A Cascade of Structure in a Drop Falling from a Faucet

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A drop falling from a faucet is a common example of a mass fissioning into two or more pieces. The shape of the liquid in this situation has been investigated by both experiment and computer simulation. As the viscosity of the liquid is varied, the shape of the drop changes dramatically. Near the point of breakup, viscous drops develop long necks that then spawn a series of smaller necks with ever thinner diameters. Simulations indicate that this repeated formation of necks can proceed ad infinitum whenever a small but finite amount of noise is present in the experiment. In this situation, the dynamical singularity occurring when a drop fissions is characterized by a rough interface.

What happens when liquid drips from a faucet? As it falls, its topology changes from a single mass of fluid into two or more drops. This common phenomenon is one of the simplest hydrodynamic examples of a singularity (*1*) in which physical quantities become discontinuous in a finite time. Here, we investigate the shape of this singularity for fluids of varying viscosity dripping through air from a cylindrical nozzle. Scientific studies of this system date back to Lord Rayleigh's stability analysis of a liquid cylinder (*2*) and Plateau's analysis of a hanging pendant droplet (*3*). More recent work has begun to address the shape of the interface near the singularity. Eggers and Dupont (*4*) simulated falling droplets by means of the Navier-Stokes equations and showed that their solutions agreed with the experimental shapes of water photographed by Peregrine *et al.* (*5*).

Although our study focuses on a specific experimental system, the dynamics near a singularity should be insensitive to changes in the initial conditions and external forcing within a wide range of parameters. Thus, the shape of the interface near the breakup point should not depend on whether the drop is falling in a gravitational field or being pulled apart by shear forces (*6*). This expectation arises from the realization that as the interface breaks, its thickness must eventually become much smaller than any other length in the problem (until microscopic atomic scales are reached). In this regime, the only length (*7*) that can affect the dynamics is the thickness of the fluid itself, so that the simplest assumption for the dynamics is self-similarity—that is, the shape near the singularity changes in time only by a change in scale. The mathematical definition of a similarity solution is

$$h(z,t) = f(t)H\left[\frac{z - z_0}{f(t)^{\beta}}\right] \qquad (1)$$

where h describes the radius of the drop as a function of the vertical coordinate z and time t; $f(t)$ is an arbitrary function of time; β is a constant; and z_0 is the position where the droplet breaks. This scaling hypothesis has worked well in describing other types of singularities, ranging from critical phenomena (*8*) to droplet breakup in a two-dimensional Hele-Shaw cell (*9, 10*). Also, for our problem here Eggers (*11*) constructed a similarity solution that showed good agreement with numerical solutions to the Navier-Stokes equation at low viscosities. On the other hand, Pumir, Siggia, and co-workers (*12*) have analyzed several mathematical models of singularities with a high Reynolds number that showed nonsteady corrections to Eq. 1. However, to the best of our knowledge those models did not have an experimental realization. For the case of the dripping faucet, we will show that both views have some validity: the similarity solution provides a basis for the underlying structure, but there are time-dependent corrections to this solution that alter the shape dramatically.

There are three independent length scales that characterize the hydrodynamics of the dripping faucet (*4*): (i) the diameter of the nozzle D; (ii) the capillary length

$$L_{\gamma} = \left(\frac{\gamma}{\rho g}\right)^{1/2}$$

which gives the balance between surface tension, γ, and the gravitational force ρg, where ρ is the fluid density and g is the acceleration of gravity; and (iii) the viscous length scale

$$L_{\eta} = \frac{\eta^2}{\rho\gamma}$$

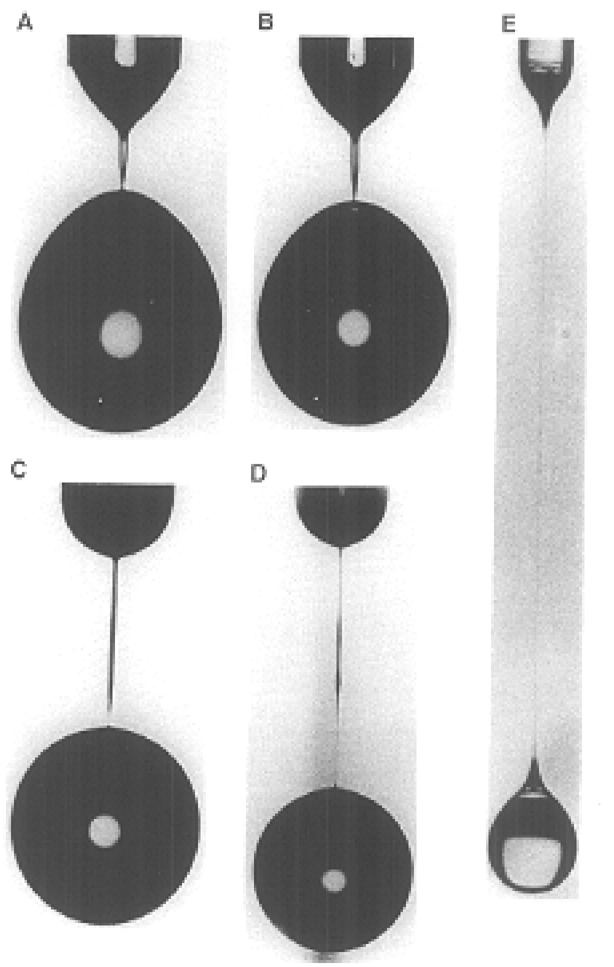

Fig. 1. The shape close to the time of breakup of five drops with different viscosities. The liquids, with $\eta = 10^{-2}$ P (**A**), 10^{-1} P (**B**), 1 P (**C**), 2 P (**D**), and 12 P (**E**), were allowed to drain slowly through a glass tube with a nozzle diameter D of 1.5 mm. (A) and (E) show pure water and pure glycerol, respectively. We used an 80-mm Hasselblad lens attached to a bellows and a still camera; the drop was illuminated from behind by a fast (≈5 μs) flash from a strobe (EG&G model MVS 2601, Salem, Massachusetts) that was triggered with a variable delay from the time the drop intersected a laser beam incident on a photodiode.

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