BRIEF COMMUNICATIONS

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On the bursting of viscous films

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In this Brief Communication we consider the shape of the rim of a very viscous film retracting with velocity U_0 under surface tension. We demonstrate that the film has a growing rim if and only if the Stokes length ν/U_0 is smaller than the radial extent of the film. If the Stokes length is larger than the radial extent of the film the retracting edge has no rim and the film is perfectly flat. These results are discussed in light of recent experiments on the bursting of viscoelastic films. [S1070-6631(99)01202-7]

In 1959 Taylor[1] and Culick[2] independently examined the disintegration of a thin liquid sheet by the nucleation of a hole. They discovered that the rim of liquid at the edge of the film retracts at the velocity,

$$U_0 = \sqrt{\frac{2\gamma}{\rho e}}, \tag{1}$$

where γ is the surface tension, ρ the density and e the thickness of the liquid film. This formula follows by balancing surface tension forces against inertia. In his original experiments, Taylor used an ingenious method to test the validity of this formula, by measuring the radius of the thin sheet formed by the impact of a jet onto a solid surface. The radius of the sheet is controlled by the competition between the outward volume flux and the inward retraction of Eq. (1). The edge of the retracting film is terminated by a rim, whose volume grows in time with the mass of fluid swept up. Keller and collaborators[3,4] analyzed the shape and rate of growth of the rim, finding that it is a cylindrical cap which grows in time like $\sqrt{t}$. The stability of this retracting rim was recently studied.[5]

These analyses neglect viscosity, since the Reynolds number of the retraction is typically very high. Recently, Debrégeas, de Gennes and Brochard-Wyart[6] examined the disintegration of an air bubble in a very viscous liquid (of order 10^6 times the viscosity of water), and found a number of novel features: (1) First, the retraction velocity grows exponentially in time, with a characteristic timescale $\eta e/(2\gamma)$, with η the viscosity of the film; (2) second, the retracting fluid is *not* collected into a rim, as in the Taylor–Culick problem. Instead, the film remains perfectly flat through the retraction, with the film thickness increasing uniformly in time. The exponential retraction velocity is a consequence of the balance between viscous stresses and surface tension. On the other hand, it was argued that the absence of a rim in the retracting film is a consequence of viscoelasticity: the idea is that the surface tension forces acting on the edge of the film are elastically propagated into the film at a velocity $\sqrt{\mu/\rho}$, where μ is the shear modulus of the film. This propagation smooths out the rim, and leads to a flat film. Since both the fluids used in the experiments (a polymer melt and a molten glass) are also viscoelastic, it was argued in Refs. 7, 6 that viscoelasticity explains the absence of a rim during the retraction.

We demonstrate in this Brief Communication that the absence of a rim can also result from a purely viscous effect. A flat film occurs as long as

$$\mathrm{Re}_f = \frac{U_0 L}{\nu} \ll 1,$$

where ν is the kinematic viscosity, U_0 the retraction velocity, L the axial extent of the film and Re_f is the Reynolds number based on these parameters. The retraction of an infinite viscous film (which is not viscoelastic) indeed has a rim, whose shape is described by a self-similar evolution similar to that uncovered by Keller.[3]

Generally, a thin film is characterized by two dimensionless parameters: $l_\nu/e = \eta^2/(\gamma\rho e)$ and Re_f. The parameter l_ν/e determines whether capillary waves precede the retraction of the film; Re_f determines whether or not there is a rim. There are thus three different regimes: (i) $\mathrm{Re}_f > 1$, $l_\nu/e < 1$; (ii) $\mathrm{Re}_f > 1$, $l_\nu/e > 1$; (iii) $\mathrm{Re}_f < 1$, $l_\nu/e > 1$. (The fourth

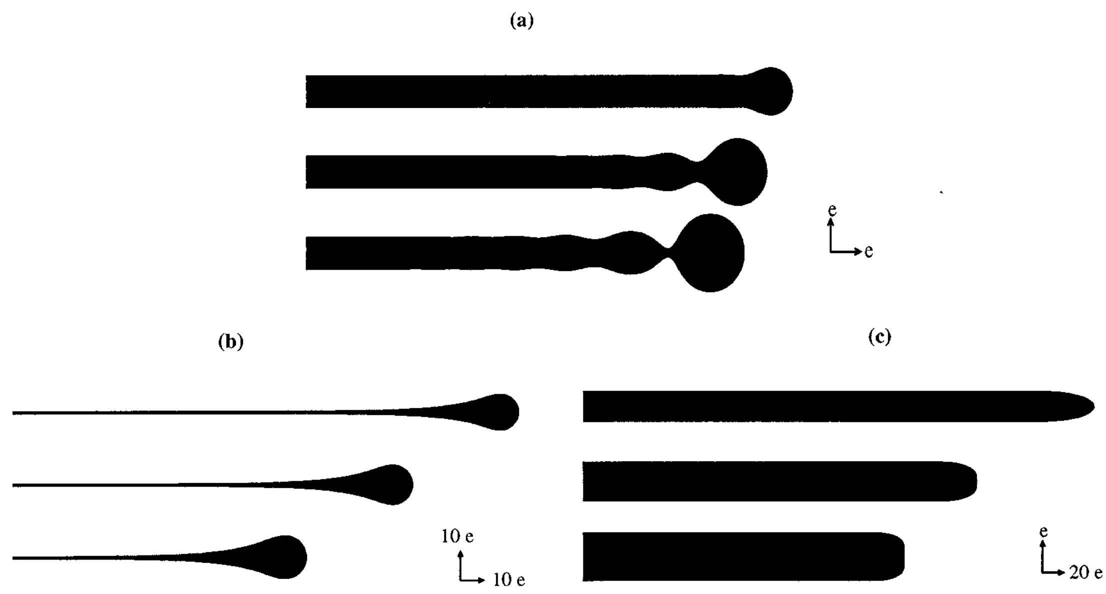

FIG. 1. Film profile during the retraction process for the three different regimes. (a) regime (i), $l_\nu/e=1.37\times10^{-6}$, $\mathrm{Re}_f=34130$; (b) regime (ii), $l_\nu/e=137$, $\mathrm{Re}_f=1208$; (c) regime (iii), $l_\nu/e=4.94\ 10^8$, $\mathrm{Re}_f=0.06$. The aspect ratio is shown using arrows; arrow lengths are given in units of the film thickness e.

limit $\mathrm{Re}_f<1$, $l_\nu/e<1$ cannot occur if $e\ll L$.) If the axial film length is infinite ($L\rightarrow\infty$) then the third regime (iii) does not exist. In the following we present simulations of the film shape for each of these three regimes.

We commence by describing the thin viscous film by a thickness $h(x,t)$. If the film thickness varies slowly with x the dynamics is well described by the lubrication equations (for a review, see Ref. 8)

$$\partial_t h+\partial_x(hv)=0, \tag{2}$$

$$\partial_t v+v\,\partial_x v=\frac{4\nu}{h}\partial_x(h\partial_x v)-\frac{\gamma}{\rho}\partial_x\kappa. \tag{3}$$

Here, $v(x,t)$ is the velocity and $\kappa=-h''(1+h'^2)^{-3/2}$ is the mean curvature of the film. In the present context, these equations were derived by Erneux and Davis,[9,10] expanding the Navier Stokes equations for long wavelength modulations of the film. Our only modification of the equations is the usage of the complete formula for the mean curvature. The reason for this modification is that Eqs. (2) and (3) then both capture long wavelength modulations of the film and the spherical cap at the edge of the retracting rim. Although this is not strictly asymptotically correct it is physically reasonable, and has successfully reproduced experimental shapes for axisymmetric jets.[11,12] Our simulations solve Eqs. (2)–(3) with a standard second order finite difference scheme with implicit timestepping. Figures 1(a)–(c) show simulations of the equations in the various regimes.

The first set shows a retracting low viscosity film in the case $l_\nu/e<1$ and $\mathrm{Re}_f>1$. The retraction velocity is observed to be constant, and in agreement with the Taylor–Culick law. The rim of the film is a cylindrical cap which grows in time, and capillary waves are visible preceding the front [Fig. 1(a)].

The second set shows a retracting film with viscosity twenty times that of water. In this case $l_\nu/e>1$ and $\mathrm{Re}_f>1$. Although the rim still exists, it is not cylindrical and there are no capillary waves [Fig. 1(b)]. Again, after a transient period, the retraction velocity is observed to be consistent with the Taylor–Culick law. The rim is not cylindrical, but has a horizontal extent controlled by the Stokes length $\nu/U_0=\sqrt{l_\nu e}$.

The final set of simulations shows a retracting film with viscosity one million times the viscosity of water and the horizontal extent of the film is smaller than the Stokes length ($l_\nu/e>1$ and $\mathrm{Re}_f<1$). Parameters were chosen to agree with the experiments of Debrégeas, de Gennes and Brochard-Wyart. In this situation, the rim at the edge of the film has completely disappeared, and as in the experiments the film thickness grows uniformly in time [Fig. 1(c)]. The film in the simulation is uniform to high precision (less than 0.02 percent variation in the thickness).

Our simulations do not show the exponential regime found by Debrégeas *et al.* The reason for this is that our flow is one dimensional, and the exponential velocity law requires a radial profile. However, we have verified that the characteristic timescale for reaching the Taylor–Culick law is $\eta e/(2\gamma)$ as predicted and measured by Refs. 7,6.

In conclusion, we have shown three different regimes for retracting thin films; two of the regimes essentially correspond to the retraction laws found by Taylor and Culick. The third regime occurs when the Stokes scale ν/U_0 is larger than the radial extent of the film. This regime is characterized by a perfectly flat film without a rim, as observed in the recent experiments of Debrégeas *et al.*

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