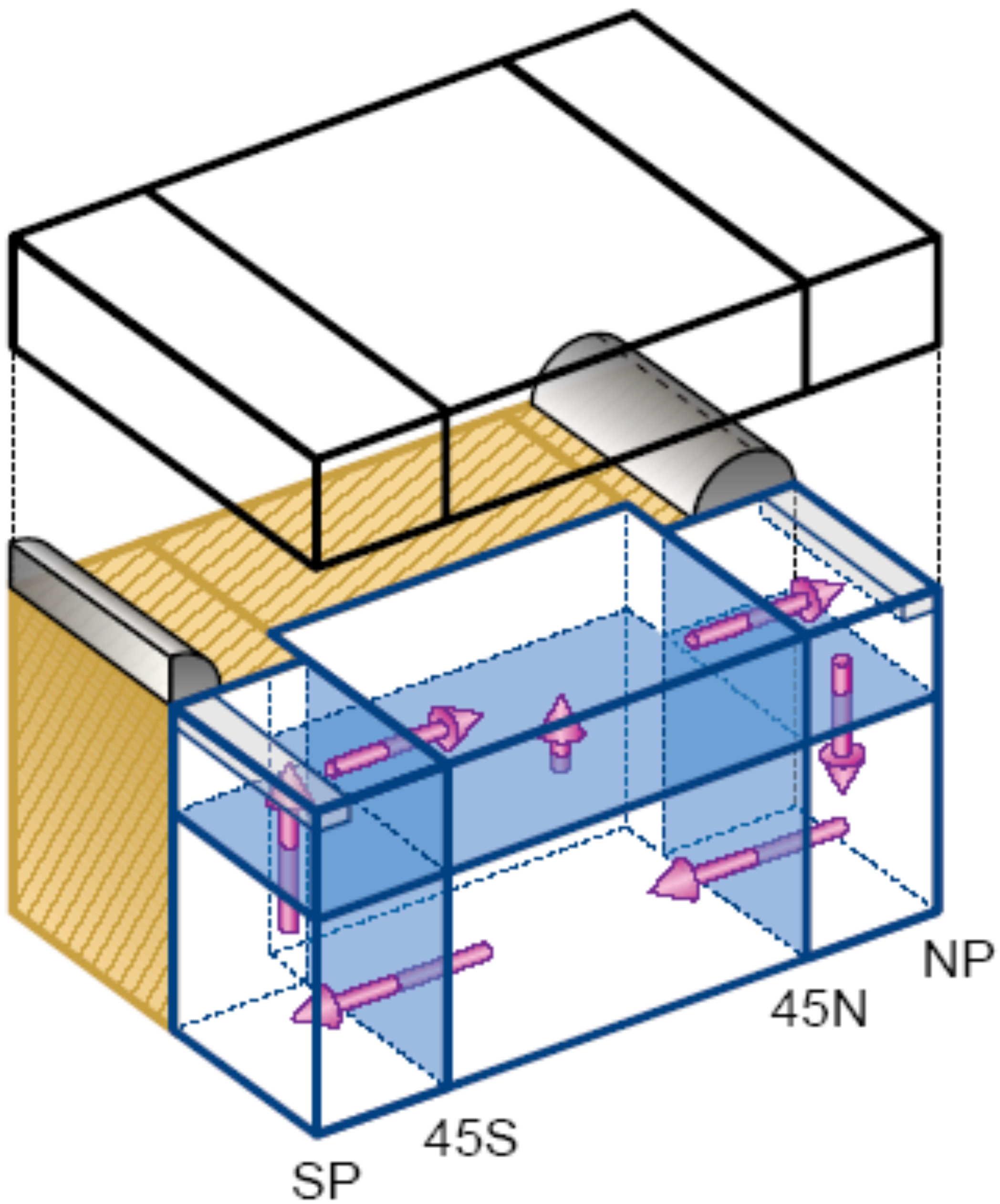
- ➢Paillard 1998: 3 steady states, one equation, transition between steady states based on Milankovitch forcing.
- ➢Glacial cycles are due to jumps between steady states (of THC?) forced by Milankovitch

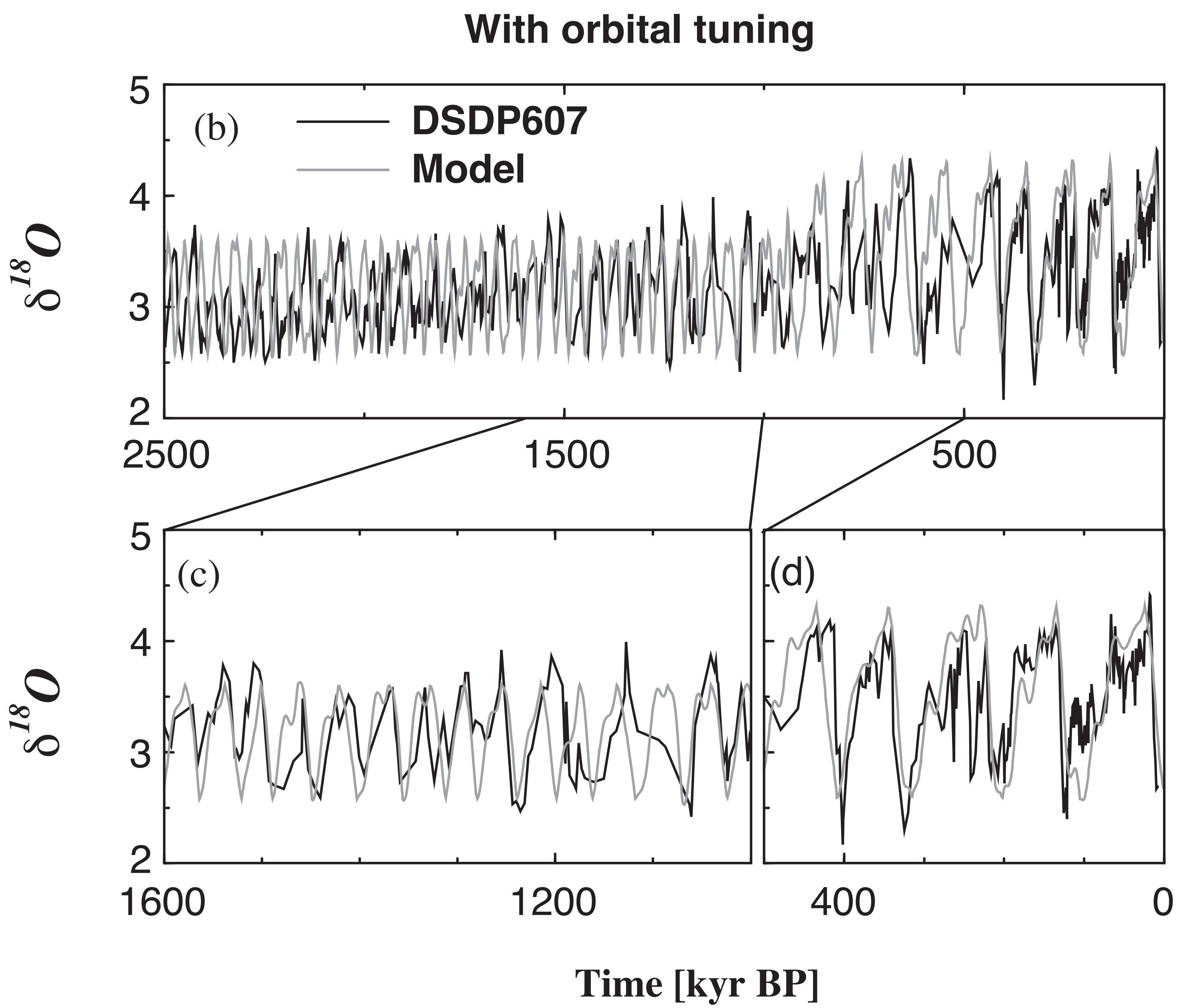




Three “successful” glacial models: 3

Gildor and Tziperman 2000: “sea ice switch”: land ice grows during warm periods (small sea ice cover) and retreats during cold periods (large sea ice cover);

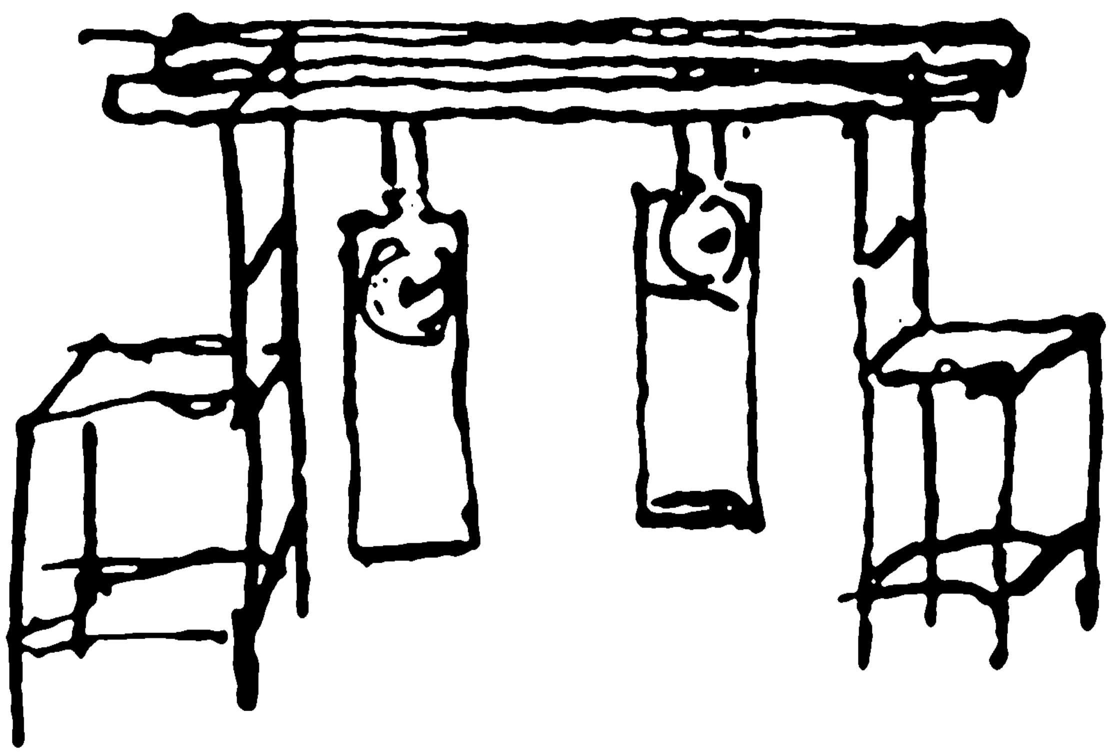




(Ashkenazy & Tziperman 2004)

Why is it so `simple' to fit glacial ice volume?

The question (Saltzman, Hansen & Maasch 1984): “How does small amplitude periodic forcing control phase in a complex nonlinear oscillatory system, and is there a good physical interpretation for this phase locking phenomenon?”



The answer: It's “nonlinear phase locking”: (Hyde&Peltier 1987; Gildor&Tziperman 2000; Ashkenazy&Tziperman 2004)

1665, Christiaan Huygens, Dutch mathematician, astronomer and physicist. While working on design of precise pendulum clocks, suitable for determination of a ship coordinates in the sea, he observed and described synchronization of two clocks placed on a common support.

http://www.agnld.uni-potsdam.de/~mros/synchro.html

Examples of nonlinear phase locking: 1/3

Moon always faces Earth: makes one rotation around its axis for every rotation around Earth.

https://www.britannica.com/list/where-did-the-moon-come-from
© Paul Morley/Fotolia

NASA,
https://www.cronodon.com/PlanetTech/mercury.html

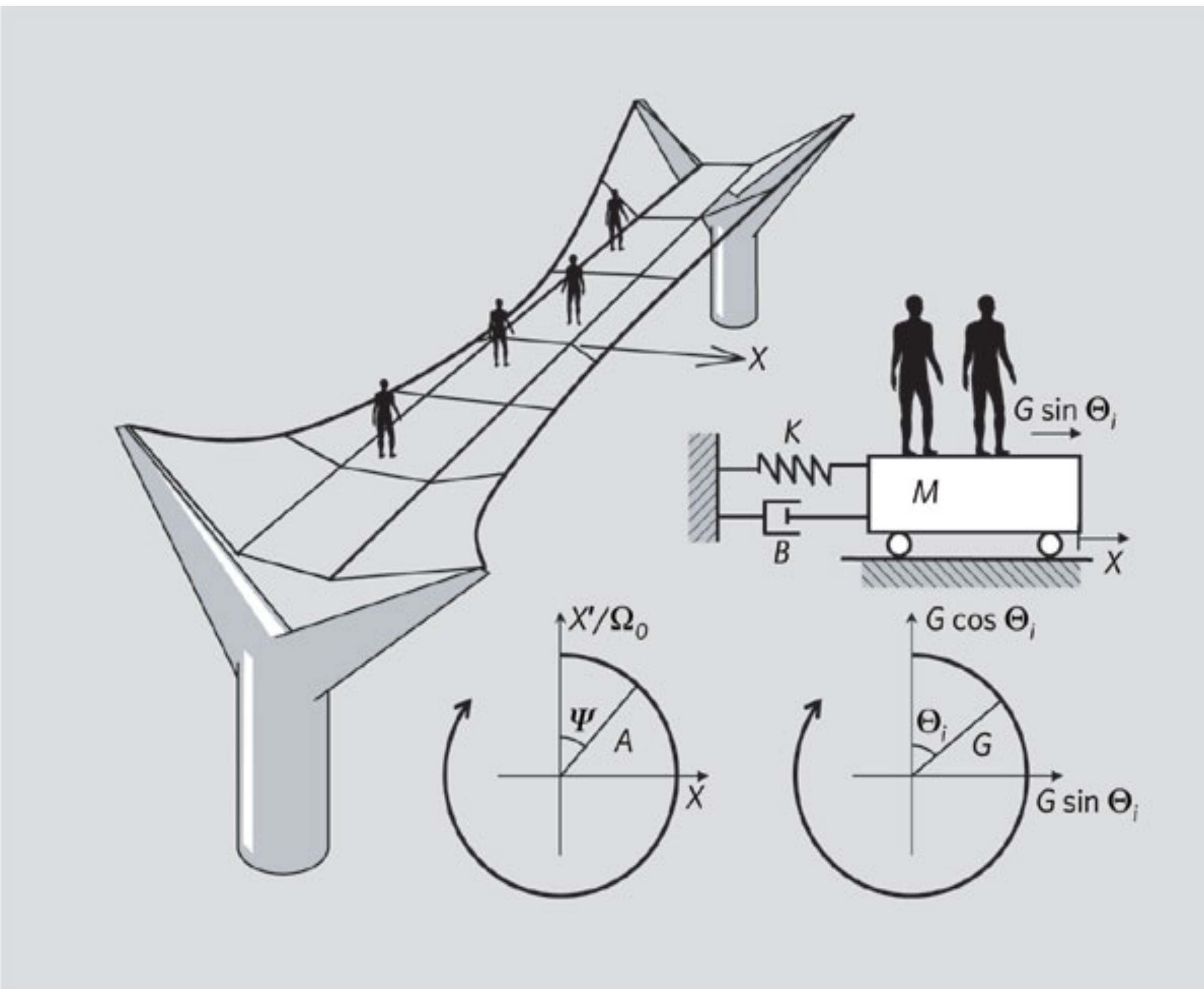
The planet Mercury turns three times for every two orbits around the Sun, this is a 2:3 nonlinear resonance

- Coincidences? No… It's nonlinear phase locking/ nonlinear resonance aided by tidal friction.
- Linear resonance: 1:1 ratio between forcing and response;
- Nonlinear resonance: p:q ratio with p,q any integers.

Examples of nonlinear phase locking: 2/3

Crowd synchrony on the Millennium Bridge

Footbridges start to sway when packed with pedestrians falling into step with their vibrations. (Strogatz et al 2005)

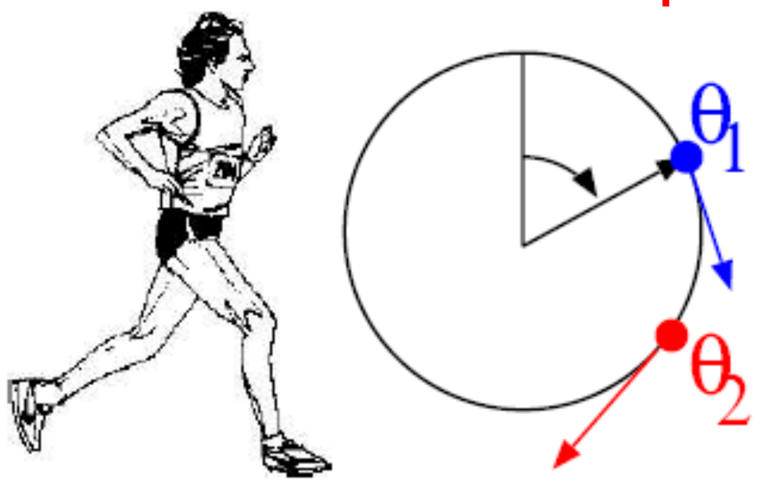


$$M\frac{d^2X}{dt^2}+B\frac{dX}{dt}+KX=G\sum_{i=1}^{N}\sin\Theta_i$$

$$\frac{d\Theta_i}{dt}=\Omega_i+CA\sin(\Psi-\Theta_i+\alpha)$$

Bridge
and pedestrians
affecting each other nonlinearly

Examples of nonlinear phase locking: 3/3



Two "coupled" runners in a circular stadium

$$\dot{\theta}_1 = \omega_1 + K_1 \sin(\theta_2 - \theta_1)$$

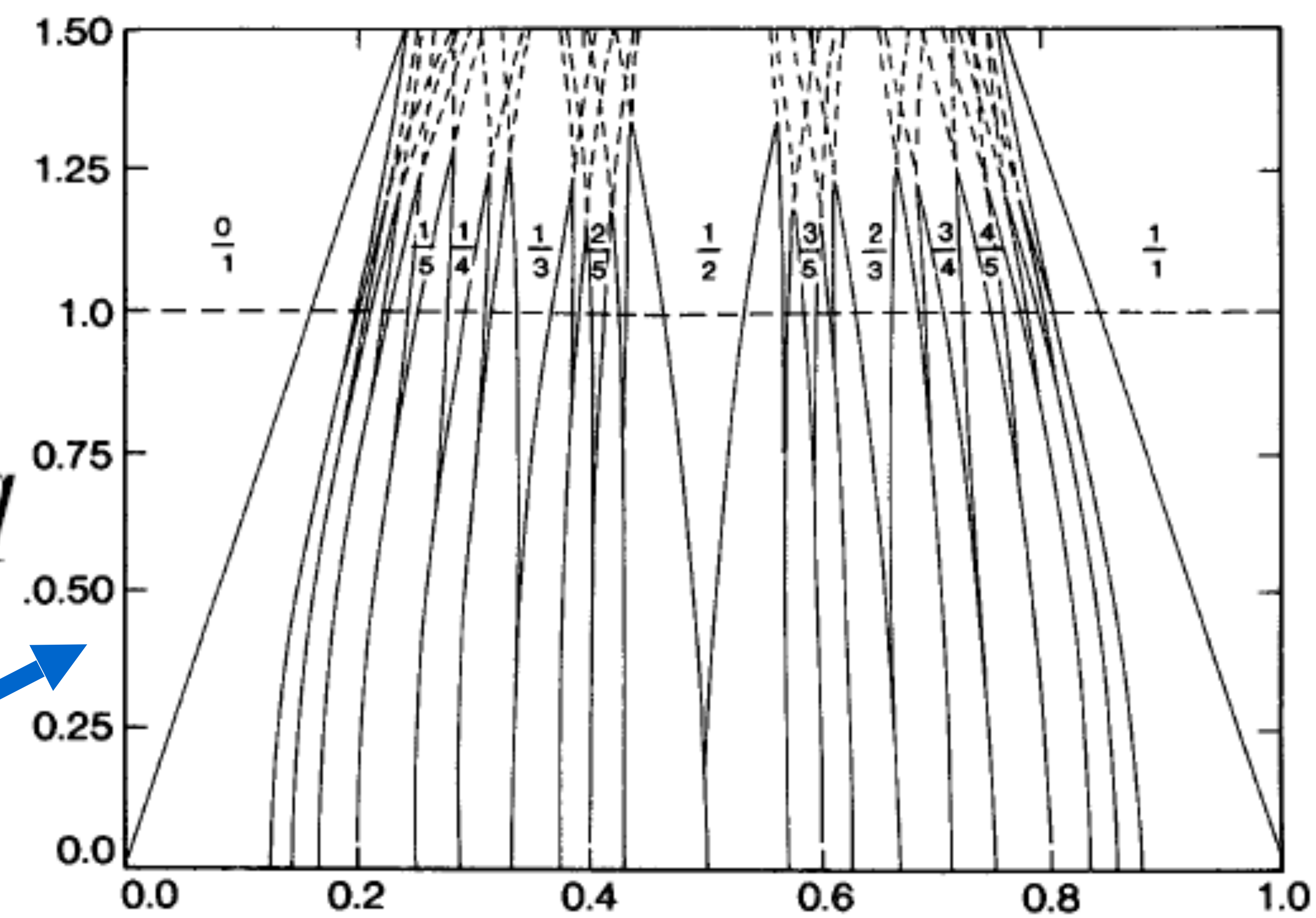
$$\dot{\theta}_2 = \omega_2 + K_2 \sin(\theta_1 - \theta_2)$$

$\omega_1 = \omega_2$: two run together, side by side

$0 < |\omega_1 - \omega_2| < K_1 + K_2$: run @ same speed, but separated by a constant distance (Strogatz 94)

Simplest model of a periodically forced pendulum:

$$\omega/\omega_f = p/q$$



Circle map & Arnold Tongues

$$\theta_{n+1} = f_\Omega(\theta_n) = \theta_n + \Omega - \frac{K}{2\pi} \sin(2\pi\theta_n)$$

Another demonstration of nonlinear phase locking

https://www.youtube.com/watch?v=W1TMZASCR-I
Alireza Bahraminasab

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Phase locking of glacial cycles

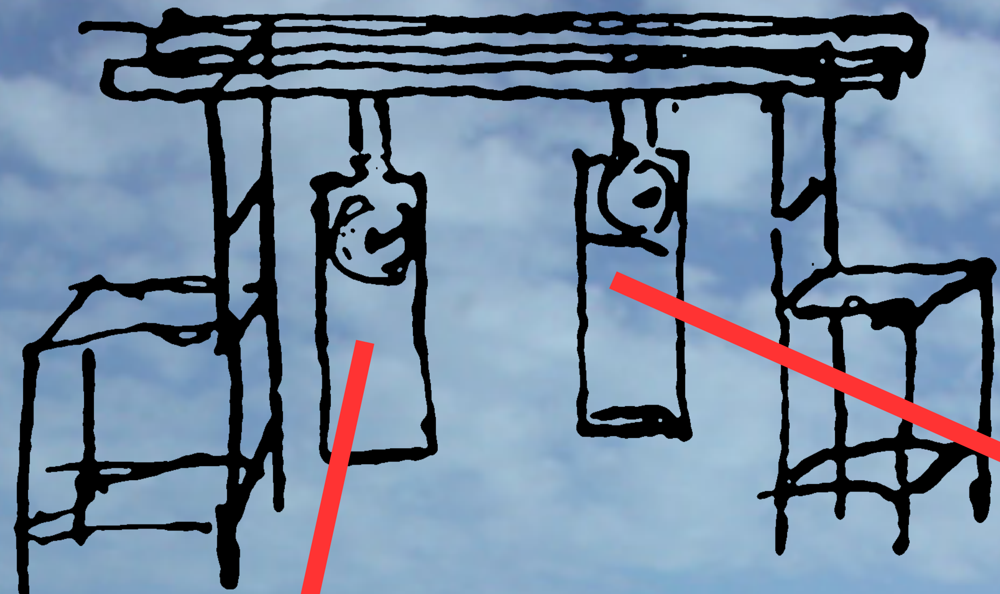
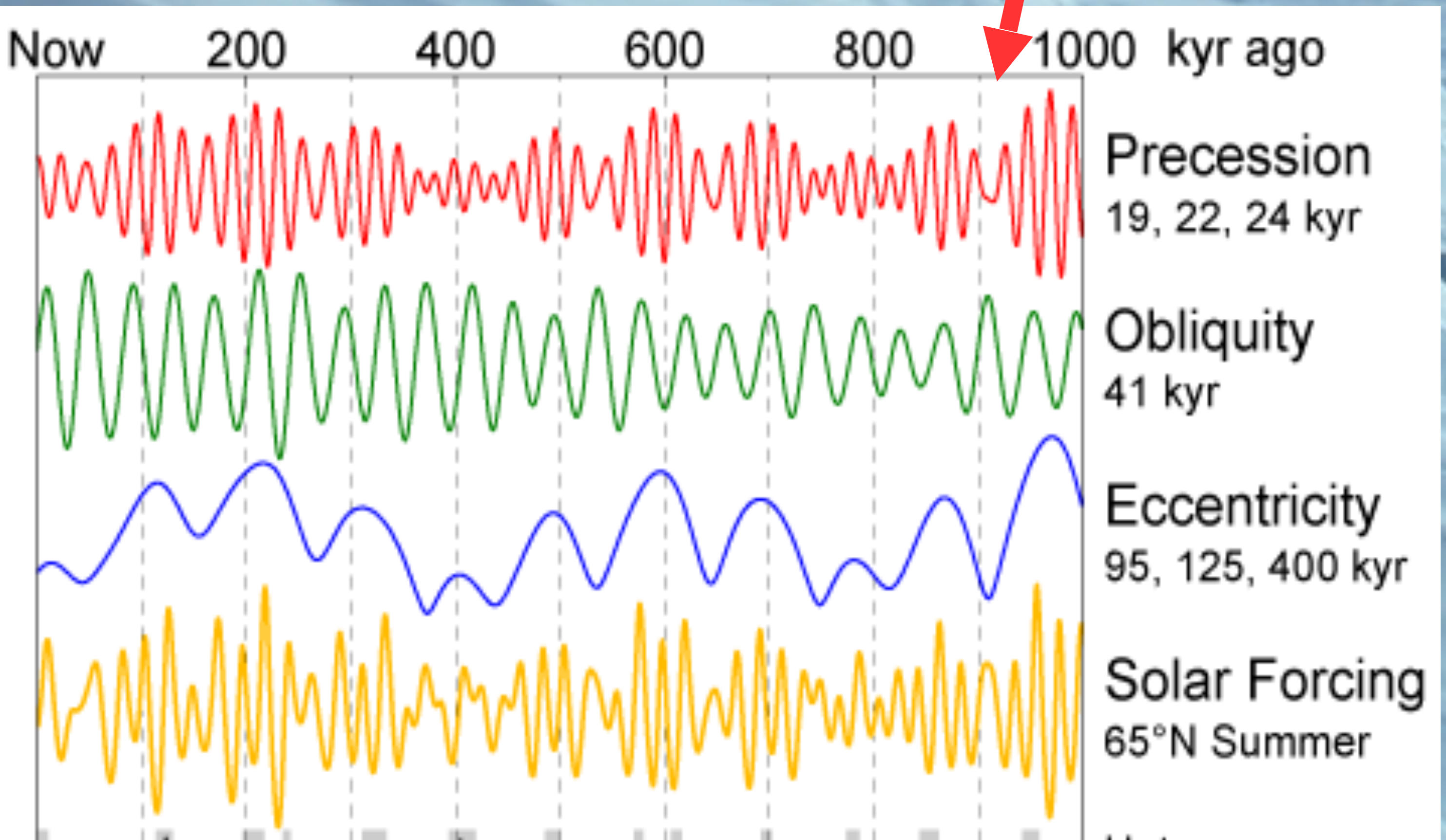


Diagram by Christian Huygens (1665)

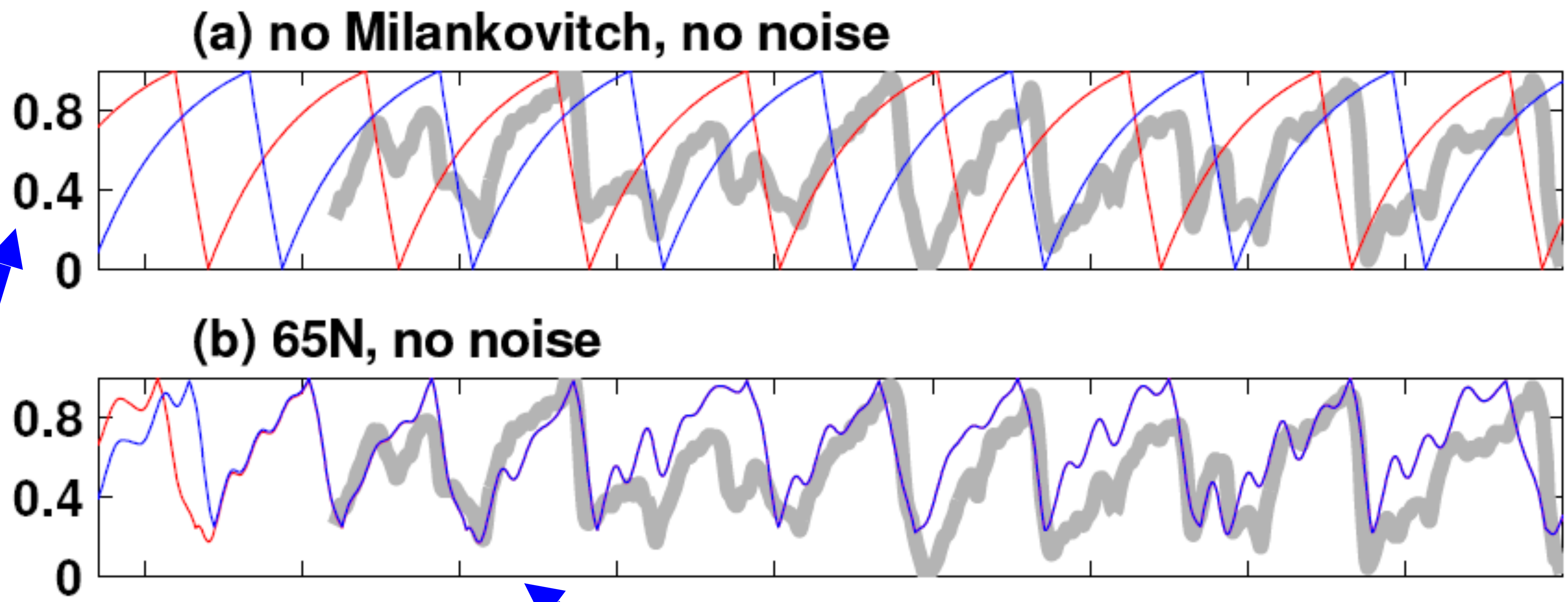


Why has it been so simple to fit SPECMAP?

Because glacial cycles are phase locked to Milankovitch forcing, and so are the models; **not** because the model's mechanism is correct…

➡ **Try to better understand nonlinear phase locking…**

Phase locking: the mechanism



(a) model solution(s) w/0 Milankovitch forcing

(b) with Milankovitch forcing, all initial conditions converge to a single time series that fits the observed ice volume record

Required ingredients:

Nonlinear oscillator(s): that can change its frequency as function of its amplitude);
Dissipation: "erases" memory of initial conditions & enables phase locking (radiative cooling in atm, glacial basal friction, ocn viscosity...)

Tziperman, Raymo, Huybers, Wunsch 2006

Phase between Milankovitch and ice volume

“Recent well dated proxies show that the phase of Milankovitch forcing during the terminations of 18 kyr BP and 135 kyr BP are not the same…" [Gallup et al, Science 2002]

Perhaps Milankovitch forcing doesn't *have* to be the same during all terminations even if Milankovitch forcing paces the cycles?

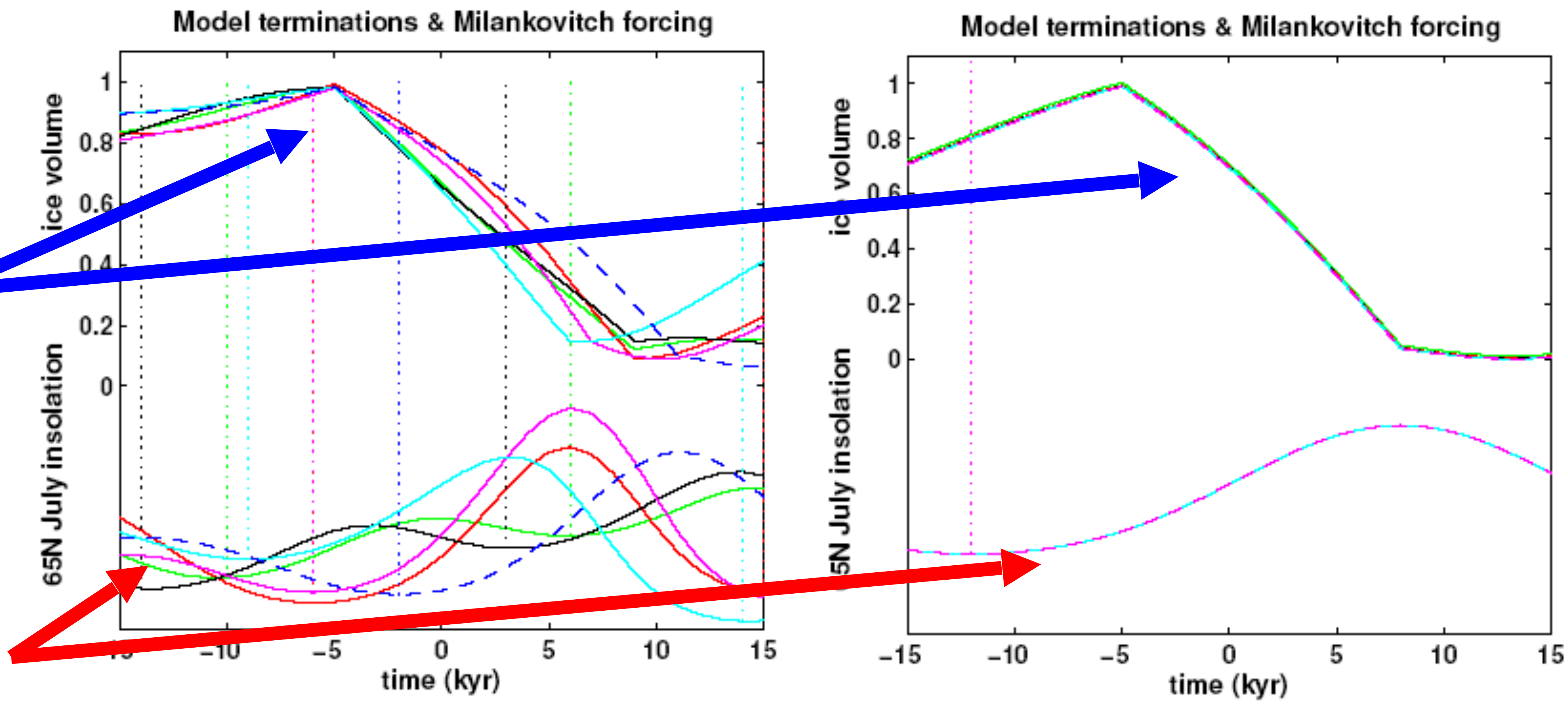
Phase between Milankovitch forcing and glacial terminations

phases during a model run forced by 65N July radiation:

And by a pure 40 kyr sine wave insolation curve

Model ice volume during all terminations

Milankovitch forcing during all terminations



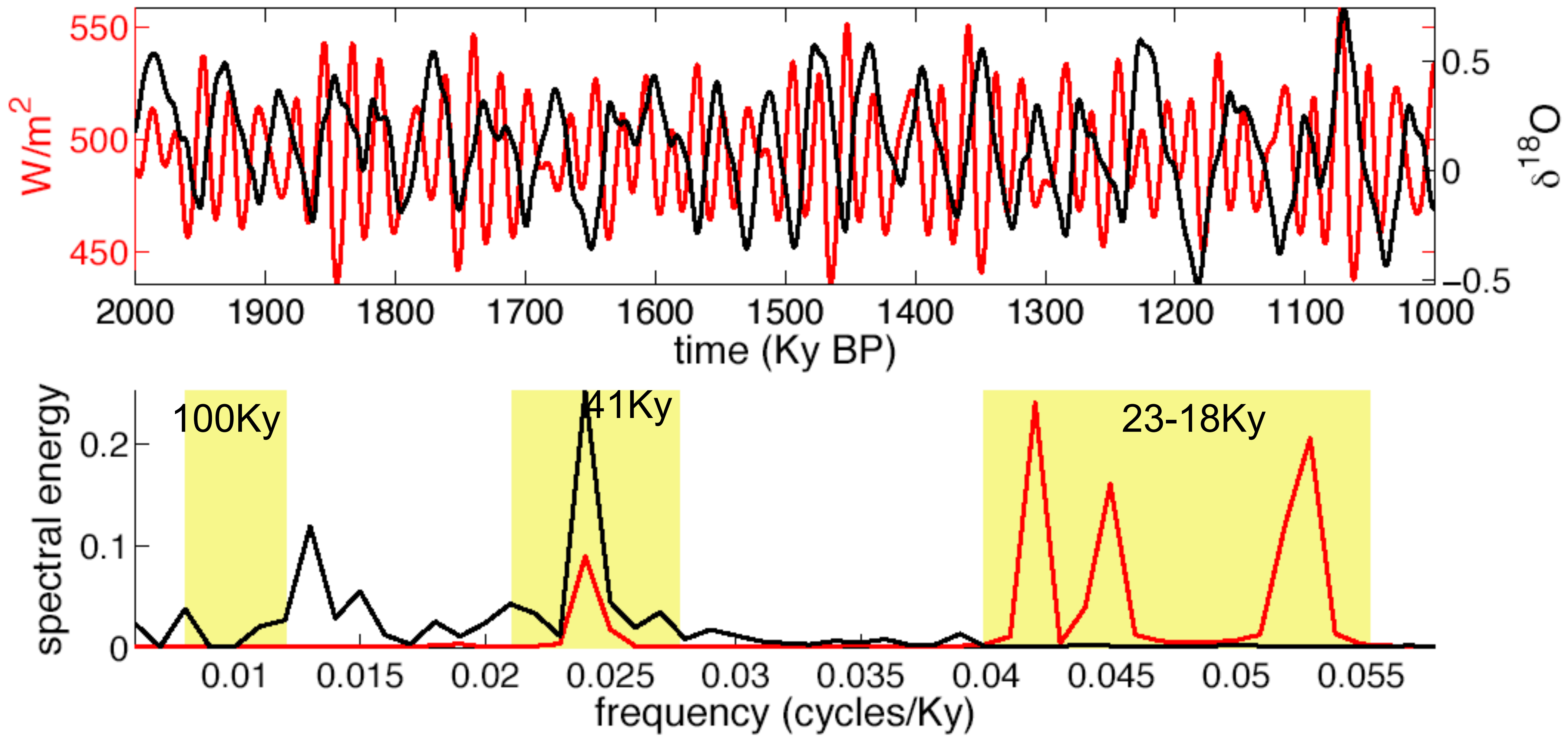
➨ Phase of Milankovitch forcing is not the same during different terminations, due to the irregular structure of the Milankovitch forcing. Still, Milankovitch forcing sets time of terminations via phase locking!

Tziperman, Raymo, Huybers, Wunsch 2006

The 41Ky problem and integrated insolation

The 41Ky problem

Imbrie et al (1992), Muller and MacDonald (2001), Nisancioglu (2004), and many others



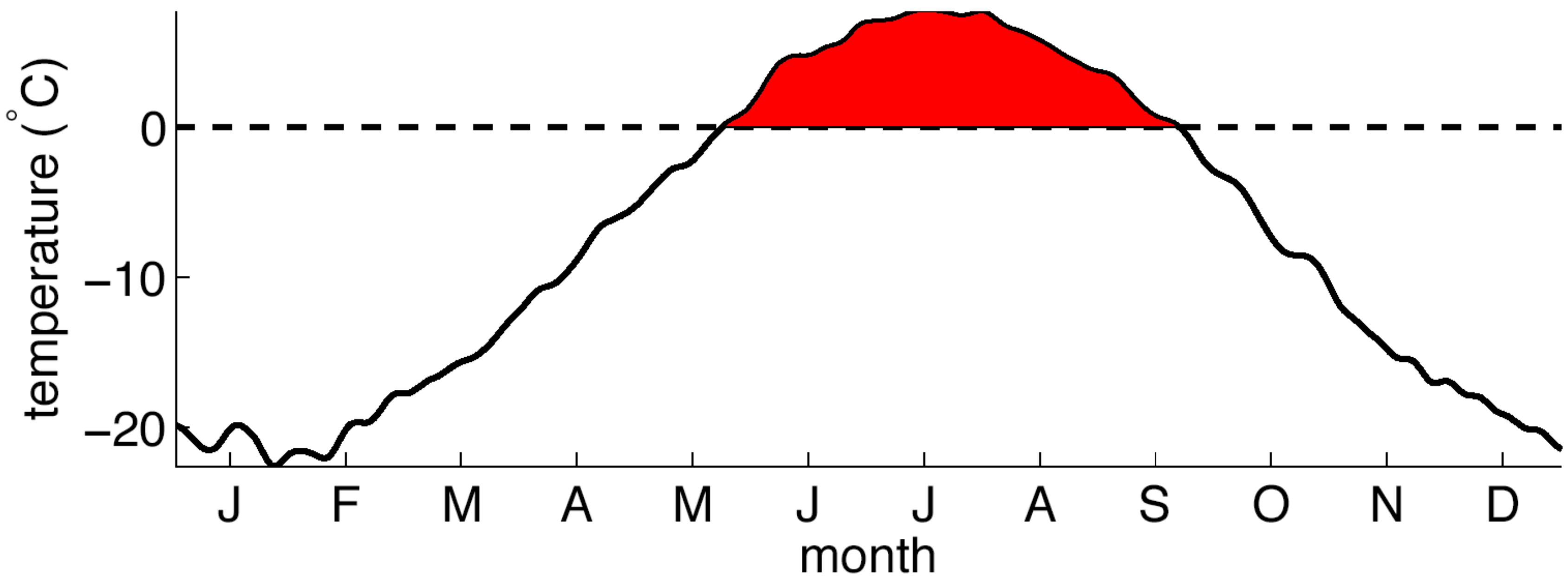
Peter Huybers

Representing the relationship between insolation, temperature and ablation

Braithwaite (1984), Paterson (1994)

ablation is known to be correlated with PDD

$$\text{Positive Degree Days} = \sum_i \alpha_i T_i \begin{cases} \alpha_i & = 0 \text{ if } T_i \leq 0 \\ \alpha_i & = 1 \text{ if } T_i > 0 \end{cases}$$



assuming insolation determines surface ice temperature, define summer energy similarly to PDD,

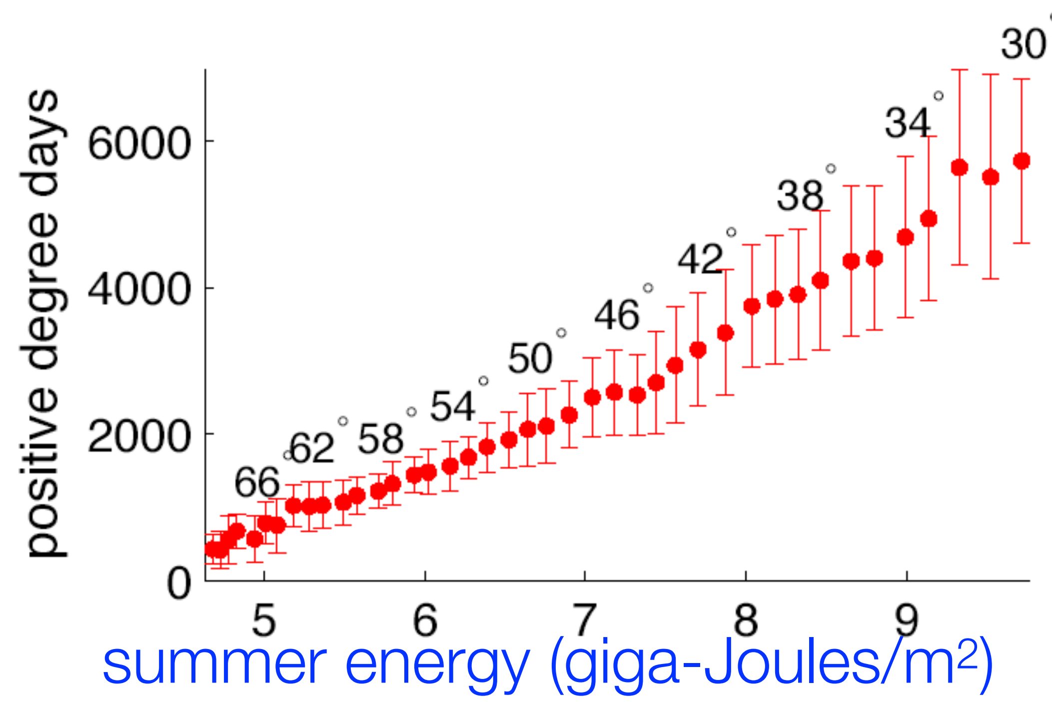
$$\text{Summer Energy} = \sum_i \beta_i W_i \begin{cases} \beta_i & = 0 \text{ if } W_i \leq \tau \\ \beta_i & = 1 \text{ if } W_i > \tau \end{cases}$$

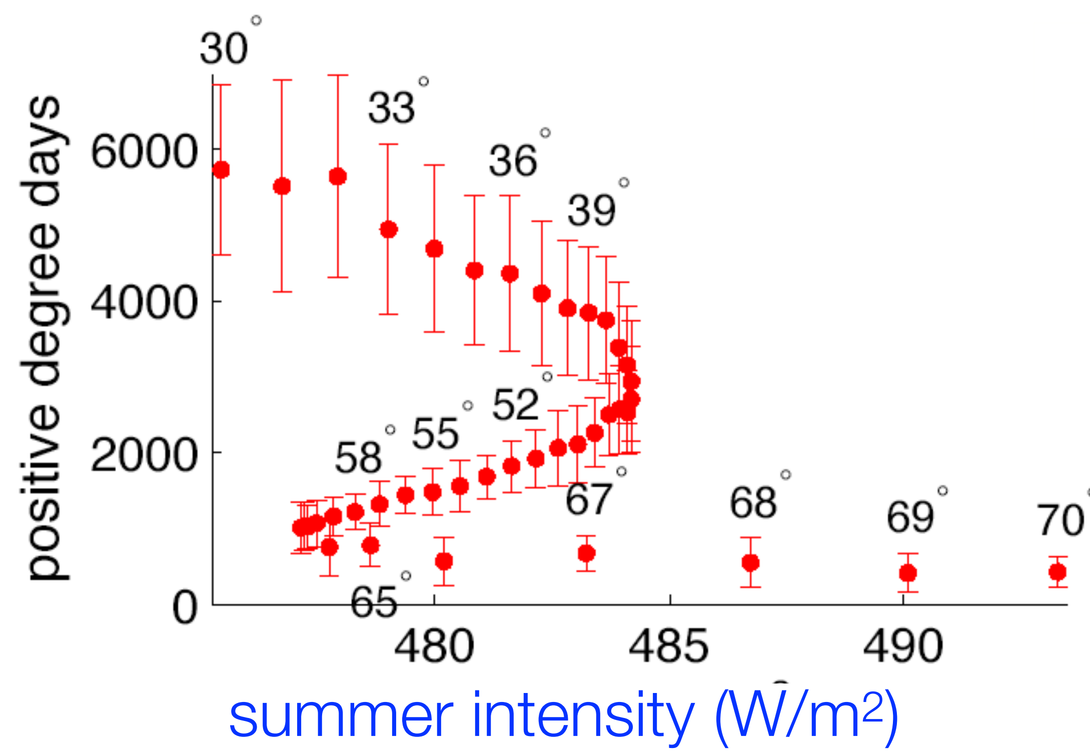
Peter Huybers

In class workshop

Do we expect the insolation threshold to be larger or smaller for a warmer climate?

Summer energy is strongly correlated with Positive degree days and insolation, but maximum summer insolation is not!





Peter Huybers

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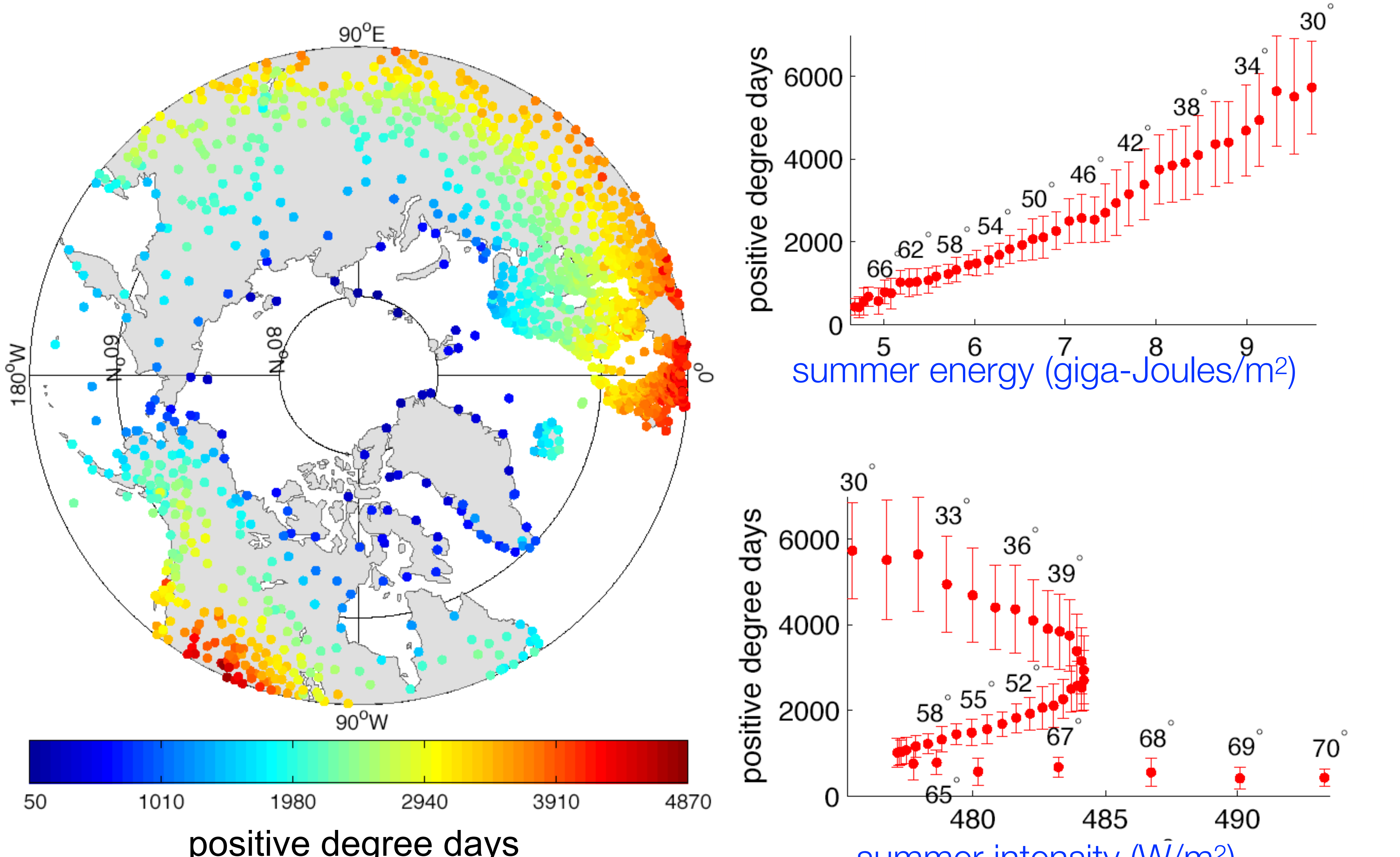


Peter Huybers

Milankovitch forcing: precession+eccentricity

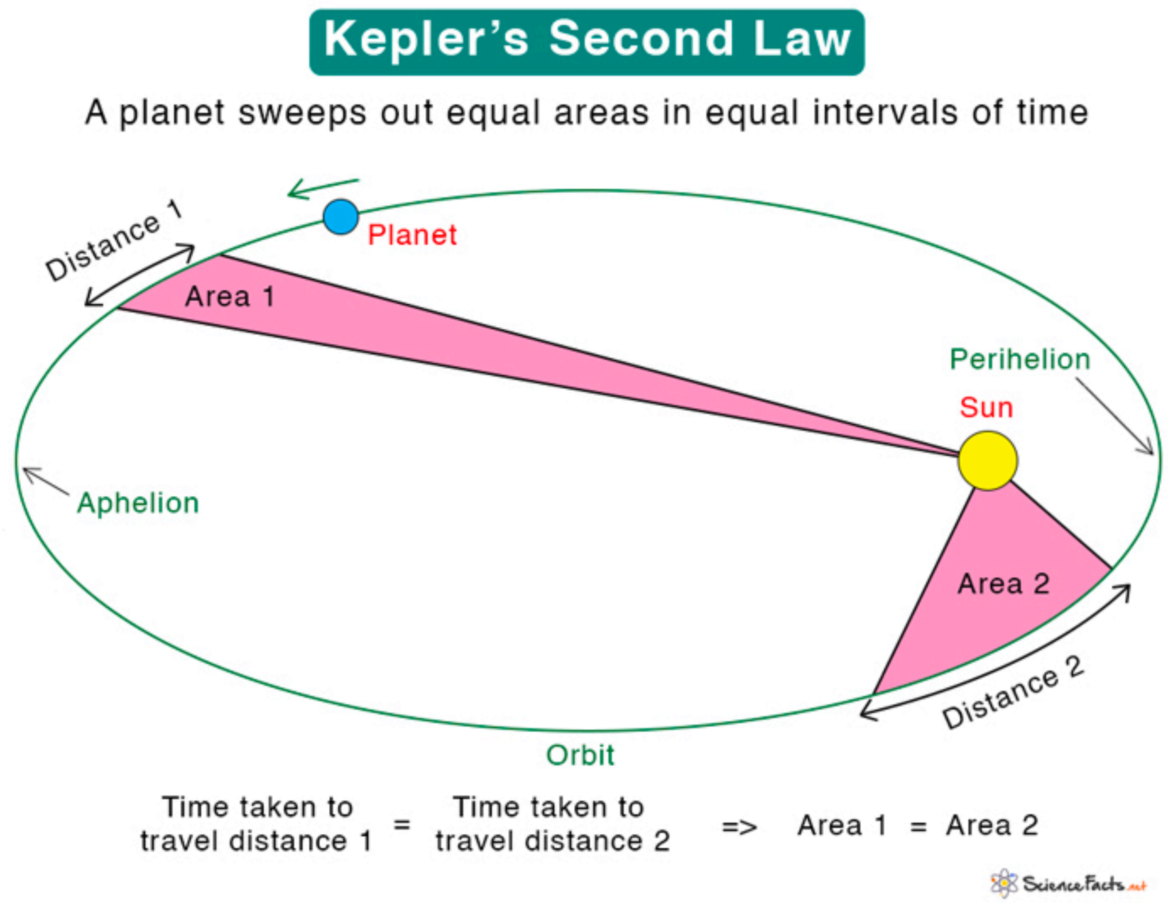
In-class workshop

Use Kepler's laws to explain *qualitatively* why the integrated summer insolation above a threshold is expected to be independent of precession

Kepler Laws:

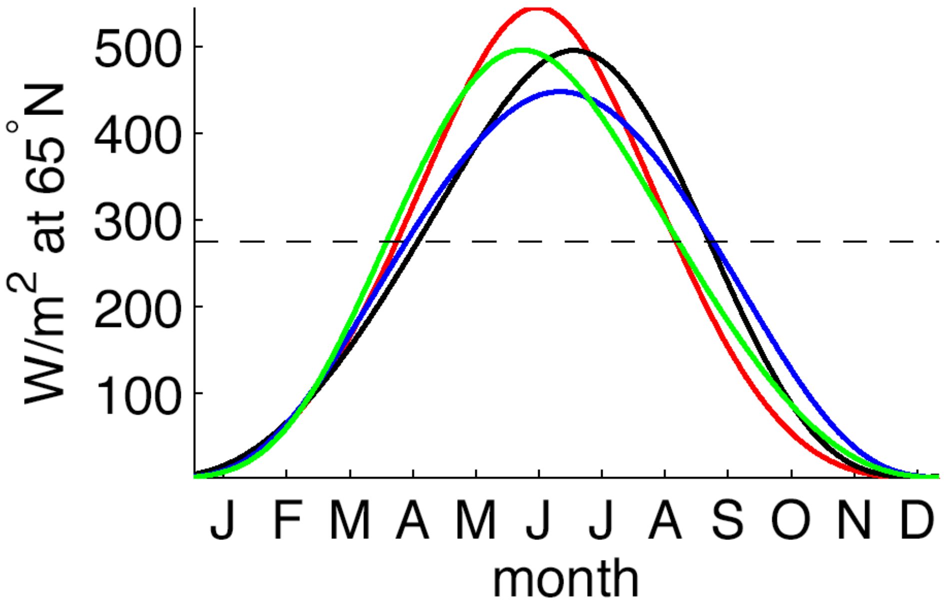
(1) planets move in elliptical orbits with the Sun as a focus,

(2) a planet covers the same area of space in the same amount of time no matter where it is in its orbit



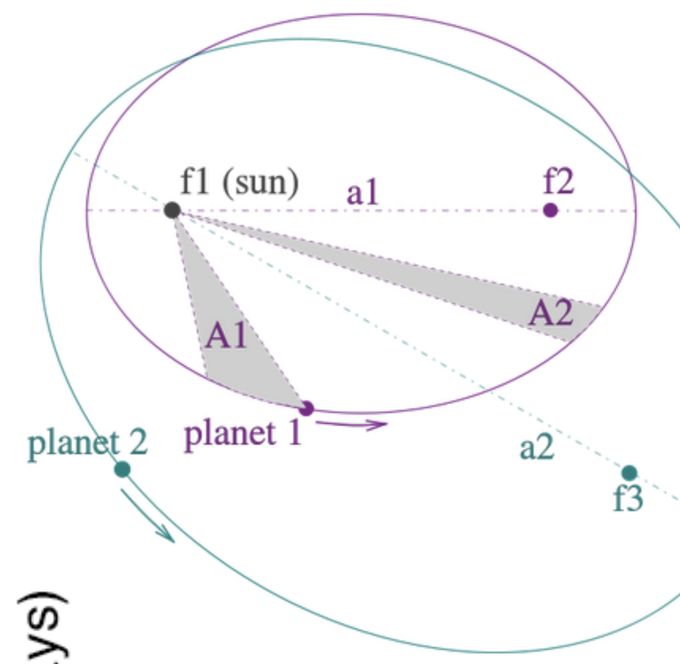
schematic from https://www.sciencefacts.net/keplers-laws.html

Summer intensity and duration

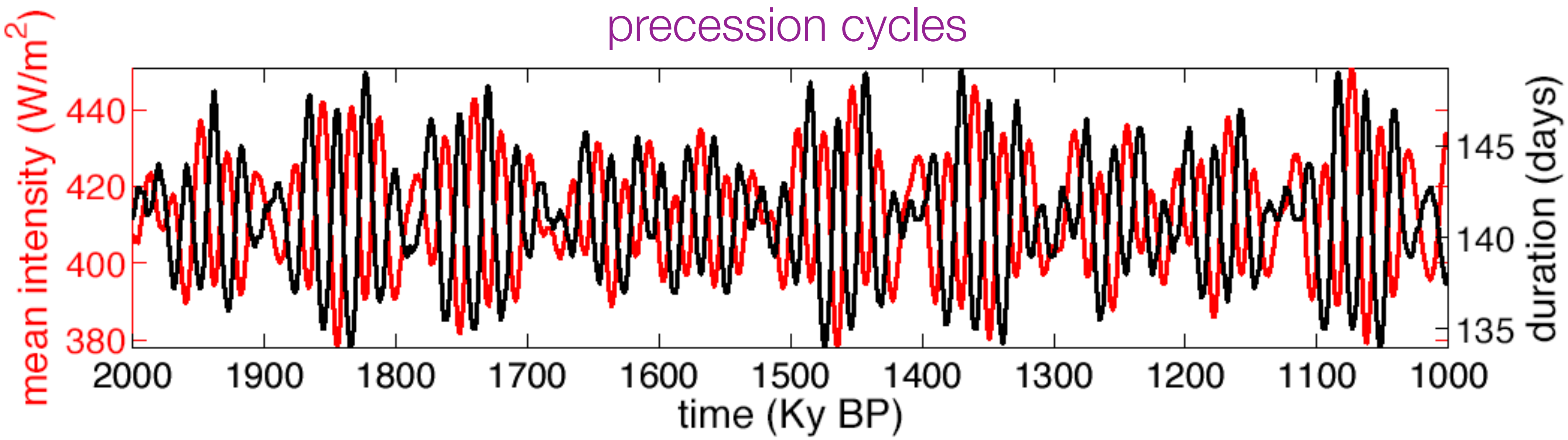


Summer duration & intensity are anti-correlated: just when Earth is closest to the sun during summer, summertime is the shortest (Kepler*). When intensity is integrated over the summertime, precession-related changes in duration and intensity nearly balance one another, and the obliquity component is dominant.

(*) a line connecting a planet & the sun sweeps out equal areas during equal intervals of time, due to angular momentum conservation

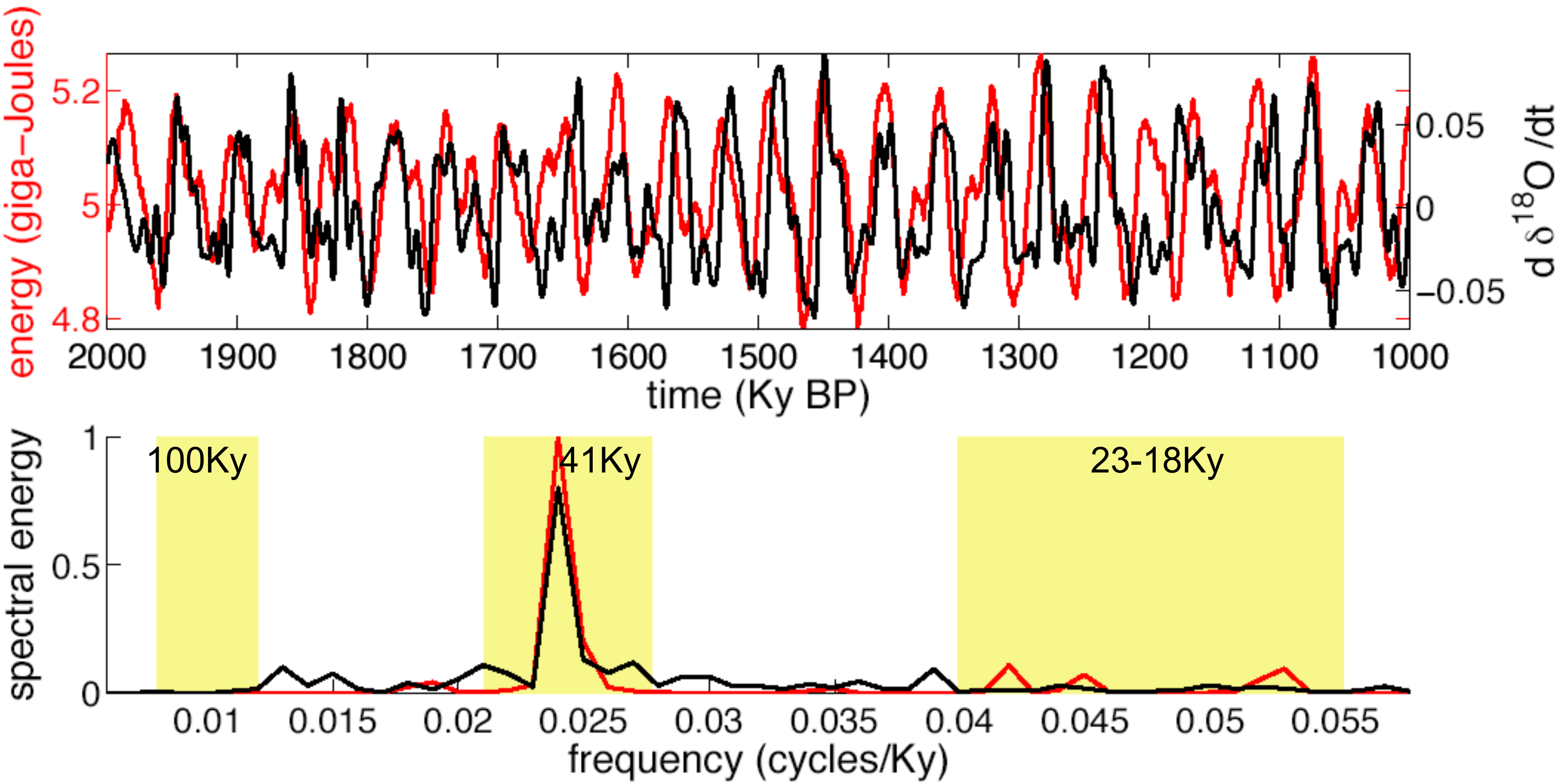


Insolation intensity & summer duration are anti-correlated during precession cycles



Peter Huybers

Early Pleistocene summer energy and the rate of ice-volume change



High threshold: integrated insulation=peak summer intensity, precession dominates; **Low threshold:** obliquity dominates.

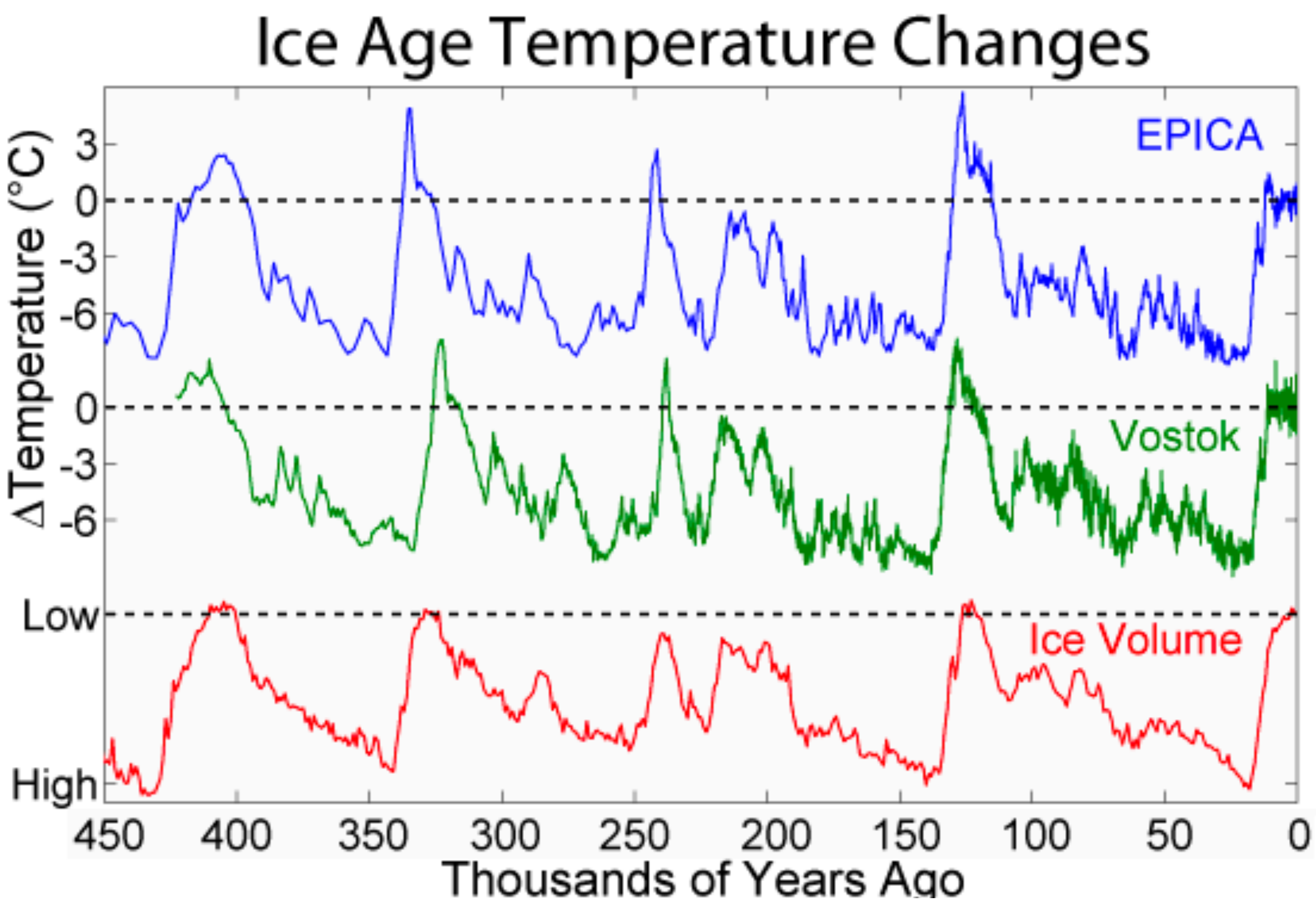
Cold climate: need a high threshold for representing melting temperature;

Warm climate: need only a low threshold.

➔ Relevant to the mid-Pleistocene transition from 41kyr to 100kyr ??

Peter Huybers

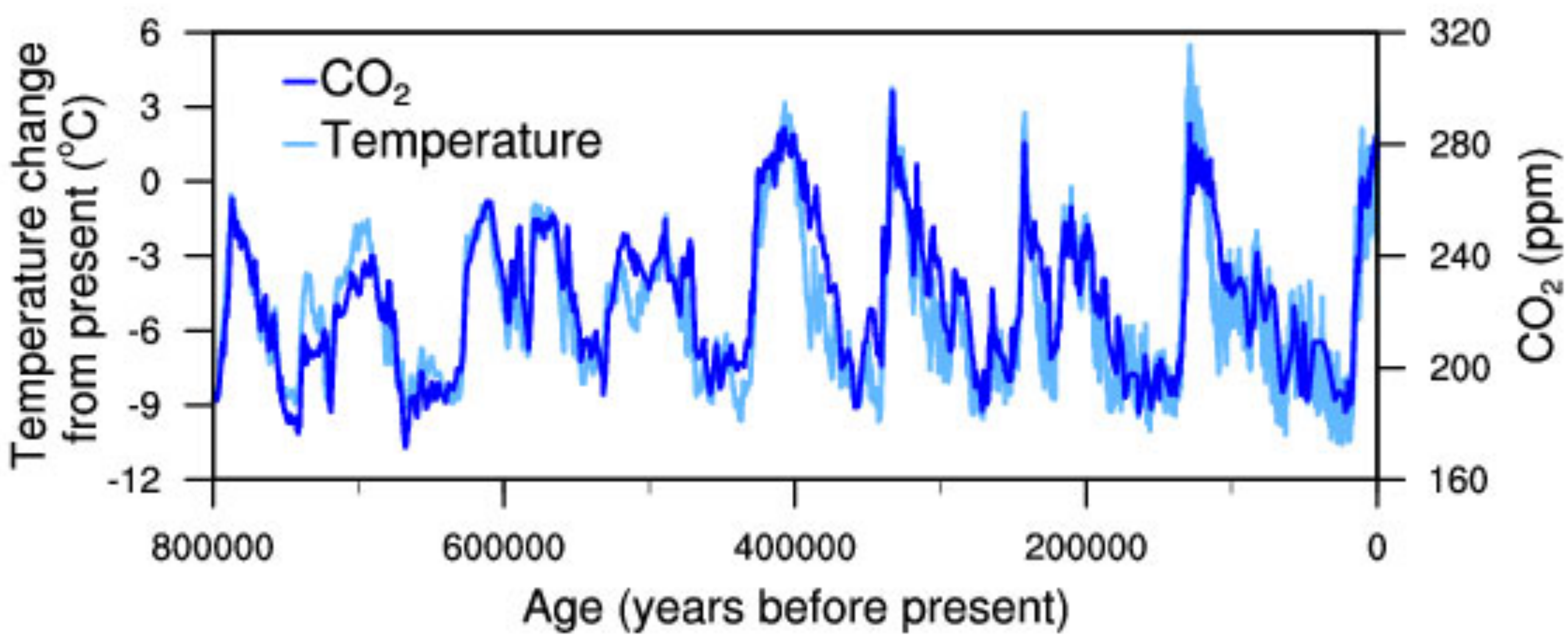
Glacial CO_2 variations



Variations of temperature, ice volume and CO_2

https://commons.wikimedia.org/wiki/File:Ice_Age_Temperature.png

https://commons.wikimedia.org/wiki/File:Temperature-change-and-carbon-dioxide-change-measured-from-the-EPICA-Dome-C-ice-core-in-Antarctica-v2.jpg



notes
The ocean carbonate system

The ocean carbonate system

Atmospheric CO_2(g) is in equilibrium with dissolved CO_2(aq), (Henry's Law)

$$\mathrm{CO_2(g)} \rightleftharpoons \mathrm{CO_2(aq)},$$

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treated together as a single variable

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Carbonic acid releases two protons

$$\mathrm{H_2CO_3^* \rightleftharpoons H^+ + HCO_3{}^-}$$

$$\mathrm{HCO_3^- \rightleftharpoons H^+ + CO_3{}^{2-}}.$$

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Water dissociation

$$\mathrm{H_2O \rightleftharpoons H^+ + OH^-}$$

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$$K_H \equiv K_0(T,S,p) = \frac{[\mathrm{H_2CO_3^*}]}{[\mathrm{CO_2(g)}]}$$

Carbonic acid releases two protons

$$\mathrm{H_2CO_3^*} \rightleftharpoons \mathrm{H^+} + \mathrm{HCO_3}^-$$

$$K_1(T,S,p) = \frac{[\mathrm{H^+}][\mathrm{HCO_3^-}]}{[\mathrm{H_2CO_3^*}]}$$

$$\mathrm{HCO_3^-} \rightleftharpoons \mathrm{H^+} + \mathrm{CO_3}^{2-}.$$

$$K_2(T,S,p) = \frac{[\mathrm{H^+}][\mathrm{CO_3^{2-}}]}{[\mathrm{HCO_3^-}]}$$

Water dissociation

$$\mathrm{H_2O} \rightleftharpoons \mathrm{H^+} + \mathrm{OH^-}$$

$$K_w(T,S,p) = [\mathrm{H^+}][\mathrm{OH^-}]$$

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➨ **6 unknown** $[\mathrm{CO_2(g)}], [\mathrm{H_2CO_3^*}], [\mathrm{OH^-}], [\mathrm{H^+}], [\mathrm{HCO_3^-}], [\mathrm{CO_3^{2-}}]$, **and only four equations**

Complete equations for the ocean carbonate system

Charge conservation (alkalinity)

Carbonate alkalinity Alk_C

$$Alk = [HCO_3^-] + 2[CO_3^{2-}] + [OH^-] - [H^+]$$

T, S, p: the ocean temperature, salinity, and pressure where the carbonate system is solved.

Complete equations for the ocean carbonate system

Charge conservation (alkalinity)

Carbonate alkalinity Alk_C (spanning $[HCO_3^-] + 2[CO_3^{2-}]$)

$$Alk = [HCO_3^-] + 2[CO_3{}^{2-}] + [OH^-] - [H^+]$$

Mass conservation (total CO_2)

$$C_T = [HCO_3^-] + [CO_3^{2-}] + [H_2CO_3^*].$$

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Mass conservation (total CO_2)

$$C_T = [\mathrm{HCO}_3^-] + [\mathrm{CO}_3^{2-}] + [\mathrm{H}_2\mathrm{CO}_3^*].$$

plus the carbonate equations

$$K_H \equiv K_0(T,S,p) = \frac{[\mathrm{H}_2\mathrm{CO}_3^*]}{[\mathrm{CO}_2(\mathrm{g})]}$$

$$K_1(T,S,p) = \frac{[\mathrm{H}^+][\mathrm{HCO}_3^-]}{[\mathrm{H}_2\mathrm{CO}_3^*]}$$

$$K_2(T,S,p) = \frac{[\mathrm{H}^+][\mathrm{CO}_3^{2-}]}{[\mathrm{HCO}_3^-]}$$

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T, S, p: the ocean temperature, salinity, and pressure where the carbonate system is solved.

➨ **6 unknown** $[CO_2(g)], [H_2CO_3^*], [OH^-], [H^+], [HCO_3^-], [CO_3^{2-}]$, **and six equations**

Complete equations for the ocean carbonate system

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Mass conservation (total CO_2)

$$C_T = [HCO_3^-] + [CO_3^{2-}] + [H_2CO_3^*].$$

The next slide shows these to be small at the oceanic pH range

plus the carbonate equations

$$K_H \equiv K_0(T,S,p) = \frac{[H_2CO_3^*]}{[CO_2(g)]}$$

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Carbonate system solution

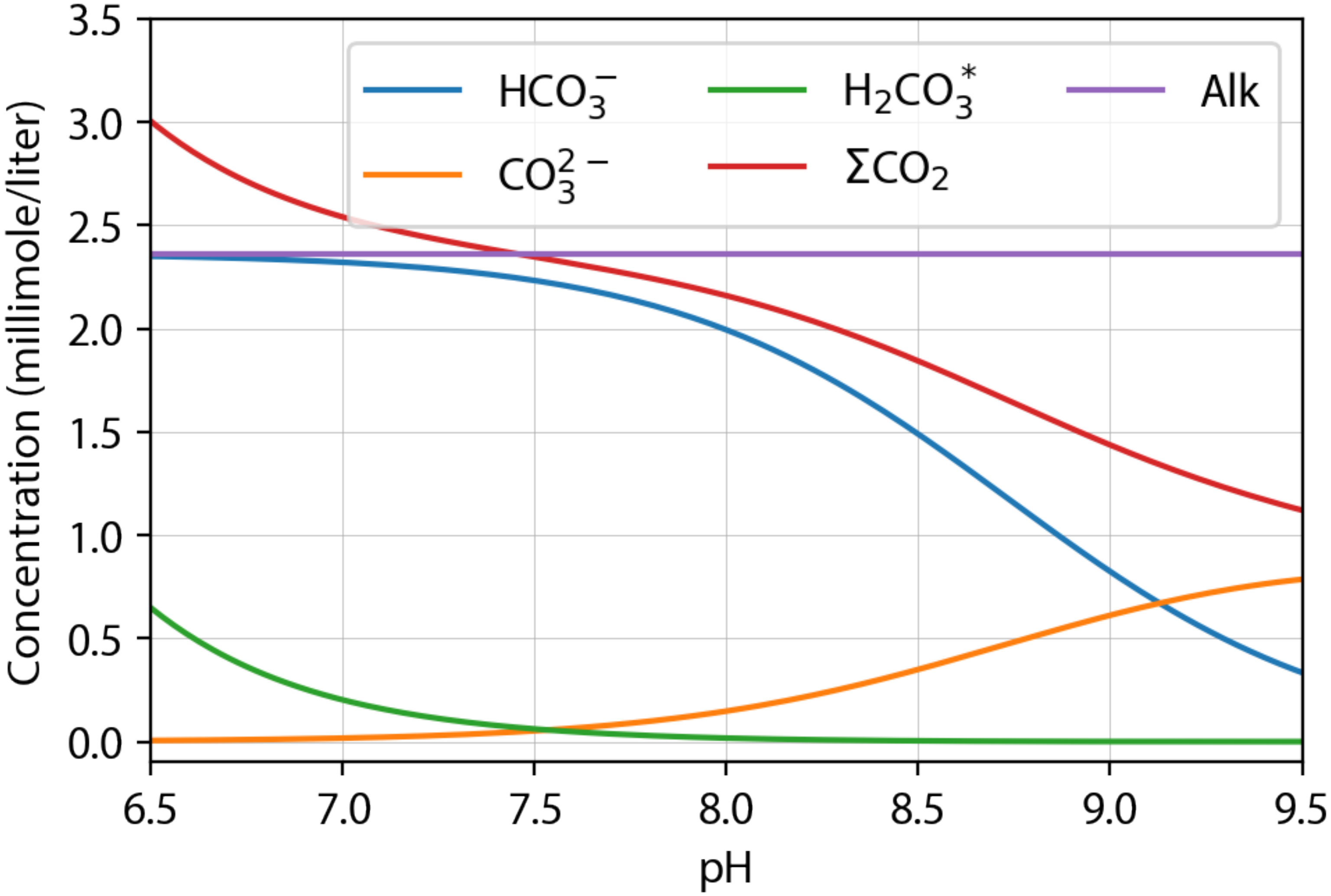


Figure 5.3: The solution of the carbonate system,
showing the concentration of carbonate species as a function of pH for a fixed alkalinity.

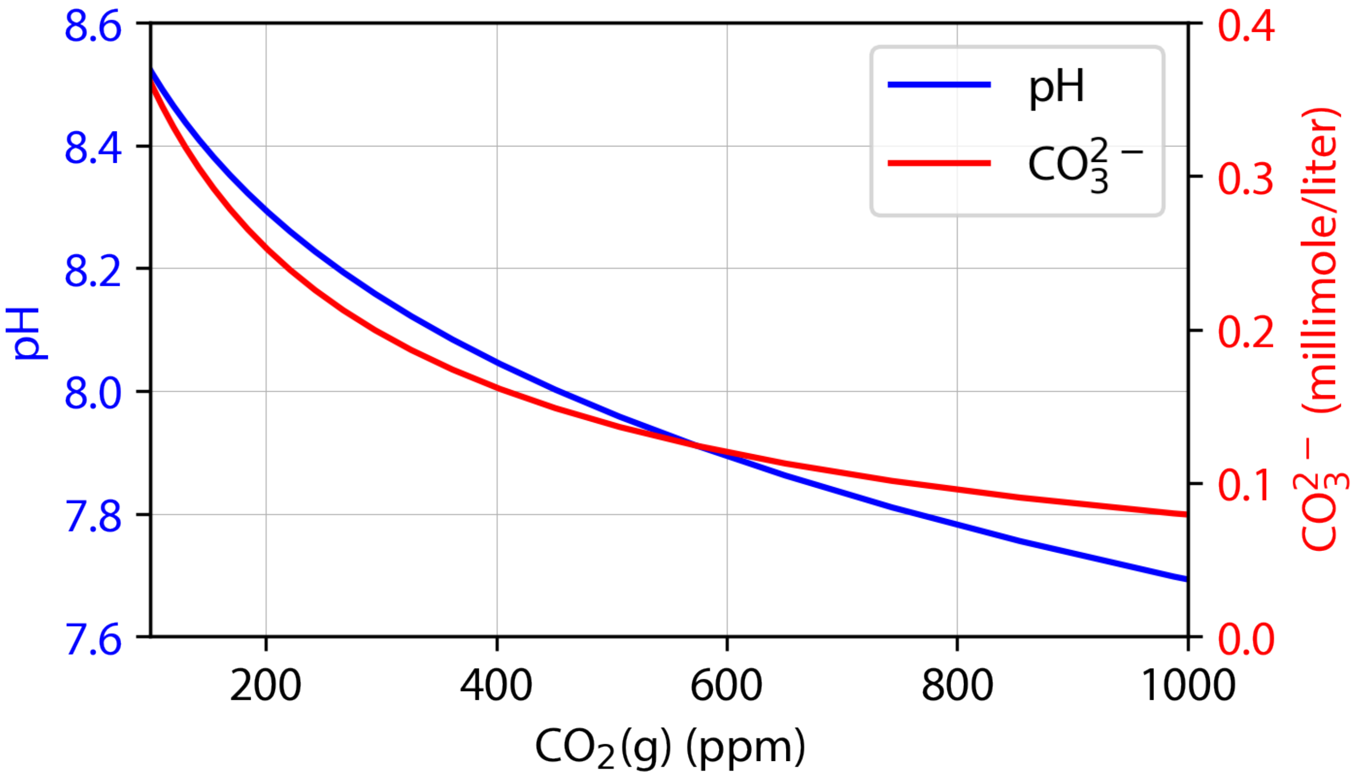
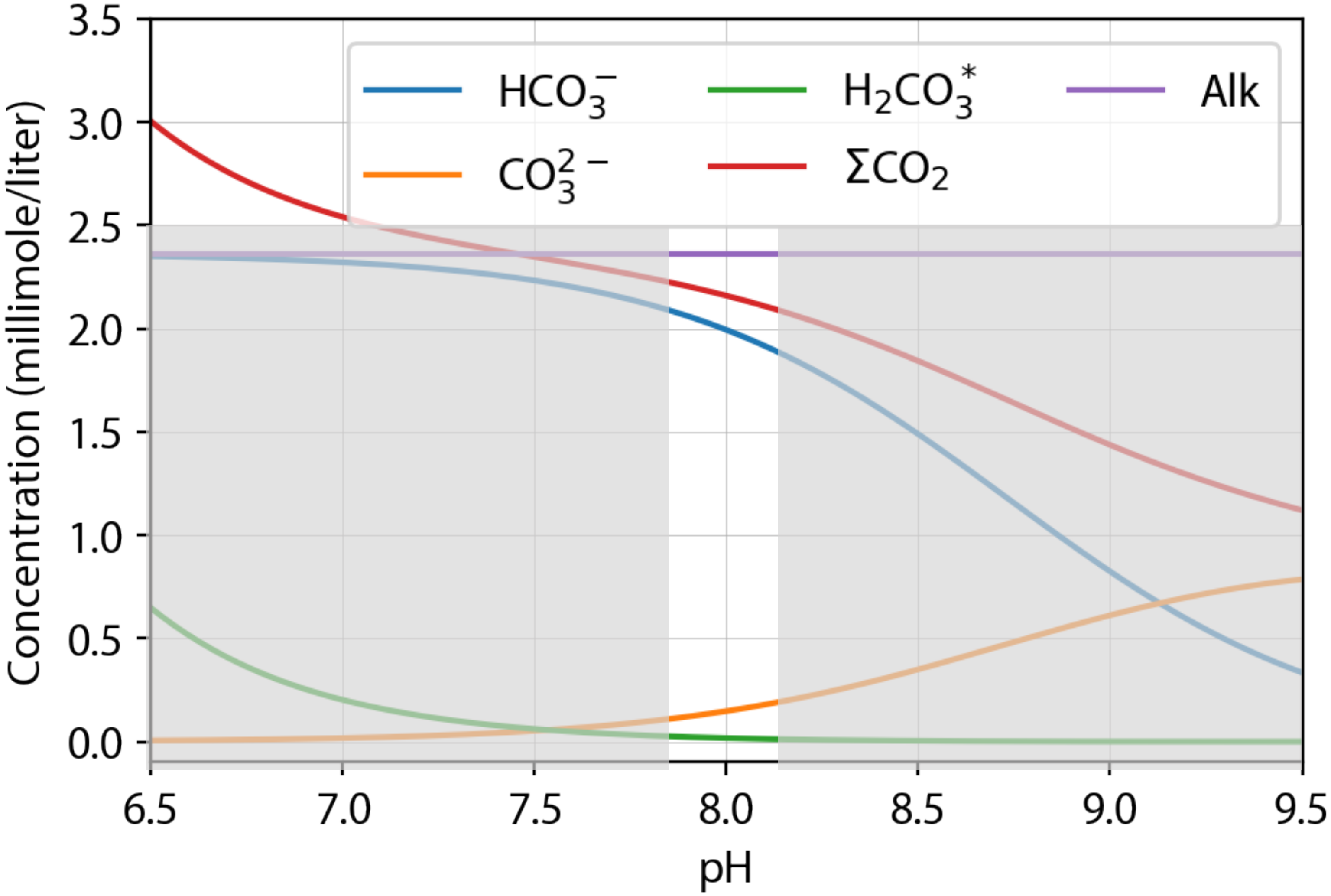


Figure 5.4: The response of pH and carbonate ion to CO_2 increase.

The solution of the carbonate system for a fixed alkalinity as in Figure 5.3, showing the ocean pH (blue) and the carbonate ion CO_3^{2-} concentration (red) as a function of atmospheric CO_2.

Carbonate system solution



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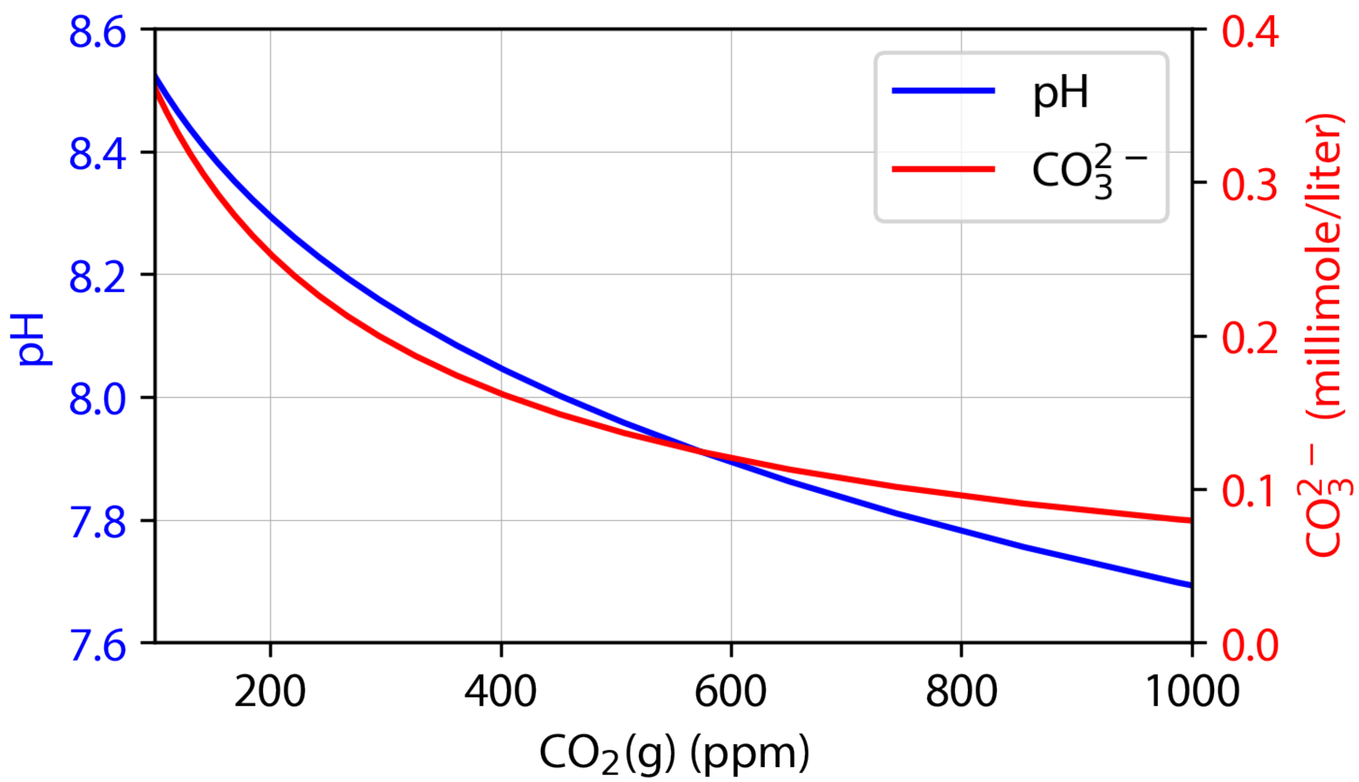

In-class workshop

Solve for HCO_3^-, CO_3^{2-} and H^+ using the approximate equations.

Approximate solution of the carbonate system

For the values of pH at present/near future:

$$[HCO_3^-]\,,[CO_3^{2-}] \gg [H^+]\,,[OH^-]\,,[H_2CO_3^*]\,.$$

Approximate solution of the carbonate system

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So that

$$Alk_C = [HCO_3^-] + 2[CO_3{}^{2-}] + \cancel{[OH^-]} - \cancel{[H^+]}$$

$$C_T = [HCO_3^-] + [CO_3^{2-}] + \cancel{[H_2CO_3^*]}.$$

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$$C_T = [HCO_3^-] + [CO_3^{2-}] + [H_2CO_3^*].$$

(the $[OH^-]$, $[H^+]$ and $[H_2CO_3^*]$ terms are crossed out)

➡ only 5 unknowns ($\mathbf{OH^-}$ drops out) & the eqns become

$$K_H = \frac{[H_2CO_3^*]}{[CO_2(g)]},$$

$$K_1 = \frac{[HCO_3^-][H^+]}{[H_2CO_3^*]},$$

$$K_2 = \frac{[CO_3^{2-}][H^+]}{[HCO_3^-]},$$

$$Alk_C = [HCO_3^-] + 2[CO_3^{2-}]$$

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Approximate solution of the carbonate system

For the values of pH at present/near future:

$$[\mathrm{HCO_3^-}],[\mathrm{CO_3^{2-}}] \gg [\mathrm{H^+}],[\mathrm{OH^-}],[\mathrm{H_2CO_3^*}].$$

So that

$$Alk_C = [\mathrm{HCO_3^-}] + 2[\mathrm{CO_3}^{2-}] + [\mathrm{OH^-}] - [\mathrm{H^+}]$$

$$C_T = [\mathrm{HCO_3^-}] + [\mathrm{CO_3^{2-}}] + [\mathrm{H_2CO_3^*}].$$

➡ only 5 unknowns ($\mathbf{OH^-}$ drops out) & the eqns become

$$K_H = \frac{[\mathrm{H_2CO_3^*}]}{[\mathrm{CO_2(g)}]},$$

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and can be solved as

$$[\mathrm{HCO_3^-}] = 2\mathrm{C_T} - \mathrm{Alk_C},$$

$$[\mathrm{CO_3^{2-}}] = \mathrm{Alk_C} - \mathrm{C_T}.$$

$$[\mathrm{H^+}] = K_2 \frac{2C_T - Alk_C}{Alk_C - C_T}$$

$$[\mathrm{H_2CO_3^*}] = \frac{K_2}{K_1}\frac{(2C_T - Alk_C)^2}{Alk_C - C_T}$$

$$[\mathrm{CO_2(g)}] = \frac{K_2}{K_1 K_H}\frac{(2C_T - Alk_C)^2}{Alk_C - C_T}.$$

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what could be simpler 😳

Response to warming

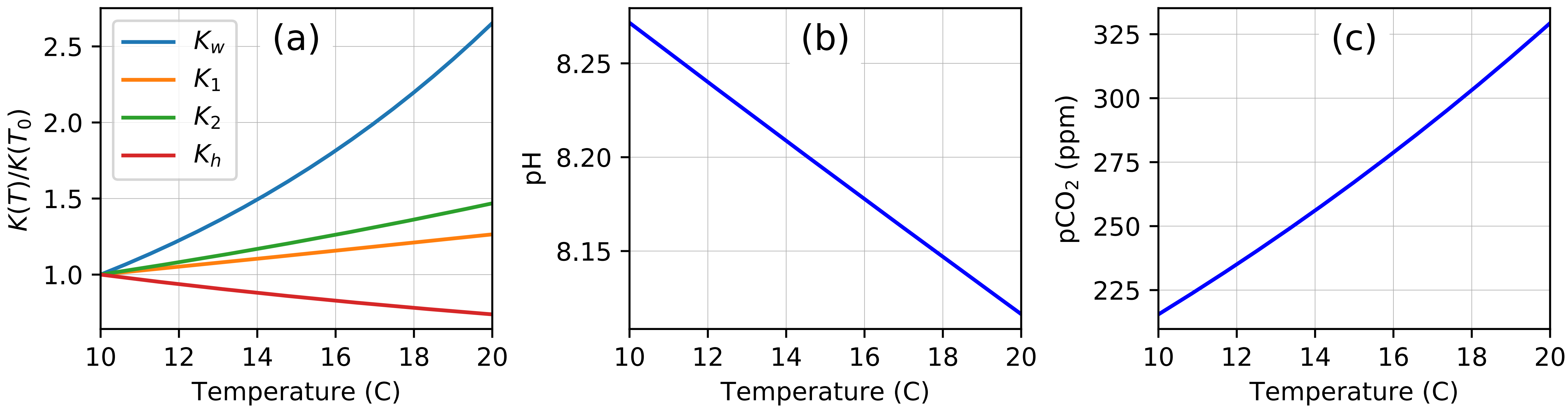

Figure 5.5: Response of the carbonate system to warming, as a function of the ocean temperature. The DIC and alkalinity are assumed fixed. (a) Reaction constants normalized by their values at 10 °C. (b) pH. (c) Atmospheric pCO_2.

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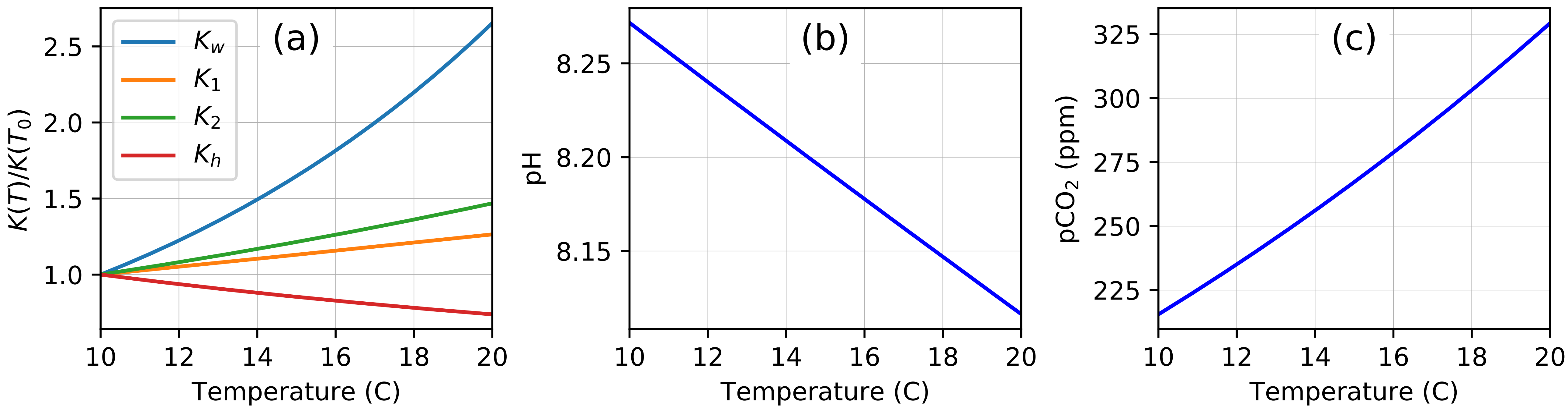

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$$[H^+] = K_2 \frac{2C_T - Alk_C}{Alk_C - C_T}, \qquad [CO_2(g)] = \frac{K_2}{K_1 K_H} \frac{(2C_T - Alk_C)^2}{Alk_C - C_T}.$$

K_1, K_2 and K_H all play a role, not only Henry's constant responsible for the dissolution of CO_2. In solution for the atmospheric CO_2 concentration, $K_2/(K_1K_H)$, Henry's constant K_H decreases with temperature, while the other two increase. The ratio overall increases, leading to the increase in atmospheric CO_2 with warming.

Finally, attempting to understand the glacial CO_2 problem using a box model

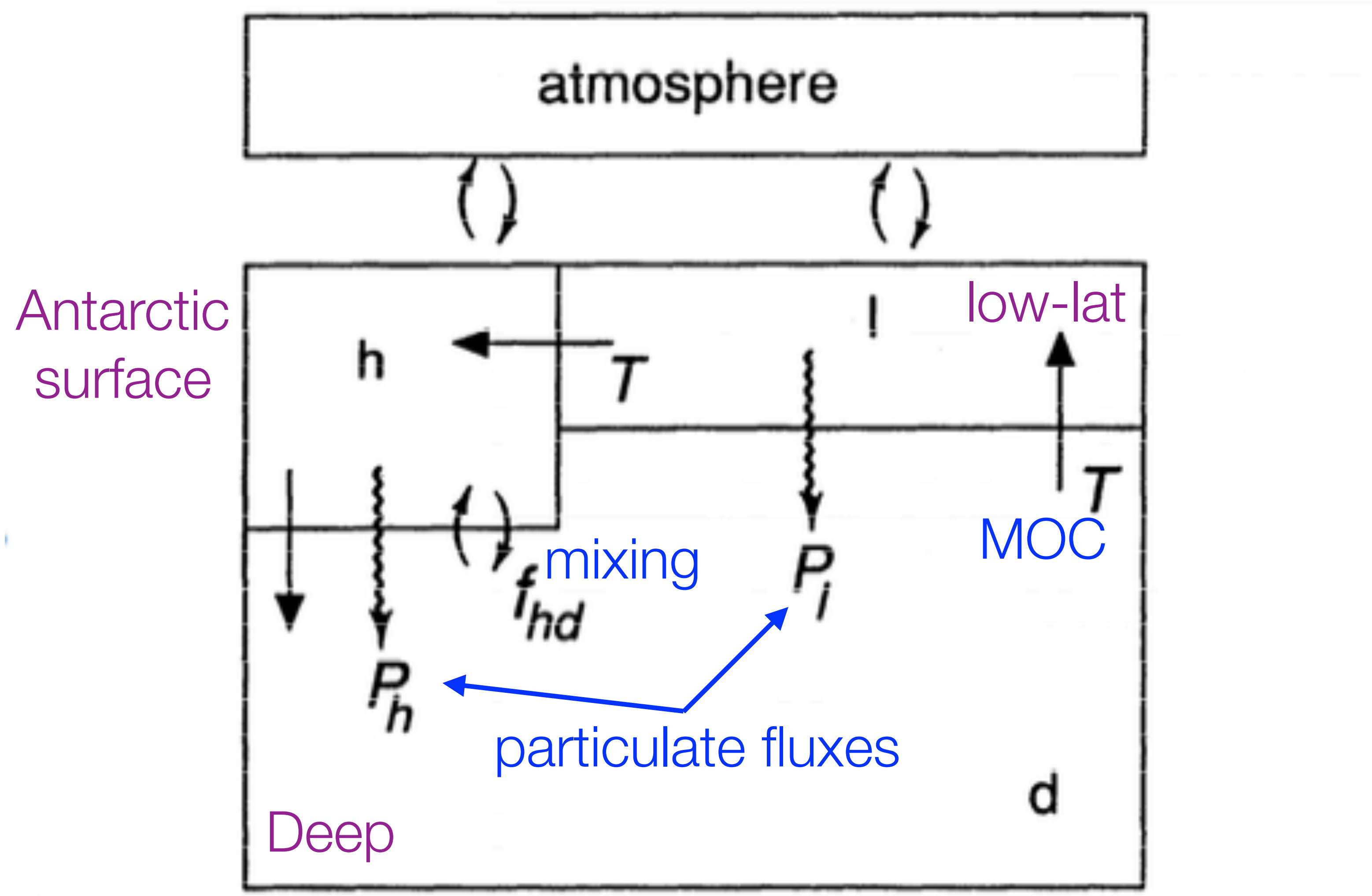


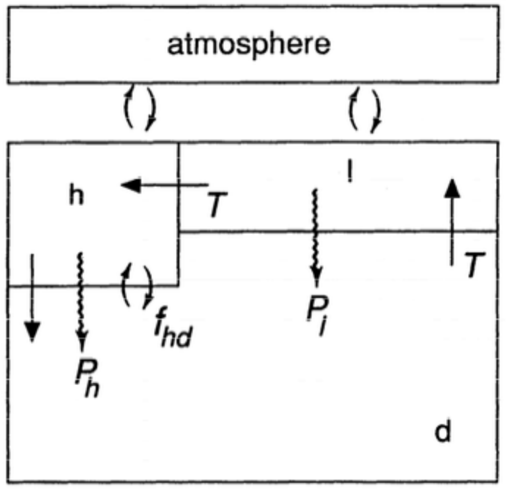
Figure 1. Schematic diagram of the three-box model of *Sarmiento and Toggweiler* [1984] and *Siegenthaler and Wenk* [1984].

Toggweiler 1999 model

Finally, attempting to understand the glacial CO_2 problem using a box model

In the low-latitude surface box, assume that the upwelling nutrient flux is completely utilized by the biology. The downward carbon particulate flux (in moles of carbon) is then,

$$P_l = r_{c:p} \times T \times PO_{4,d}.$$



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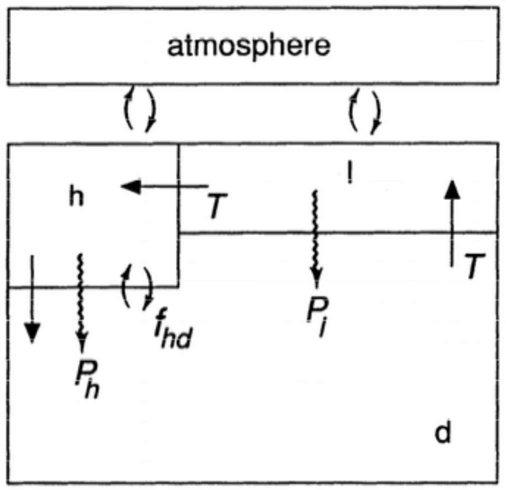

The deep box Dissolve Inorganic Carbon budget ($\sum CO_2 \equiv C_T \equiv DIC \equiv$ Total CO_2)

$$\frac{d}{dt}(\Sigma CO_{2d}) = (f_{dh} + T)(\Sigma CO_{2h} - \Sigma CO_{2d}) + (P_l + P_h)$$

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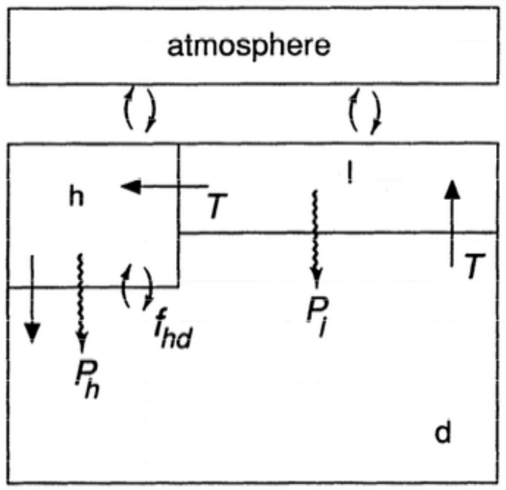

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small

$$\Delta = d - h,\ s = d + h,\ \Rightarrow h = \frac{1}{2}(s - \Delta)$$

Toggweiler 1999

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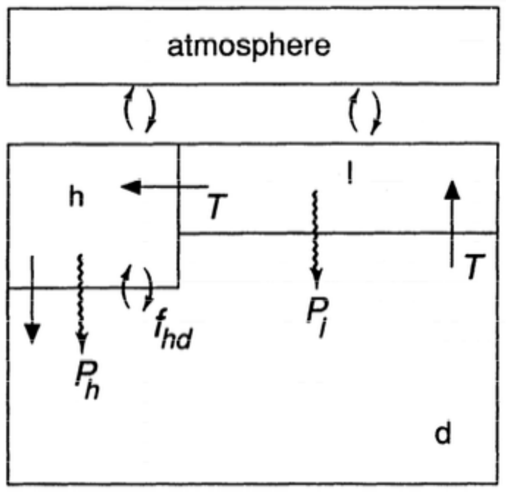

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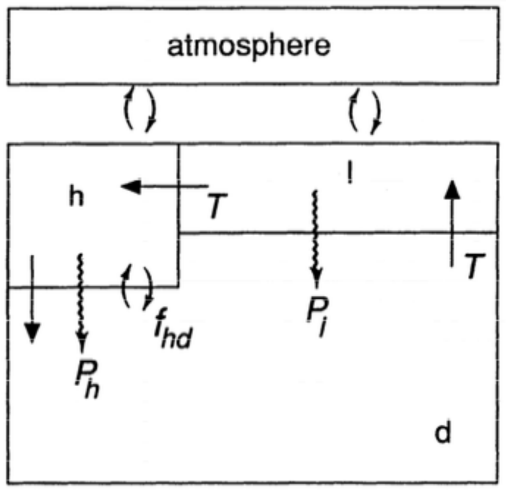

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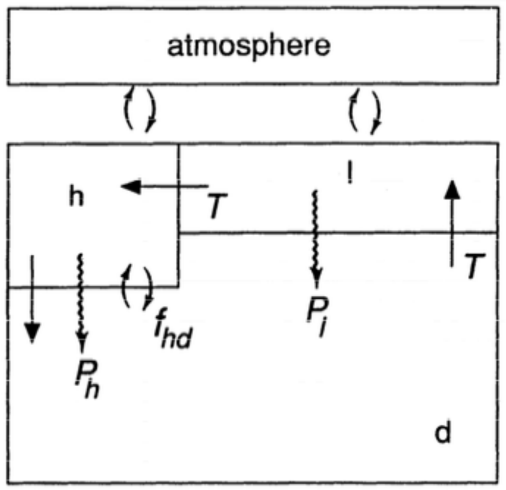

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Turns out this explanation contradicts proxy evidence of SO productivity, back to square 1

Glacial cycles, summary

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- 100 kyr time scale
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Mechanism for glacial cycles is still unresolved, although this is the largest climate variability signal over the past 1 Myr; Mechanism for CO_2 variations also still not clear

The End