7.3 Equivalent potential temperature/ moist static energy and convective instability

The moist stability is determined by following a parcel raised from the ground level as shown in Figure 2. Initially, the parcel could be heated by the ground/ ocean, and rises spontaneously, or it could just be forced up by large-scale boundary layer turbulent motions. As the parcel rises, it initially cools adiabatically. It reaches saturation at its Lift Condensation Level (LCL). At this point condensation starts, and now the parcel cools less rapidly as it is raised because of the release of latent heat by the condensing vapor. When the parcel temperature, increased by the latent heat release, reaches that of the environment, the two densities are equal. This is the level of free convection: a further lifting of the parcel will lead to further latent heat release, its temperature will be warmer than that of the environment, and free convection can finally start.

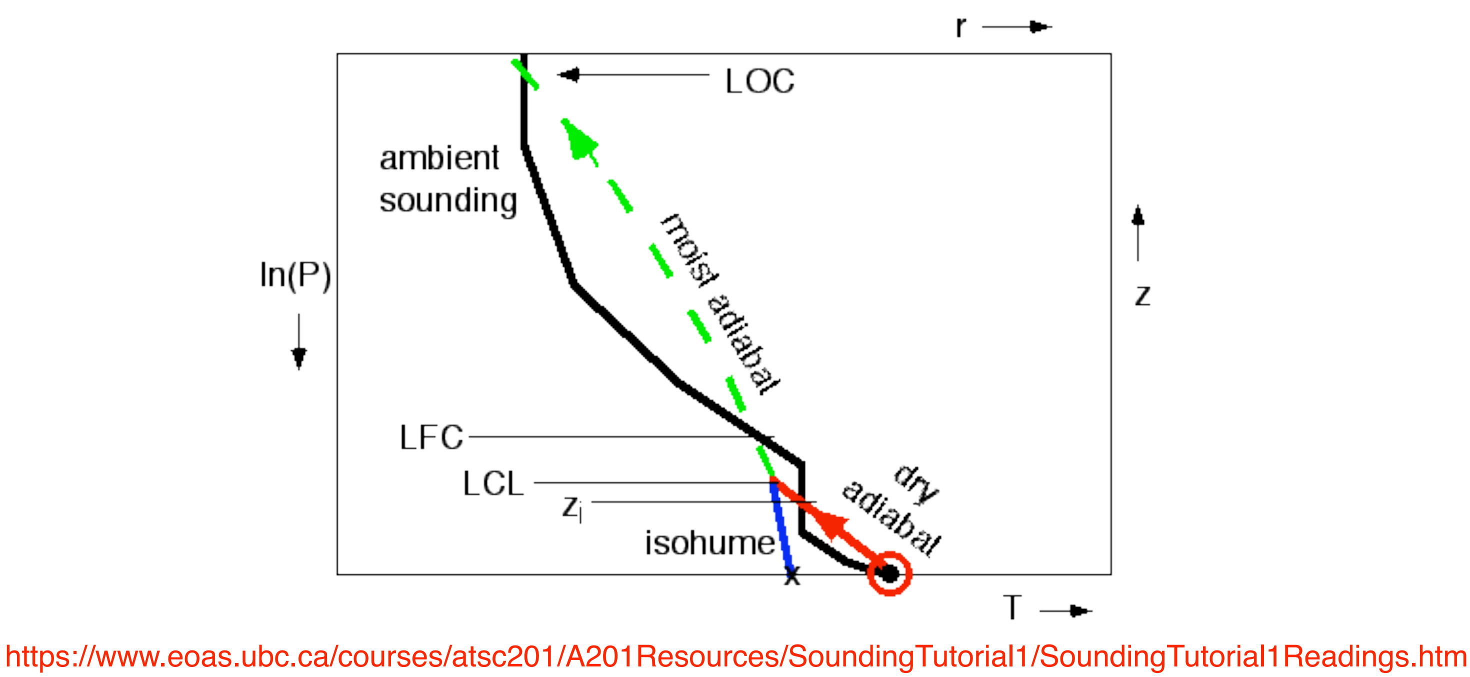

Figure 2: A rising air parcel example: rises initially by its own buoyancy along a dry adiabat (red) to the top of the boundary layer (z_i). From that point it needs to overcome the stability from z_i to the LFC by being mechanically forced upward by boundary layer turbulence, for example; along the way becomes saturated at LCL and rises from there along a moist adiabat (green). It keeps rising due to convective instability starting at the LFC (Figure from UBC).

The level at which convection starts (LFC) is where the parcel density reaches that of the environment. Instead of comparing densities, we can compare the temperature of the lifted parcel and of the environment, because their pressure is the same, so that the density is a function of the temperature only. Instead, again, we can compare the saturation equivalent potential temperature (or the saturation MSE) of the parcel and of the environment, as these are a function of temperature only. Note that because the parcel's $\theta_e(z)$ (or its MSE) is a function of both the temperature and mixing ratio at z, we cannot simply compare the parcel equivalent potential temperature to the equivalent potential temperature $\theta_e^{env}(z)$ of the environment at the new level in order to determine their density difference.

We showed above that θ_e and MSE are conserved for a rising parcel even in the presence of condensation. That's because as the parcel rises, it looses internal energy in the form of Lq, but gains it due to the release of heat of condensation which increases c_pT, keeping the parcel's total MSE constant. The parcel will be saturated once it reaches the LCL, so that the θ_e/ MSE of the parcel at the new level are at their saturation values and are equal to the θ_e/ MSE at the surface where the parcel was lifted from,

$$\theta_e^{parcel}(z) = \theta_e^{*,parcel}(z) = \theta_e^{parcel}(z=0). \tag{97}$$

or equivalently,

$$MSE^{parcel}(z) = MSE^{*,parcel}(z) = MSE^{parcel}(z=0).$$

Given this, we can diagnose the temperature difference between the parcel and environment at a level z above the LCL, by comparing the saturation environment equivalent potential temperature (or saturation environment MSE) at z, to the equivalent temperature (or MSE) at the surface.

Summary: in order to know if a parcel that is currently at the surface will start free convection if raised to a level z above the LCL, we need to compare its density once lifted to that level to the environment density at the same level. Instead, we can compare its temperature once lifted to that of the environment at that level. Instead again, we can compare the saturation equivalent potential temperature (or saturation MSE) of the parcel at the ground to that of the environment at level z. Given that θ_e and MSE are conserved for the parcel, we can therefore compare the equivalent potential temperature (or MSE) at the surface where the parcel was lifted from, to the saturation equivalent potential temperature of the environment (or to the saturation MSE of the environment) at z. Convection will occur if,

$$\theta_e^{*,env}(z) \leq \theta_e^{*,parcel}(z) = \theta_e^{parcel}(z) = \theta_e^{parcel}(z=0),$$

or equivalently,

$$MSE^{*,env}(z) \leq MSE^{*,parcel}(z) = MSE^{parcel}(z) = MSE^{parcel}(z=0).$$

Note that we are assuming that the level z is sufficiently high that the parcel is expected to be saturated once there, and therefore the equality $\theta_e^{*,parcel}(z) = \theta_e^{parcel}(z)$. This explains the usefulness of the equivalent potential temperature and MSE for diagnosing moist stability.