POPs, A brief reminder, for EPS231

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Consider linear system driven by zero-correlated noise,

$$\dot{\mathbf{P}} = A\mathbf{P} + \mathbf{f}(t)$$

solution is

$$\mathbf{P}(t) = e^{At}\mathbf{P}(0) + \int_0^t ds\, e^{A(t-s)}\mathbf{f}(s)$$

assume damped system, so the first term that is driven by the initial conditions decays. Use this solution to calculate the delayed covariance of the state vector,

$$\langle \mathbf{P}(t+\tau)\mathbf{P}(t)^T \rangle = \int_0^t ds \int_0^{t+\tau} ds'\, e^{A(2t+\tau-s-s')}\langle \mathbf{f}(s)\mathbf{f}(s')^T \rangle$$

using $\langle \mathbf{f}(s)\mathbf{f}(s')^T \rangle = I\sigma^2\delta(s-s')$,

$$C(\tau) = \langle \mathbf{P}(t+\tau)\mathbf{P}(t)^T \rangle = e^{A\tau}\int_0^t ds\, e^{A(2t-2s)}\sigma^2 = C(0)e^{A\tau}$$

therefore,

$$e^{A\tau} = C(0)^{-1}C(\tau).$$

This implies that the eigenvectors of A and of the normalized delayed-covariance matrix are the same, and the eigenvectors of the covariance matrix are $e^{\lambda_i \tau}$ where λ_i are those of A. Therefore one can reconstruct the dynamic operator from the delayed covariance matrix of the state vector.

When the above analysis is carried out in the reduced space of the EOFs of the system, such that $\mathbf{P}(t)$ contains the amplitudes of a few of these EOFs, then the eigenvectors of A are referred to as principal oscillatory patterns (POPs, von Storch et al., 1988; Penland, 1989). Unlike the EOFs, they reflect the actual dynamical operator.

References

Penland, C. (1989). Random forcing and forecasting using principal oscillation pattern analysis. *Mon. Weath. Rev.*, 117:2165–2185.

von Storch, H., Bruns, T., Fischer-Bruns, I., and Hasselmann, K. (1988). Principal Oscillation Pattern analysis of the 30-60 day oscillation in a GCM equatorial troposphere. *J. Geophys. Res.*, 93:11015–11021.