the equator. Convergence is best for the thermocline deviation on the equator. Application of the eastern boundary condition and equating term by term to zero gives

$$
\begin{aligned}
(r_0 + b_0)e^{-\phi(1-x_0)} &= \pi^{\frac{1}{4}} h_E^G \\
b_{2j+1} e^{\phi(4j+3)(1-x_0)} &= 2\alpha_{2j+1}\, \pi^{\frac{1}{4}} h_E^G \\
b_{2j} &= 0
\end{aligned}
$$

from which the coefficients b_j can be solved. Eventually, the complete solution to the pulse forcing at $x = x_0$, i.e. the Green's function for the problem, is found as

$$\mathbf{G}(x, y, \phi; x_0) = \pi^{\frac{1}{4}} h_E^G \mathbf{K}(\phi(1-x), y) - \mathbf{L}(\phi(x_0 - x), y)\mathcal{H}(x_0 - x) \quad (7.45)$$

where vector functions $\mathbf{K}$ and $\mathbf{L}$ are defined as

$$\mathbf{K}(\eta, y) = e^{\eta}\mathbf{\Phi}_0(y) + 2\sum_{j=0}^{\infty} \alpha_{2j+1} e^{-\eta(4j+3)} \mathbf{\Phi}_{2j+1}(y) \quad (7.46a)$$

$$\mathbf{L}(\eta, y) = r_0 e^{\eta}\mathbf{\Phi}_0(y) - \sum_{j=0}^{\infty} (2j+1) r_j e^{-\eta(2j+1)} \mathbf{\Phi}_j(y) \quad (7.46b)$$

Up to this point, only the eastern boundary amplitude of the thermocline h_E^G is still unknown, but it can be determined from the western boundary condition (7.33) and becomes

$$\pi^{\frac{1}{4}} h_E^G(\phi; x_0) = \frac{\int_{-\infty}^{\infty} L_u(\phi x_0, y)dy}{\int_{-\infty}^{\infty} K_u(\phi, y)dy} \quad (7.47)$$

where K_u and L_u are the first components of $\mathbf{K}$ and $\mathbf{L}$, respectively. This completes the basic machinery needed in the next sections to understand the response of the ocean to varying wind stress forcing.

7.3. Physics of Coupling

Anomalies in sea surface temperature somehow manage to change the winds, and in the first subsection a model is sketched how to compute the low level wind response due to SST anomalies. Next, wind stress anomalies induce changes in the ocean circulation and examples are shown in 7.3.2, using the results of Technical box 7.1. Finally, a model is considered in 7.3.3 to determine how changes in ocean circulation induce SST anomalies.

7.3.1. Atmospheric response to diabatic heating

A class of simple models to analyse the low level wind response due to heating anomalies in the tropics was proposed by Matsuno (1966). These models are also of shallow water type, following the same approach as in section 7.2. The steady response of one of these models was analysed in detail in Gill (1980) and since then, this type of model is referred to as a Gill model. The equations are

$$\frac{\partial U_*}{\partial t_*} - \beta_0 y_* V_* - \frac{\partial \Theta_*}{\partial x_*} + a_M U_* = 0 \tag{7.48a}$$

$$\frac{\partial V_*}{\partial t_*} + \beta_0 y_* U_* - \frac{\partial \Theta_*}{\partial y_*} + a_M V_* = 0 \tag{7.48b}$$

$$\frac{\partial \Theta_*}{\partial t_*} - c_a^2\left(\frac{\partial U_*}{\partial x_*} + \frac{\partial V_*}{\partial y_*}\right) + a_M \Theta_* = Q_* \tag{7.48c}$$

where (U_*, V_*) are the low level winds, Θ_* the geopotential height (with dimension m^2/s^2), a_M is a damping coefficient and c_a is the phase speed of the first baroclinic Kelvin wave in the atmosphere. The flow is forced by a representation of the adiabatic heating term Q_* (having dimension m^2/s^3). Note the similarities with the reduced gravity ocean model with the difference being in the forcing terms. More accurate derivations of these type of models can be found in Holton (1992).

To study the response, it is convenient to scale the equations with

$$t_* = \frac{L}{c_o} t \; ; \; x_* = Lx \; ; \; y_* = \lambda_a y \tag{7.49a}$$

$$\Theta_* = c_a^2 \Theta \; ; \; U_* = c_a U \; ; \; V_* = \frac{\lambda_a}{L} c_a V \tag{7.49b}$$

$$Q_* = q_0 Q \; ; \; \lambda_a = \sqrt{\frac{c_a}{2\beta_0}} \tag{7.49c}$$

Note that the factor 2 in the definition of λ_a is different from the scaling of the ocean model. On the other hand, already anticipating coupling, the time is scaled with the advective time scale in the ocean. The dimensionless equations become

$$c\frac{\partial U}{\partial t} - \frac{y}{2} V - \frac{\partial \Theta}{\partial x} + \epsilon_a U = 0 \tag{7.50a}$$

$$c\zeta_a^2 \frac{\partial V}{\partial t} + \frac{y}{2} U - \frac{\partial \Theta}{\partial y} + \zeta_a \epsilon_a V = 0 \tag{7.50b}$$

$$c\frac{\partial \Theta}{\partial t} - \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) + \epsilon_a \Theta = \mu_0 Q \tag{7.50c}$$

with $\epsilon_a = a_M L/c_a$, $\mu_0 = q_0 L/c_a^3$, $c = c_o/c_a$ and $\zeta_a = \lambda_a/L$. Of these parameters, both c and ζ_a are small and to a good approximation, the atmospheric time derivatives can be neglected, as well as the damping in the meridional momentum balance. All fields must be bounded far from the equator. The solution of this linear problem is provided in Technical box 7.2.

Technical box 7.2: Solution of the Gill model

With $c \to 0$, $\zeta_a \to 0$ and by introducing new independent variables $S = \Theta + U$ and $R = \Theta - U$ the problem (7.50) becomes

$$-(\frac{y}{2}V + \frac{\partial V}{\partial y}) - \frac{\partial S}{\partial x} + \epsilon_a S = \mu_0 Q \quad (7.51a)$$

$$\frac{y}{2}(S - R) - (\frac{\partial S}{\partial y} + \frac{\partial R}{\partial y}) = 0 \quad (7.51b)$$

$$\frac{y}{2}V - \frac{\partial V}{\partial y} + \frac{\partial R}{\partial x} + \epsilon_a R = \mu_0 Q \quad (7.51c)$$

Subsequently, the variables S, R and the forcing Q are expanded into parabolic cylinderfunctions $D_n(y)$ with coefficients depending on x,

$$Q(x,y,t) = \sum_{n=0}^{\infty} Q_n(x) D_n(y) e^{i\omega t} \quad (7.52a)$$

$$R(x,y,t) = \sum_{n=0}^{\infty} R_n(x) D_n(y) e^{i\omega t} \quad (7.52b)$$

$$S(x,y,t) = \sum_{n=0}^{\infty} S_n(x) D_n(y) e^{i\omega t} \quad (7.52c)$$

where a periodic time dependence in the forcing has been assumed with frequency ω. The parabolic cylinderfunctions $D_n(y)$ are related to the Hermite polynomials through

$$D_n(y) = 2^{\frac{-n}{2}} e^{-\frac{y^2}{4}} H_n(\frac{y}{\sqrt{2}}) \quad (7.53)$$

For all n, the relations

$$\frac{y}{2}D_n + D_n' = nD_{n-1} \; ; \; \frac{y}{2}D_n - D_n' = D_{n+1} \quad (7.54)$$

are valid. Substitution of the expansions (7.52) into the equations (7.51) gives a system of ordinary differential equations for the coefficient functions S_n and R_n. For $n = 0$,

$$\epsilon_a R_0 + R_0' - \mu_0 Q_0 = 0 \quad (7.55a)$$

$$R_1 = 0 \quad (7.55b)$$

$$\epsilon_a S_0 - S_0' - V_1 - \mu_0 Q_0 = 0 \quad (7.55c)$$

from which R_0 and R_1 are directly determined. For $n = 1$, one obtains

$$\begin{aligned} \epsilon_a S_1 - S_1' - 2V_2 - \mu_0 Q_1 &= 0 & (7.56a)\\ 2R_2 &= S_0 & (7.56b)\\ V_0 - \mu_0 Q_1 &= 0 & (7.56c) \end{aligned}$$

from which V_0 directly follows. For $n > 1$, the equations become

$$\begin{aligned} \epsilon_a S_n - S_n' - (n+1)V_{n+1} - \mu_0 Q_n &= 0 & (7.57a)\\ \epsilon_a R_n + R_n' + V_{n-1} - \mu_0 Q_n &= 0 & (7.57b)\\ (n+1)R_{n+1} - S_{n-1} &= 0 & (7.57c) \end{aligned}$$

Using (7.57c) to eliminate the terms involving S_n in (7.57a) and adding the results to (7.57b) for $n \to n+2$ gives a single equation for R_{n+2}, $n > 0$, i.e.

$$(2n+3)\epsilon_a R_{n+2} - R_{n+2}' - \mu_0(Q_n + (n+1)Q_{n+2}) = 0 \qquad (7.58)$$

from which R_{n+2} and eventually the total solution for U, V and Θ can be calculated. The results for U and Θ are

$$\begin{aligned} U(x,y,t) = & \frac{e^{i\omega t}}{2}[(2R_2(x) - R_0(x))D_0(y) + 3R_3(x)D_1(y)]\\ + & \frac{e^{i\omega t}}{2}\left[\sum_{n=2}^{\infty}((n+2)R_{n+2}(x) - R_n(x))D_n(y)\right] \qquad (7.59a)\\ \Theta(x,y,t) = & \frac{e^{i\omega t}}{2}[(R_0(x) + 2R_2(x))D_0(y) + 3R_3(x)D_1(y)]\\ + & \frac{e^{i\omega t}}{2}\left[\sum_{n=2}^{\infty}((n+2)R_{n+2}(x) + R_n(x))D_n(y)\right] \qquad (7.59b) \end{aligned}$$

where

$$\begin{aligned} R_0(x) &= \mu_0 \int_0^x e^{-\epsilon_a(x-s)} Q_0(s)\, ds\\ R_1(x) &= 0 \end{aligned}$$

$$R_{n+2}(x) = \mu_0 \int_x^1 e^{(2n+3)\epsilon_a(x-s)}((n+1)Q_{n+2}(s) + Q_n(s))\, ds$$

for $n = 0, 1, \cdots,$. This completes the full solution of the Gill model.

An example of the steady response of the Gill model is considered with the forcing described by

$$Q(x,y) = \psi_0(\frac{\lambda_a}{\lambda_o}y)\ \sin\pi x\ ;\ \omega = 0 \qquad (7.61)$$

with ψ_0 the Hermite function defined in (7.18). Note that since $y = y_*/\lambda_a$, the argument in the Hermite function is y_*/λ_o and hence the meridional scale of the forcing is the Rossby radius of deformation of the ocean. With $c_a = 30\ m/s$, $c_o = 2\ m/s$, the ratio of the Rossby deformation radii of atmosphere and ocean is about 3 ($\lambda_a \approx 826\ km$).

The forcing (7.61) is shown Fig. 7.15a and in subsequent panels, the stationary ($\omega = 0$) zonal wind response (7.59a) is plotted for $\epsilon_a = 0, 2.5$ and 5.0. For each value of ϵ_a, there are westerly (easterly) winds to the west (east) of the maximum heating. The signal west of the heating maximum is mainly due to Rossby waves, while that to the east is due to the Kelvin wave. The zonal wind response becomes more local as the value of ϵ_a increases. This is also clear physically, since ϵ_a is a ratio of the basin length L and an atmospheric damping length scale c_a/a_M. When the damping a_M increases, the length scale over which anomalies are damped decreases and hence the response is more localized to the forcing.

An approximation to the equatorial zonal wind response U is obtained by truncating the solution (7.59a) for only the first three parabolic cylinderfunctions, i.e. with $D_0(0) = 1, D_1(0) = 0, D_2(0) = -1$, the equation (7.59a) gives

$$U(x, 0, t) = e^{i\omega t}(\frac{3}{2}R_2(x) - \frac{1}{2}R_0(x)) \qquad (7.62)$$

where the R_4 contribution is also neglected. As we will show later on, this expression turns out to be useful when considering reduced models which only take the equatorial response into account.

When the diabatic heating structure is known, the low level wind response can be computed from the Gill model. However, this leaves the problem to relate the diabatic heating structure and the SST anomalies. The simplest connection (Zebiak, 1982) is that convection mostly occurs over the warmest water which leads to a direct coupling with SST anomalies $\tilde{T}_*$ and those in latent heat $\tilde{Q}_*$ through

$$\tilde{Q}_* = \alpha_T \tilde{T}_* \qquad (7.63)$$

with some constant coefficient α_T (with dimension $m^2/(s^3 K)$). If a typical scale of the temperature anomaly is ΔT, then $q_0 \doteq \alpha_T \Delta T$. The dimensionless parameter measuring the amount of heating per SST anomaly is then given by

$$\mu_0 = \frac{\alpha_T \Delta T L}{c_a^3} \qquad (7.64)$$

which will be part of the main coupling parameter introduced in the ocean-atmosphere model in subsequent sections. The relation between SST anomalies and diabatic forcing above is far from perfect and many improvements based on detailed atmospheric modelling have been suggested (see e.g., Neelin et al. (1998) and references therein).

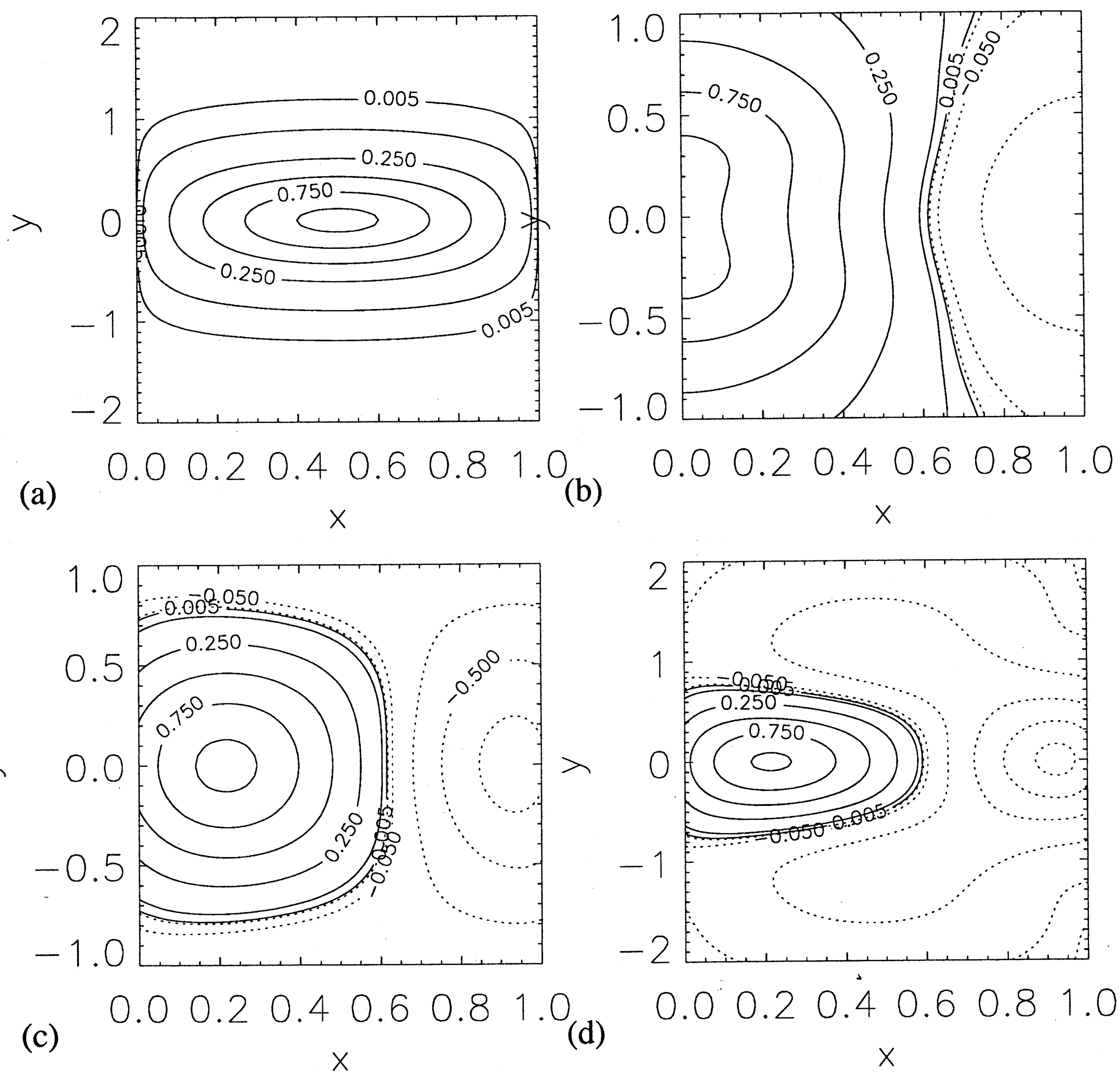

Figure 7.15. *(a) Pattern of the diabatic forcing $Q(x,y)$ given by (7.61). The zonal wind response (7.59b) is plotted for three different values of ϵ_a in subsequent panels, (b) $\epsilon_a = 0$, (c) $\epsilon_a = 2.5$ and (d) $\epsilon_a = 5.0$. Note that y is scaled with $\lambda_a = 826$ km and that x is scaled with the basin length $L = 1.5 \times 10^4$ km. The zonal velocity is scaled with $c_a = 30$ m/s and the factor $\mu_0 = 1$.*

7.3.2. Adjustment of the ocean

The low level surface winds exert a wind stress on the ocean surface according to the bulk formula

$$(\tau^x, \tau^y) = C_d \rho_a |\mathbf{U}| \mathbf{U} \tag{7.65}$$

where C_d is the drag coefficient, ρ_a the density of air and $\mathbf{U} = (U, V)$. Considering perturbations $\tilde{\mathbf{U}}$ from some reference state $\bar{\mathbf{U}}$, the perturbation wind stress can be taken proportional to the perturbation velocity in the lower atmospheric layer,