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Reviewed work(s):
Source: *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. 197, No. 1051 (Jul. 7, 1949), pp. 433-453
Published by: The Royal Society
Stable URL: http://www.jstor.org/stable/98250
Accessed: 11/12/2012 12:04

The magnetic field within the earth

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(*Received* 23 *November* 1948)

The paper discusses the magnetic effects of internal motions in the core of the earth. It is shown that tidal friction, fluctuations in the rate of rotation, nutation, and the variation of latitude have negligible magnetic effects. Precession is also ineffective if Poincaré's theorem on the precession of a liquid sphere in a rigid shell is applicable to the earth.

Thermal convection is shown to be likely to occur in the core. The conservation of angular momentum will require it to be associated with a radial gradient of angular velocity which will have a large magnetic effect. Its interaction with the dipole field can produce a toroidal field which is many times as intense as the dipole field.

The convective and rotational motions can interact with the dipole and toroidal field in a way that tends to reproduce the dipole field. The complete theory has not been worked out, but it seems likely that the interaction is strong enough to maintain the field. The whole process resembles that occurring in a self-exciting dynamo.

The existence of a strong field in the core removes the difficulties previously found in the theory of the secular variation.

1. Introduction

Many years ago Larmor (1919) suggested that the magnetic field of the earth might be maintained in a way analogous to that of a dynamo. He supposed that motions in the interior interact with the field to produce the electric currents necessary to maintain the field. In the 30 years since Larmor made this suggestion knowledge of the earth's interior has advanced, and it is now certain that there is a central liquid core having a radius of about half that of the earth. It is probable that this core is composed of molten iron and can thus provide the conducting medium required by Larmor.

To establish such a theory of the main field it is necessary to show that the motion produced by known agencies is of a kind that can maintain a field. This is the main problem of the present paper.

A clue to what is required is provided by the secular variation. A simple and natural explanation of this can be given if it is assumed to be due to electromagnetic induction by motions in the outer part of the core (Elsasser 1946; Bullard 1948, referred to hereafter as I). It is shown in I that such an explanation is only possible if the field in the core is substantially greater than that which would be obtained by applying the inverse cube law to the field observed at the surface. This makes it likely that a large field in the interior of the core is a necessary part of the mechanism that maintains the external field.

It has been pointed out by Elsasser (1947) that a large field can be produced in the core by the relative rotation of its parts. This is discussed quantitatively in § 2. In §§ 3, 4 and 5 possible astronomical causes of such a motion are discussed. The results are entirely negative, all the apparently adequate astronomical causes proving incapable of maintaining a sufficient differential rotation in the presence of the electromagnetic forces.

In § 6 the possibility of thermal convection in the core is examined. Convection will certainly occur if the temperature gradient required to transport the heat generated by radioactivity exceeds the adiabatic gradient. The rate of generation of heat required for this is about 1 % of that occurring in the surface rocks and is of the same order as that in iron meteorites. It is found that the Corioli forces associated with convection would produce a radial variation in angular velocity of the kind required to give a large field in the core by interaction with the dipole field.

In § 8 the further interactions of the convective motion and the field are discussed, and it is shown to be probable that they will reproduce the main field, and thus maintain it. The calculations needed to render the argument complete are outlined.

2. The conditions for a toroidal field

If a conducting sphere rotates in a magnetic field parallel to its axis, an electromotive force is produced directed at right angles to the axis of rotation. If the sphere rotates like a rigid body this produces no electric current. If, however, the angular velocity is not the same at all distances from the centre currents can flow. A magnetic field is then produced that runs along parallels of latitude, and circles the axis in opposite directions in the two hemispheres. This field is entirely confined within the sphere, and it is zero on the axis and in the equatorial plane. It is convenient to call it the toroidal field.

Elsasser (1946*a*, *b*, 1947) has discussed induction in a sphere with great generality and elegance. He does not, however, discuss particular models in detail. The purpose of this section is to take the simplest model of the core that will give the toroidal field and to determine the conditions necessary for the dipole field of the earth to give a toroidal field ten times as great as itself.

The model taken is one in which the outer and inner parts of the core each rotate like rigid bodies, but with slightly differing angular velocities. The details of the solution of this electromagnetic problem are somewhat complicated and are given in a separate paper (Bullard 1949, referred to as III).

Until its origin is completely understood the field within the core is uncertain. There is therefore a difficulty in knowing what field to take as the inducing field. Fortunately, the field deep in the core has only a slight effect, and it is possible to proceed by making extreme assumptions between which the truth must lie. The field outside the core is taken to be that of the centred dipole that best represents the field observed at the surface of the earth. Within the core we assume, first that the field is also that due to the centred dipole, and second that it is constant. Neither of these assumptions is plausible physically; they are intended to represent two extreme forms of field giving the same result at the surface of the core. One increases indefinitely as the centre is approached, the other is uniform throughout the core. Since both give the same result the uncertainty due to the lack of a theory of the main field is greatly reduced.

It may be shown (III, §§ 6 and 7) that if a sphere rotates in a uniform field H_0, the toroidal field produced in colatitude θ at the boundary between the rotating inner part of the core and the conducting shell is*

$$H_\phi = \tfrac{2}{5}\pi\kappa b v H_0(1-b^5/a^5)\sin 2\theta, \tag{1}$$

* If H_0 is the field on the axis at radius b the same expression gives the toroidal field due to a dipole at the centre.

where b is the radius of the inner sphere, a the outer radius of the shell (that is, the radius of the core, 3473 km.), v is the maximum relative velocity of the sphere and the shell, and κ is the electrical conductivity of both. H_0 will be taken as 4 gauss, the value of the dipole field at the surface of the core. For the conductivity we take 3×10^{-6} e.m.u. (3000 ohm^{-1} cm.$^{-1}$); this is the mean of the values (5000 and 1000 ohm^{-1} cm.$^{-1}$) deduced by different methods by Elsasser and the author, and is probably uncertain by a factor of about 3.

To remove the difficulties in the theory of the secular variation H_ϕ must be substantially greater than H_0, but its exact value is of no great importance. To obtain an order of magnitude for the rate of rotation required we take $H_\phi(\text{max.})/H_0 = 10$. The relative velocity needed can then be calculated as a function of b from (1). The results, expressed both as an angular and as a linear velocity at the surface of the rotating sphere, are shown in figure 1. It will be seen that the minimum velocity of 0·013 cm./sec. occurs when the conducting shell has a thickness of about 1000 km. The velocity is very insensitive to variations in this thickness, and for values between 210 to 2600 km. the velocity is in the range 0·013 to 0·03 cm./sec.

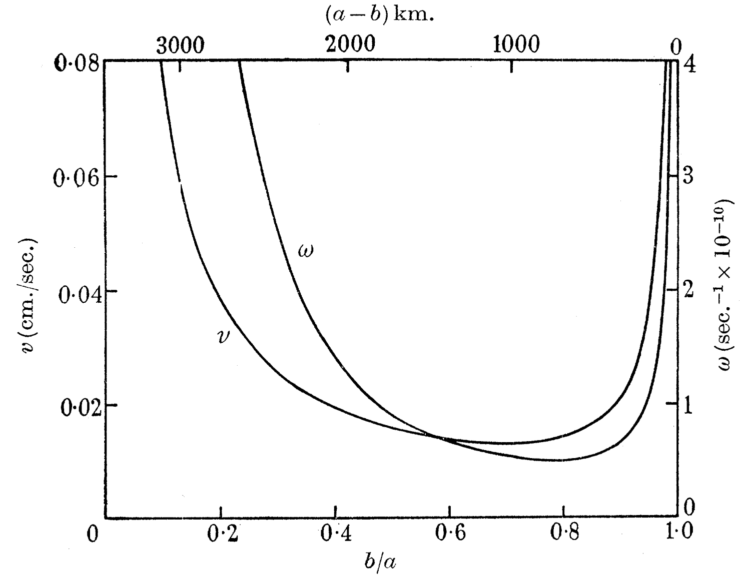


FIGURE 1. Linear and angular velocities necessary to produce a toroidal field of 40 gauss.

For a sphere of radius equal to that of the whole core rotating in an infinite conducting medium the peripheral velocity needed to give $H_\phi/H_0 = 10$ would be 0·0076 cm./sec., which is only about three times less than the velocities found in the last paragraph. Thus

$$H_\phi(\text{max.}) = 2\pi\kappa v a H_0/15 \tag{2}$$

may conveniently be taken as a rough approximation to H_ϕ except when $(a-b)$ is very small or very large. For shells thinner than 200 km. the velocity required increases rapidly and is given approximately by

$$v = 5{\cdot}3/(a-b),$$

where v is in cm./sec. and $(a-b)$ in km.

The conditions for the production of a toroidal field of 40 gauss are therefore quite definite. The inner part of the core must rotate relative to the outer. If the outer part is more than 200 km. thick a relative angular velocity of less than 10^{-10} sec.$^{-1}$ and a relative linear velocity of less than 0·03 cm./sec. will suffice. If the shell is thinner than 200 km. the velocity must be increased and reaches 1 cm./sec. for a thickness of about 5 km. The upper limit to the thickness of the shell will not concern us, as the difficulty is to get a thick enough shell rather than to avoid too thick a one. In making these calculations it has been assumed that the change from the shell to the inner part of the core is discontinuous. This is an analytical convenience, but it is certain that the substitution of a gradual change in velocity with radius would not affect the results in any important way. A gradual change is, of course, to be expected in a liquid.

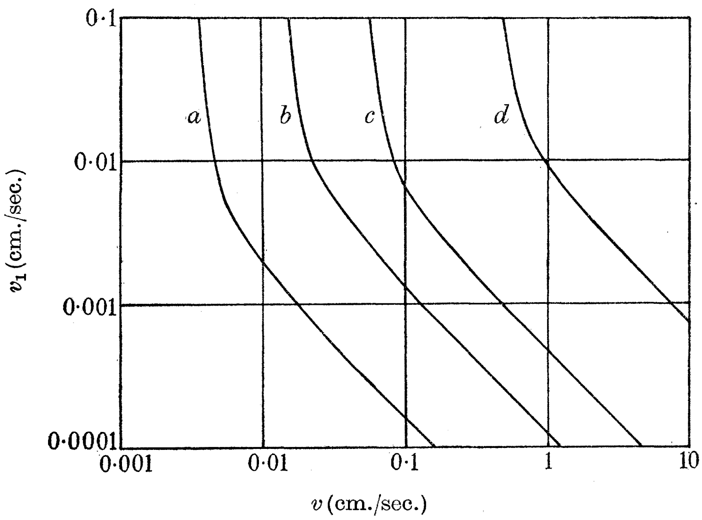


FIGURE 2. Velocities necessary to account for the secular variation. Curve a, cylinder, $a_1 = 400$ km.; curve c, sphere, $a_1 = 400$ km.; curve b, cylinder, $a_1 = 200$ km.; curve d, sphere, $a_1 = 200$ km.

The velocities necessary to produce the observed secular variation can be more closely investigated. It has been shown in I that the secular variation in South Africa during the last 100 years requires a dipole of moment $M = 2{\cdot}0 \times 10^{24}$ gauss cm.^{3} in latitude 25° S., longitude 20° E. or, alternatively, a long line of dipoles stretching from north-east to south-west through this point and having a moment of $3{\cdot}4 \times 10^{15}$ gauss cm.^{3}/cm. of its length. If the dipole is to be produced by the rotation of a sphere of radius a_1 with a peripheral velocity v_1, in a field H_ϕ, then

$$M = H_\phi a_1^3 f(\rho),$$

where $\rho^2 = 4\pi\kappa v_1 a_1$ and $f(\rho)$ is the function given in table 4 and figure 1 of III. If H_ϕ is taken as $(2/15)\,\pi v a H_0 \sin 2\Phi$, where Φ is the geomagnetic latitude, v_1 may be calculated for any assumed values of v and a_1. Similar relations hold for a cylinder. Typical results are shown in figures 2a and 2b. As would be expected a large v can be combined with a small v_1 or both may be of the same order of magnitude. There is, however, a lower limit below which v must not fall. For a sphere of radius 400 km.

the limit is 0·05 cm./sec., for a cylinder it is 0·0032 cm./sec. If v and v_1 are of the same order of magnitude both must be about 0·06 cm./sec. for a spherical eddy or 0·005 cm./sec. for a cylindrical eddy each 400 km. in radius. These figures may be somewhat underestimated, as no allowance has been made for the skin effect in the changing field of the secular variation (see I, § 8).

Velocities of the same order of magnitude will account for the widespread anomalies that constitute the 'non-dipole' part of the main field, for this field is of the same general magnitude as the secular variation summed over a hundred years.

When an attempt is made to fit the direction as well as the magnitude of the secular variation and non-dipole fields difficulties are found; these are discussed in §§ 7 and 8 following the development of the theory of the rotation of the core.

3. Tidal friction

The most obvious cause of a rotation of the inside of the core relative to the outer part is the slowing down of the whole earth by tidal friction. It might be expected that the slowing of the core would lag behind that of the solid part, and that the inner part of the core would, therefore, rotate more quickly than the rest. A boundary layer of the liquid core would be expected to cling to the outer solid part of the earth and to rotate with it. The theory of this process has been worked out in detail by Bondi & Lyttleton (1948) on the assumption that viscosity is the only force that has to be taken into account. They find that there is a boundary layer of thickness $\sqrt{(\nu/\Omega \,|\cos\theta\,|)}$, where ν is the kinematic viscosity, Ω is the angular velocity of rotation of the earth ($7{\cdot}29 \times 10^{-5}$ radian/sec.) and θ is the colatitude. The part of the core inside the boundary layer rotates with an angular velocity which varies radially. The maximum angular velocity occurs on the axis and is $\dot{\Omega}a/\sqrt{(\nu\Omega)}$ above that of the outer part of the earth, where $\dot{\Omega}$ is the secular deceleration of the earth ($\dot{\Omega} = 2{\cdot}2 \times 10^{-22}\,\text{sec.}^{-2}$) and a is the radius of the core. The maximum linear velocity occurs at a radius $a/\sqrt{2}$ and is $\dot{\Omega}a^2/4\sqrt{(\nu\Omega)}$ above that for a rigid body. At the junction with the boundary layer the velocity falls to that characteristic of a rigid body rotating with the outer part of the earth. There is also a much slower motion towards the equatorial plane and away from the axis with a velocity of about $\dot{\Omega}a/\Omega$. In the boundary layer there is a drift from equator to poles with a velocity of about $\dot{\Omega}a^2/\sqrt{(\nu\Omega)}$. If we take* $\nu = 10^{-3}$ (I, § 9) these quantities become:

thickness of boundary layer	$= 4$ cm.
maximum angular velocity	$= 3 \times 10^{-10}\,\text{sec.}^{-1}$
maximum linear velocity	$= 0{\cdot}025$ cm./sec.
drift towards equator	$= 1{\cdot}0 \times 10^{-9}$ cm./sec.
drift to poles in boundary layer	$= 0{\cdot}10$ cm./sec.

The boundary layer in which the poleward motion occurs is too thin to have any perceptible magnetic effect. If its thickness is increased, as it might be if the laminar motion were unstable, the mean velocity in it would be reduced approximately in

* Bondi & Lyttleton take $\nu = 10^6$, a value which they obtain by extrapolating from laboratory conditions allowing for the effect of pressure, but neglecting the effect of temperature. Such a high value seems to me highly improbable.

proportion, and would be only 4×10^{-7} cm./sec. for a thickness of 10 km. The velocity towards the equatorial plane is so small as only to produce a motion of 100 km. since the Cambrian. It seems, therefore, that the motion in meridian planes can be ignored.

The motion of rotation is at first sight hopeful, as its velocity is of the required order of magnitude and its possible magnetic effects have been suggested by several workers in this field. The hopeful appearance is, however, deceptive as may be shown by considering the energy available. The rate of loss of energy by the core is $I\Omega\dot{\Omega}$, where I is the moment of inertia of the core. Putting $I = 8{\cdot}6 \times 10^{43}$ g.cm.^{2}, this gives $1{\cdot}4 \times 10^{18}$ erg/sec. The energy in the magnetic field is shown in III, § 6 to be about $0{\cdot}13a^3H_\phi^2$(max.). With $H_\phi = 40$ gauss this gives 9×10^{27} erg. This energy will need to be replenished in a time comparable with the time in which the field would decay to $1/e$ in the absence of rotation. This time is about $4a^2\kappa/\pi$ or $4{\cdot}6 \times 10^{11}$ sec. (14,000 yr.), which gives 2×10^{16} erg/sec. as the energy needed to maintain the field. If, therefore, 1·5 % of the energy lost by the core could be used to maintain the toroidal field we should have discovered an adequate source of power. This is, however, impossible, as all but a minute fraction of the energy lost by the core is transferred to the solid part of the earth. For consider a state in which the core is rotating faster than the outer part of the earth, and ignore for the moment the action of the moon. The rate of loss of energy by the core, $-\dot{E}_1$, is $-I\Omega\dot{\Omega}$, whilst the rate of loss of energy by the whole system, $-\dot{E}$, is

$$-\dot{E} = I_1\Omega_1\dot{\Omega}_1 + I\Omega\dot{\Omega},$$

where I_1 and Ω_1 are the moment of inertia and angular velocity of the solid part of the earth. This energy is that dissipated in the core. The conservation of angular momentum requires

$$I_1\dot{\Omega}_1 + I\dot{\Omega} = 0.$$

Thus
$$\dot{E} = I\dot{\Omega}(\Omega_1 - \Omega) = \dot{E}_1(\Omega_1 - \Omega)/\Omega,$$

since $(\Omega_1 - \Omega)/\Omega$ is a fraction of the order of 10^{-5} the proportion of the energy dissipated is much too small to be of any importance. The lunar forces exerted through the tides on the outer part of the earth will not affect this argument; they merely serve to absorb the energy transferred to the outer part of the earth.

The matter may be looked at from another point of view. The couple G tending to prevent relative rotation between the core and the outer part of the earth is (III, § 9)

$$G = \tfrac{4}{15}b^3H_0H_\phi(\text{max.}),$$

if $H_0 = 4$ gauss, $H = 40$ gauss, and $b = 3{\cdot}4 \times 10^8$ cm. then $G = 1{\cdot}8 \times 10^{27}$ dyne cm. The couple acting on the core is $I\dot{\Omega}$, which is $1{\cdot}9 \times 10^{22}$ dyne cm. and is smaller than G by a factor of about 10^5. Thus the electromagnetic forces compel the rotation of the core to follow the rest of the earth with only a very small lag, and the forces of tidal friction could only produce about 4×10^{-4} gauss of toroidal field.

Elsasser (1947, p. 831) has suggested that the inner part of the core may rotate slower than the outer part owing to the electromagnetic dissipation of tidal energy in it. The problem has not been worked out in detail, but it seems improbable that there can be a perceptible effect of this sort. It is shown in III, § 11 that a magnetic field extending throughout the core takes a time of the order of 14,000 years to establish itself. The periods of the tides are so short compared with this that only very small currents will be produced. In the somewhat analogous problem of the oscillating sphere treated in III, § 10 this reduced the field by a factor equal to the square root of the ratio of the period to the time of relaxation of the field. For a semi-diurnal tide this factor is 1/3000. The maximum velocity of the equilibrium lunar semi-diurnal tide at the surface of the core is 2×10^{-3} cm./sec.; there is, therefore, no large velocity to offset the unfavourable period. If induction by the tidal motion cannot produce an appreciable field the energy dissipated will be small, and the field produced indirectly, by the slowing of the core, will be smaller still. Viscous dissipation of tidal energy in the core is easily shown to be negligible.

4. Fluctuations in the earth's rotation

It has long been known that the angular velocity of the earth is not constant. The irregularities in its rate of rotation appear as apparent errors in the positions of the moon and planets. From a comparison of the results for different bodies it has been shown by Spencer Jones (1939) and de Sitter (1927) that the variations are almost wholly due to changes in the moment of inertia of the earth. It is noteworthy that these fluctuations are the only phenomenon with an origin inside the earth that has a time scale comparable with that of the secular variation; a connexion between the two is, therefore, not unreasonable.

Reliable observations extend from 1640 to the present day. The largest known sudden change occurred in 1897, when the day increased by about 0·0034 sec., corresponding to a decrease of about 3×10^{-12} sec.$^{-1}$ in the angular velocity of the earth. If the inner part of the core retained the angular velocity that it had before the change, a toroidal field of 3 gauss could be produced.

This is too small to be of much interest, but in view of the uncertainty in the true value of κ it is desirable to consider the matter in rather more detail. Suppose that the earth has been rotating for a very long time with a difference of angular velocity ω between the inner and outer parts of the core. If the couple maintaining the relative angular velocity is removed there will (III, § 9) be a decelerating electromagnetic couple

$$G = -\tfrac{4}{15}a^3 H_0 H_\phi \doteqdot -8\pi\kappa a^5 H_0^2 \omega/225; \tag{4}$$

H_ϕ will decay with a time constant $\tau_1 = 4\kappa b^2/\pi$, which is about 14,000 years. For times short compared to this the couple will be given by (4) and

$$I\dot{\omega} = -8\pi\kappa a^5 H_0^2 \omega/225.$$

The angular velocity therefore decays exponentially with a time constant

$$\tau_2 = 225 I/8\pi\kappa a^5 H_0^2 = 15D/\kappa H_0^2,$$

where D is the density. Putting $D = 10{\cdot}7$ g./cm., $\kappa = 3 \times 10^{-6}$ e.m.u. and $H_0 = 4$ gauss, we get

$$\tau_2 = 39 \text{ days}.$$

The initial conditions assumed in this problem are, however, not what is required for considering the effects of a sudden change in the earth's rotation. If the earth has been rotating for a long time as a rigid body there will be no toroidal field. After a sudden change ω in the angular velocity of the outer part, the toroidal field will grow with a time constant τ_1 provided the relative angular velocity is maintained. In fact ω will be reduced by electromagnetic forces. A detailed treatment of the interaction between the growing field and the decreasing rotation would be complicated, but limits to the time τ required to reduce the angular velocity to $1/e$ are easily obtained by noting that for $0 < t < \tau$ the toroidal field is less than if ω had been maintained at its initial value, but greater than if it had its value at $t = \tau$. The result is

$$\tau = \beta\sqrt{(\tau_1\tau_2)}, \quad \text{where} \quad 1 < \beta < 2;$$

if $\tau_1 = 14{,}000$ years and $\tau_2 = 39$ days this gives $\tau \doteqdot 39$ years. The field can never greatly exceed its value at this time, which is less than $\beta\sqrt{(\tau_2/\tau_1)}$ times the value it would reach if ω were maintained. Since $\sqrt{(\tau_2/\tau_1)}$ is only 0·0028, and even the equilibrium field is unpromisingly low, it is impossible that sudden changes in rotation of the order observed could have any appreciable effect on the magnetic field.

Even the large change in moment of inertia due to the melting of the ice at the close of the ice age has only a moderate effect. If this change is represented very crudely as the sudden addition to the earth of a shell of water of thickness q, the material being previously at the poles, the change δI in the moment of inertia is

$$\delta I/I = 2(MR^2/I)\,Dq/D_0R,$$

where D is the density of the water, D_0 the mean density of the earth and M and R the mass and mean density of the earth. Putting $q = 10^4$ cm. and $I/MR^2 = 0{\cdot}33$ this gives $\delta I/I = 1{\cdot}7\times10^{-5}$, and the change in the day as 1·5 sec. This is about 500 times greater than the sudden changes occurring at present and could produce a toroidal field of a few gauss. As the value taken for q is moderate and the assumptions made are very crude, the matter is perhaps worthy of further attention. We shall not discuss it further here.

The result that a sphere rotating freely in a conducting medium in the presence of a magnetic field will have its angular velocity reduced to $1/e$ in a time τ_2 if it has a fully developed toroidal field, and in a time $\beta\sqrt{(\tau_1\tau_2)}$ if it has none, throws some light on the nature of the movements possible for a fluid in a magnetic field. If rotatory motions are to persist for a time long compared to τ_1 and τ_2 they must be driven by forces, and not merely maintained by inertia. For large bodies both τ_1 and τ_2 are very short compared to the time needed for viscosity to slow down rotation. The latter time is about $0{\cdot}05a^2/\nu$, which is 160 million years for a sphere of radius 100 km. and kinematic viscosity $\nu = 0{\cdot}001$. Thus the electromagnetic forces will be a much more potent agency than viscosity in the prevention of turbulence.

It is possible, but not very likely, that part of the variation in the earth's rate of rotation is periodic. In particular, attention has been given to the representation of the errors in the moon's longitude by Newcomb's 'great empirical term'

$$10''{\cdot}71 \sin(140^\circ{\cdot}0T + 240^\circ{\cdot}7),$$

where T is measured in centuries from A.D. 1900. This term could be produced by a fluctuation

$$6{\cdot}3 \times 10^{-10} \cos(140^\circ{\cdot}0T + 240^\circ{\cdot}7) \text{ radians/sec.}$$

in the angular velocity of the earth. The period τ_0 of this term is 257 years which is much less than τ_1, the natural period of decay of the current system (about 14,000 years). It is shown in § 10 of III that if the inner part of the core is unaffected by the varying speed of rotation the toroidal field will have an amplitude $0{\cdot}225\sqrt{(\tau_0/\tau_1)} = 0{\cdot}030$ time that which would be produced by a steady difference of $6{\cdot}3 \times 10^{-10}$ sec.$^{-1}$ in angular velocity. From (2) above this gives a maximum toroidal field of 23 gauss. Since the period of 257 years is very large compared with the time of 39 days required for a fully developed toroidal field to reduce a difference of rotation to $1/e$, the inner part of the core will in fact follow closely the rotation of the outer part, and only a small fraction of the 23 gauss calculated above will be produced.

From these considerations it appears that neither the secular slowing of the earth's rotation, nor the sudden changes, nor the 'great empirical term' can have any important magnetic effects, in spite of the angular velocities being of the magnitude shown in § 2 to be necessary to produce a large toroidal field.

5. Precession and nutation

Poincaré has shown (Lamb 1932, p. 724) that the ellipticity of the earth is sufficient to ensure that the material of the core moves with the rest of the earth like a rigid body in a small motion of precession, even in the absence of viscosity. If this theorem is applicable to the earth, precession can have no magnetic effect. It has not been proved that the motion like a rigid body is stable, nor that it is possible for a precession of finite amplitude (the angle at the vertex of the cone swept out by the earth's axis is 47°). It is therefore not impossible that there may be internal motions associated with precession. If the inner part of the core failed to precess, a relative angular velocity of $3{\cdot}5 \times 10^{-5}$ sec.$^{-1}$ would be produced. This is 10^5 to 10^6 times what is required to produce a large field in the core. It is therefore clear that Poincaré's theorem must hold with great exactness if precession is not to have a large magnetic effect. We do not treat the hydrodynamical problem further here, as Bondi & Lyttleton state that they will shortly discuss it. The relative angular velocity would be roughly at right angles to the earth's axis and the field produced of the type discussed in § 6 of III. The electromagnetic forces will tend to prevent relative motion and will probably, as in § 4, prevent the field rising above $\sqrt{(\tau_2/\tau_1)} = 0{\cdot}0028$ of the value it would attain if the full relative angular velocity of the precession were possible. In view of the large angular velocity available this argument does not prove the effect to be negligible, and everything hangs on the applicability of Poincaré's theorem.

Nutation, the secular change in the obliquity of the ecliptic and the variation of latitude may be shown to have negligible magnetic effects. Thus, with the possible exception of the precession, we have shown that astronomical causes are incapable of producing a field inside the core of the earth substantially greater than the dipole field. We now consider the effects of thermally induced motions.

6. Convection currents

It is improbable that the core is entirely free from radioactivity, and the heat produced by a very slight activity may cause convective motions with important magnetic effects.

Convection will certainly occur if the temperature gradient slightly exceeds the adiabatic gradient. The adiabatic gradient is $g\alpha T/c_p$, where α is the coefficient of thermal expansion, g is the acceleration due to gravity and c_p is the specific heat at constant pressure. In the outer part of the core g is about 900 cm./sec.^{2}, T may be taken as about 10,000° C (Jeffreys 1929, p. 139). α and c_p may be roughly estimated from their values at ordinary temperatures and pressures, using theoretical results for the extrapolation (Mott & Jones 1936). The results are 9×10^{-6}/° C and 0·18 cal./g.° C. These values give an adiabatic gradient of 1·1° C/km. The thermal conductivity may be estimated as 0·18 cal./° C cm.sec.; with this conductivity and a gradient equal to the adiabatic gradient, conduction could transport $3{\cdot}0\times10^{12}$ cal./sec. from the whole core. If the total heat generation exceeds this, convection must occur. Since the mass of the core is $1{\cdot}9\times10^{27}$ g. the corresponding generation of heat in each gram of the core is $1{\cdot}6\times10^{-15}$ cal./sec. This is under 1 % of the average rate of generation of heat in acid igneous rocks and is, therefore, not unreasonably high. It is, however, considerably higher than the rate of generation of heat in iron meteorites which may represent a material similar to that of the core.

Six meteorites examined by Paneth (1942) were found to contain an average of $0{\cdot}7\times10^{-8}$ g. U/g. and $4{\cdot}3\times10^{-8}$ g. Th/g. These would generate $0{\cdot}4\times10^{-15}$ cal./g.sec.,* or $0{\cdot}75\times10^{12}$ cal./sec. in the whole core, that is, about a quarter of the heat needed to maintain the adiabatic gradient against conduction. The data used are subject to such large errors that it is not certain that the gradient in a core composed of iron meteorites would really fall below the adiabatic. Even if it does it is still likely that convection currents will occur. This follows from an extension of von Zeipel's theorem due to Verhoogen (1948). He shows that a rotating ellipsoid in hydrostatic equilibrium cannot also be in thermal equilibrium, and must necessarily be subject to convection currents.

Convection currents may also be produced by asymmetries in the distribution of temperature. Further investigation is necessary to determine how rapid a motion will be produced by a given asymmetry.

It thus appears that the existence of convection currents in the core may reasonably be postulated, and that a temperature gradient of the order of the adiabatic is not unreasonable. To be on the safe side we shall assume one-tenth of the adiabatic gradient and show that if convection occurs it can produce a large toroidal field, and will probably explain the maintenance of the main field itself. The possible magnetic effects of convection have previously been discussed by Frenkel (1945), who suggested that it may be possible to base a theory of the main field on it, but who does not describe any specific mechanism. Slichter & Bullard (1947) have objected that convective motions on a large scale are unstable and would break up into a turbulent

* This figure is given incorrectly by Slichter & Bullard (1947).

motion. This difficulty is removed by the arguments of § 4 which show that the magnetic field will exert a stabilizing influence on the motion.

The general effect of any convective motion is to interchange the material near the outside of the core with that inside. The conservation of angular momentum will then require that the inside of the core is continually accelerated and the outside retarded. In the presence of a dipole magnetic field a toroidal field will be produced and the accelerations will be opposed by the electromagnetic forces. The whole system constitutes a heat engine for converting the heat produced by radioactivity into the energy of a system of currents and a magnetic field. By the second law of thermodynamics the efficiency of such an engine cannot exceed $\delta T/T$, where δT is the difference of temperature over which it is working. We take this difference of temperature to be that produced by a gradient of one-tenth of the adiabatic gradient over a distance of the order of the radius of the core. Over 1000 km. one-tenth of the adiabatic gradient of 1·1° C/km. gives a temperature difference of 110° C. If $T = 10{,}000°$ C the efficiency cannot therefore exceed 1·1 %. The energy required to maintain a toroidal field of 40 gauss is about 2×10^{16} ergs/sec. or 5×10^{8} cal./sec. The efficiency required is therefore only 0·07 % or one-sixteenth of the ideal thermodynamic efficiency. In fact, there seems every reason why the process contemplated should give an efficiency not greatly below that for an ideal engine. Viscosity will dissipate an amount of energy that is negligible compared with that used in inducing the field, and eddy diffusion will probably be largely prevented by the field. Thermal conduction will be the main cause of irreversible changes, but if convection occurs at all it is likely to carry a large part of the heat. We conclude therefore that there is no difficulty in providing energy for convection currents in the core sufficient to produce a large magnetic effect.

In order to estimate roughly the radial velocities necessary to produce an appreciable toroidal magnetic field we consider the transport of momentum across the spherical surface that divides the core into two equal halves. This has a radius 0·79 time that of the core. If the core is rotating like a rigid body with an angular velocity Ω, the angular momenta M_1 and M_2 in the two halves are

$$M_1 = 0{\cdot}126 Ma^2\Omega, \quad M_2 = 0{\cdot}273 Ma^2\Omega,$$

where M and a are the mass and radius of the core. If the material of the two halves is interchanged in the absence of a magnetic field the conservation of angular momentum requires that the inside and outside halves should have angular velocities ω_1 and ω_2 after the change given by

$$0{\cdot}273 Ma^2\Omega = 0{\cdot}126 Ma^2\omega_1, \quad 0{\cdot}126 Ma^2\Omega = 0{\cdot}273 Ma^2\omega_2,$$

which gives $$\omega_1 - \Omega = 1{\cdot}17\Omega, \quad \Omega - \omega_2 = 0{\cdot}54\Omega, \quad \omega_1 - \omega_2 = 1{\cdot}71\Omega. \tag{5}$$

If the mean radial velocity over half the surface of radius $0{\cdot}79a$ is v_r inwards and over the other half v_r outwards, the time taken for the interchange is $0{\cdot}53a/v_r$. The rate of change of angular momentum of either half is

$$0{\cdot}126 Ma^2(\omega_2 - \Omega)\, v_r/0{\cdot}53a = 0{\cdot}28 Ma\Omega v_r.$$

This must be equal to the electromagnetic couple which is $\frac{4}{15}a_3H_0H_\phi$, whence, if $H_\phi = 40$ gauss,

$$v_r = a^2H_0H_\phi/1{\cdot}0M\Omega = 1{\cdot}4\times10^{-4}\,\text{cm./sec.} \tag{6}$$

This is the mean velocity.

The relative tangential velocity, v, of the two halves of the core corresponding to any given radial velocity can be found by substituting for H_ϕ in (2) from (6). The result is

$$v/v_r = 2{\cdot}4M\Omega/\kappa a^3H_0^2 = 160.$$

The rotation is, therefore, rapid compared with the radial motion.

The time to travel 1000 km. at a velocity v_r is $0{\cdot}7\times10^{12}$ sec. or 23,000 years. This gives plenty of time for the electromagnetic forces to redistribute the angular momentum before a complete circuit is completed.

If T_1 is the difference in temperature between the ascending and descending currents the rate of transport of heat across a surface of radius r is $2\pi r^2Dv_rcT_1$, where D is the density of the core and c its specific heat. If most of the heat is carried by convection, the temperature difference required to transport the $0{\cdot}75\times10^{12}$ cal./sec. assumed earlier in this section is $0{\cdot}006°$ C. As the assumed heat is considerably more than is required to provide the energy for the field, this may be an overestimate. It is essential to the argument that this small difference of $0{\cdot}006°$ C is *not* the temperature difference between which the engine works. It is the difference in temperature between the descending and ascending currents at a given level. The temperature difference occurring in the expression for the efficiency is the radial difference provided by the adiabatic gradient. Elsasser (1947) has not made this distinction, and has thus rejected radioactive heat as a factor in the problem. Frenkel (1945) makes a distinction similar to that made in this paper.

The mechanism described above decelerates the outer part of the core and tends to make it rotate more slowly than the rigid rocky mantle. Since the eddies causing the secular variation lie in the outer part of the core they may be expected to drift slowly westwards. This drift will cause the centres of rapid secular change to drift westwards with a velocity $(\Omega-\omega_2)\,R = 0{\cdot}32(\omega_1-\omega_2)\,R$, where R is the radius of the earth. By (2) this velocity is about $3{\cdot}0H_\phi R/\pi\kappa a^2H_0$. For $H_\phi = 40$ gauss this gives $0{\cdot}017$ cm./sec. or $5{\cdot}4$ km./yr. A drift in the predicted direction is shown by Vestine's maps (Vestine, Laporte, Cooper, Lange & Hendrin 1947; Elsasser 1949); the observed velocity is about 7 km./yr. corresponding to $H_\phi = 50$ gauss and $\omega = 1{\cdot}4\times10^{-10}$ sec.$^{-1}$. If the uncertainty in κ can be reduced, this method may give a reliable determination of H_ϕ and ω.

The boundary conditions for a viscous fluid require it to move with the solid with which it is in contact at the surface of the core. The work of Bondi & Lyttleton (1948) makes it likely that the transition layer will be so thin as to have no direct magnetic effect. The viscous forces in it will, however, exert a couple on the solid part of the earth. If this couple is large enough it may prevent the outer part of the core from rotating slower than the solid part of the earth and may thus invalidate the argument of the last paragraph. The viscosity and the thickness of the boundary layer are both so uncertain as to render a useful calculation impossible.

The ultimate fate of the $0{\cdot}75 \times 10^{12}$ cal./sec. that we have assumed to be generated in the core requires some consideration. It represents a small fraction of the whole flow of heat reaching the surface of the earth, which is about 7×10^{12} cal./sec., and its arrival at the surface would cause no difficulty. The difficulty is to devise a method of transporting it away from the core. Conduction cannot do this, as the whole of geological time is too short for an appreciable fraction of the heat generated in the core to be conducted to the surface. If convection currents exist in the outer part of the earth this difficulty is removed. The possibility of such currents has been discussed by many authors, but lies outside the scope of this paper.

If there are to be convection currents in the core the heat must leave it. If it did not leave, it would cause a rise in temperature. With the amounts of U and Th found by Paneth in meteorites the rise since the solidification of the earth $3{\cdot}3 \times 10^9$ years ago may be shown to be about 300° C. By the time all the U and Th have disintegrated this would rise to 1400° C. These are very moderate figures and show that the heat generated is not sufficient to have any rapid catastrophic effects. The time required to heat the whole core through the calculated difference in temperature between the ascending and descending currents is about 140,000 years. In this time the convection current would have travelled about 3600 km. or a distance of about one radius of the core.

The only feature of the convection currents that has been used in the argument of this section is the production of a difference in the angular velocity in the inner and outer parts of the core. Without a detailed theory of the motion it is difficult to go further. It is clear that the motion produced is not necessarily a rotation having the same angular velocity in all latitudes, and also that the magnetic effect of the radial motion must be considered. These factors make it likely that there will be some deviation from the simple picture of an easterly field in the northern hemisphere and a westerly one in the southern hemisphere. In the next section we show that the secular variation requires such complications, and in § 8 use them to outline a theory of the origin of the main field.

7. The observed secular variation

Figure 3 shows a simplified picture of the secular variation of the vertical component of the field in 1922·5. This figure is derived from Vestine *et al.* (1947, figure 133*a*). It shows the line of no variation and the main centres of rapid change. The analysis in I of the region near South Africa suggests that this field resembles that which would be produced by a line of horizontal dipoles situated near the surface of the core and lying directly under the line of zero rate of change of vertical field. Calculations are in progress to test the accuracy of this approximation which seems likely to give a fair representation to the true field. Such a line of dipoles could be produced by a sinuous line of rotating matter with its axis along the line of dipoles. The long eddy assumed in I to explain the South African focus of rapid secular change would form part of this line. The field required in the core will depend on the radius assumed for the eddy and will be of the same general magnitude as those found in South Africa (e.g. 20 gauss for a cylinder of 200 km. radius; see I, p. 253). The magnitude now raises no difficulty, but the direction does. In its simplest form our theory gives opposite toroidal fields in the two hemispheres and only the dipole field at the equator. It is clear, however, from figure 3 that the lines of equal secular variation cross the equator without discontinuity. Also the field must have a component to the south-east under South Africa and to the south-west under South America. The first of these is inconsistent with a purely westerly field in the southern hemisphere. The secular variation thus requires a more complicated field for its explanation.

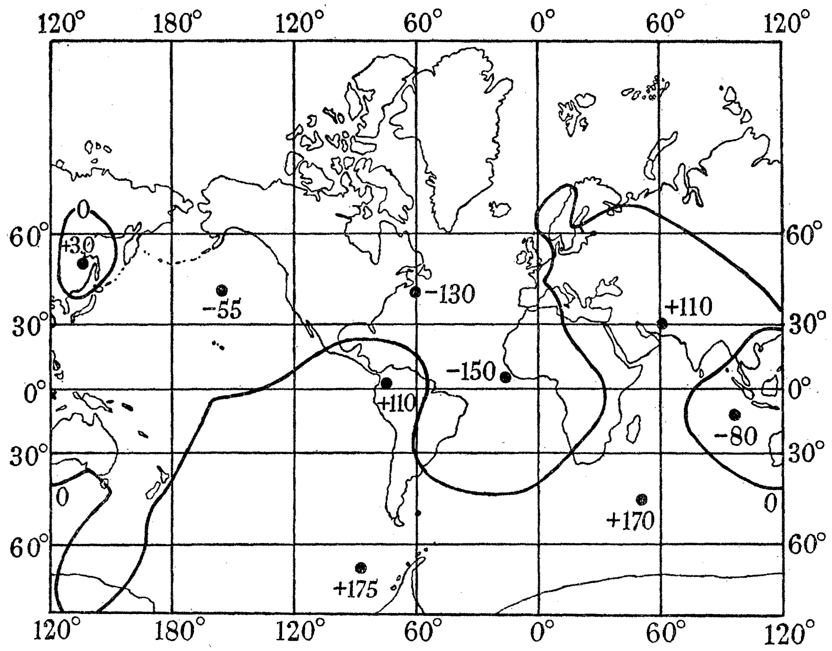


FIGURE 3. Secular change in the vertical component of the earth's magnetic field (γ/yr.) for 1922·5. The line of zero rate of change and all maxima of more than 30γ/yr. are shown.

8. THE CAUSE OF THE EARTH'S MAIN FIELD

The demonstration that it is possible in a very simple and natural way to produce a field in the interior of the earth much in excess of the dipole field leads to the reconsideration of Larmor's suggestion that the main field itself might be supported by induction. Elsasser (1947) has discussed the relation between field and motion in a very general way. He concludes that there do exist chains of relations in which a field H_1 interacts with a motion M_1 to produce a field H_2 which interacts with another component of the motion M_2 to reproduce H_1. The particular chain that he considers he rejects as 'so complicated and artificial that it would hardly seem convincing'. It suffers from the almost insuperable objection that it involves a rotation about an axis inclined to that of the earth.

The considerations of § 6 suggest that the only plausible motion in the core on a large scale is a radial convective motion, together with the rotation about the earth's axis that is necessary to conserve angular momentum. We now attempt to construct a system of motions of this type that will maintain a field. For this it is desirable to employ Elsasser's notation for vector fields. An electric or magnetic field or a current system satisfying Maxwell's equations can be derived from scalar functions ψ_n^{mc} and ψ_n^{ms}, where

$$\left.\begin{aligned}\psi_n^{mc} &= R(r)P_n^m(\theta)\cos m\phi,\\ \psi_n^{ms} &= R(r)\,P_n^m(\theta)\sin m\phi,\end{aligned}\right\} \tag{7}$$

here θ is the colatitude, ϕ the longitude and $R(r)$ a function of r (see III, §3). The fields derived from this are of two kinds called toroidal and poloidal. The toroidal field T_n^{mc} has components

$$\begin{aligned}\text{radial component of } T_n^{mc} &= 0\\ \theta \text{ component of } T_n^{mc} &= (\sin\theta)^{-1}\,\partial\psi/\partial\phi\\ \phi \text{ component of } T_n^{mc} &= -\partial\psi/\partial\theta\end{aligned}$$

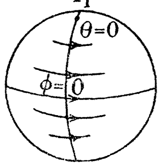
and similarly for T_n^{ms}. The corresponding poloidal vectors S_n^{mc} and S_n^{ms} are proportional to curl T_n^{mc} and curl T_n^{ms}. The general form of the fields for $n = 1$ and 2 is shown in figures 4*a* and *b*. The S fields are more difficult to visualize than the T's as they have a radial component. More than one sketch is given for some of the S fields to illustrate the behaviour both when confined within the sphere and when spreading outside it, the former apply to currents and the latter to magnetic fields.

The same scheme can be used to classify motions within a sphere. A system of convection currents in a stationary sphere might involve motions of the S_1, S_1^c, S_1^s, S_2^c, S_2^s S_2^{2c} or S_2^{2s} types. In a rotating sphere these would be accompanied by T_1 and perhaps by T_2 motions produced by the mechanism described in §6. There may also be S and T motions with higher n's, but these may probably be neglected in a preliminary discussion. It is possible, by methods devised by Elsasser, to draw up tables showing what interactions are possible between these motions and a given field (the tables given by Elsasser refer to complex vectors, but are easily adapted to the real vectors used here).

The field outside the earth is predominately S_1. It is natural therefore to start with this and to consider the network of possible interactions that stems from it. If this is done it is found that the combinations of motions $T_1 S_1$ and $T_1 S_2$ do not produce a closed chain returning to S_1, and therefore cannot maintain a field. The combinations $T_1 S_2^{2c}$, $T_1 S_2^c$ and $T_1 S_1^c$ do produce closed chains. Similar schemes would apply to the corresponding motions with s in place of c in the upper index.

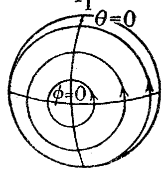
The simplest of these schemes is that associated with an S_2^{2c} motion (figure 5 and table 1). This motion consists of two rising and two sinking currents spaced evenly around the equator. The S_1 field outside the core may be taken as that of a dipole at the centre of the earth. Inside the core its radial variation is not known, but it is probably approximately constant. This field interacts with the T_1 rotation (figure 6*a*); in this motion the inner part of the core rotates from west to east relative to the outer. This gives an S_2 current system (figure 6*b*); in this the current travels inwards along the equatorial plane and outwards at both poles. This current produces a T_2 magnetic field directed from west to east in the northern hemisphere and from east to west in the southern hemisphere (figure 6*c*). This

T_1



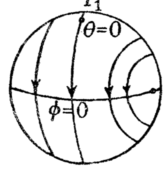
$\psi_1 = R(r)\cos\theta$
$T_r = 0$
$T_\theta = 0$
$T_\phi = R\sin\theta$

T_1^c



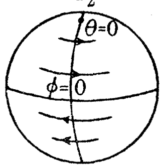
$\psi_1^c = R(r)\sin\theta\cos\phi$
$T_r = 0$
$T_\theta = R\sin\phi$
$T_\phi = R\cos\theta\cos\phi$

T_1^s



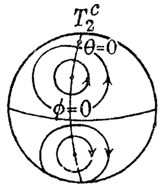
$\psi_1^s = R(r)\sin\theta\sin\phi$
$T_r = 0$
$T_\theta = R\cos\phi$
$T_\phi = -R\cos\theta\sin\phi$

T_2



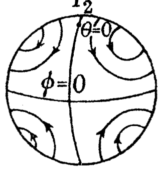
$\psi_2 = R(r)(3\cos^2\theta - 1)$
$T_r = 0$
$T_\theta = 0$
$T_\phi = \frac{3}{2}R\sin 2\theta$

T_2^c



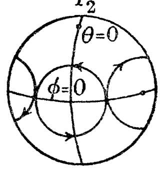
$\psi_2^c = \frac{3}{4}R(r)\sin 2\theta\cos\phi$
$T_r = 0$
$T_\theta = -\frac{3}{2}R\cos\theta\sin\phi$
$T_\phi = -\frac{3}{2}R\cos 2\theta\cos\phi$

T_2^s



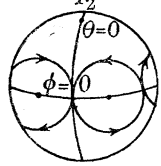
$\psi_2^s = \frac{3}{4}R(r)\sin 2\theta\sin\phi$
$T_r = 0$
$T_\theta = \frac{3}{2}R\cos\theta\cos\phi$
$T_\phi = -\frac{3}{2}R\cos 2\theta\sin\phi$

T_2^{2c}



$\psi_2^{2c} = \frac{3}{2}R(r)\sin^2\theta\cos 2\phi$
$T_r = 0$
$T_\theta = -3R\sin\theta\sin 2\phi$
$T_\phi = -\frac{3}{2}R\sin 2\theta\cos 2\phi$

T_2^{2s}

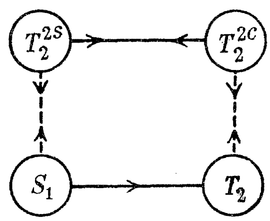


$\psi_2^{2s} = \frac{3}{2}R(r)\sin^2\theta\sin 2\phi$
$T_r = 0$
$T_\theta = -3R\sin\theta\cos 2\phi$
$T_\phi = -\frac{3}{2}R\sin 2\theta\sin 2\phi$

FIGURE 4*a*. Classification of toroidal fields. $\phi = 0$ is to the front and $\phi = \frac{1}{2}\pi$ to the right of all the diagrams (T_1^c and T_2^{2s} have accidentally been drawn for negative R, the rest have R positive).

TABLE 1. INDUCTION BY T_1 AND S_2^{2c} MOTIONS

velocity	inducing field	current	induced field
T_1	S_1	S_2	T_2
S_2^{2c}	S_1	S_2^{2s}	T_2^{2s}
S_2^{2c}	T_2	S_2^{2c}	T_2^{2c}
S_2^{2c}	T_2^{2c}	S_2	T_2
S_2^{2c}	T_2^{2s}	T_1	S_1
T_1	T_2^{2c}	S_2^{2s}	T_2^{2s}
T_1	T_2^{2s}	S_2^{2c}	T_2^{2c}



——→—— Coupling by T_1 motion
– – →– Coupling by S_2^{2c} motion

FIGURE 5. Relations between induced and inducing fields with T_1 and S_2^{2c} motions.

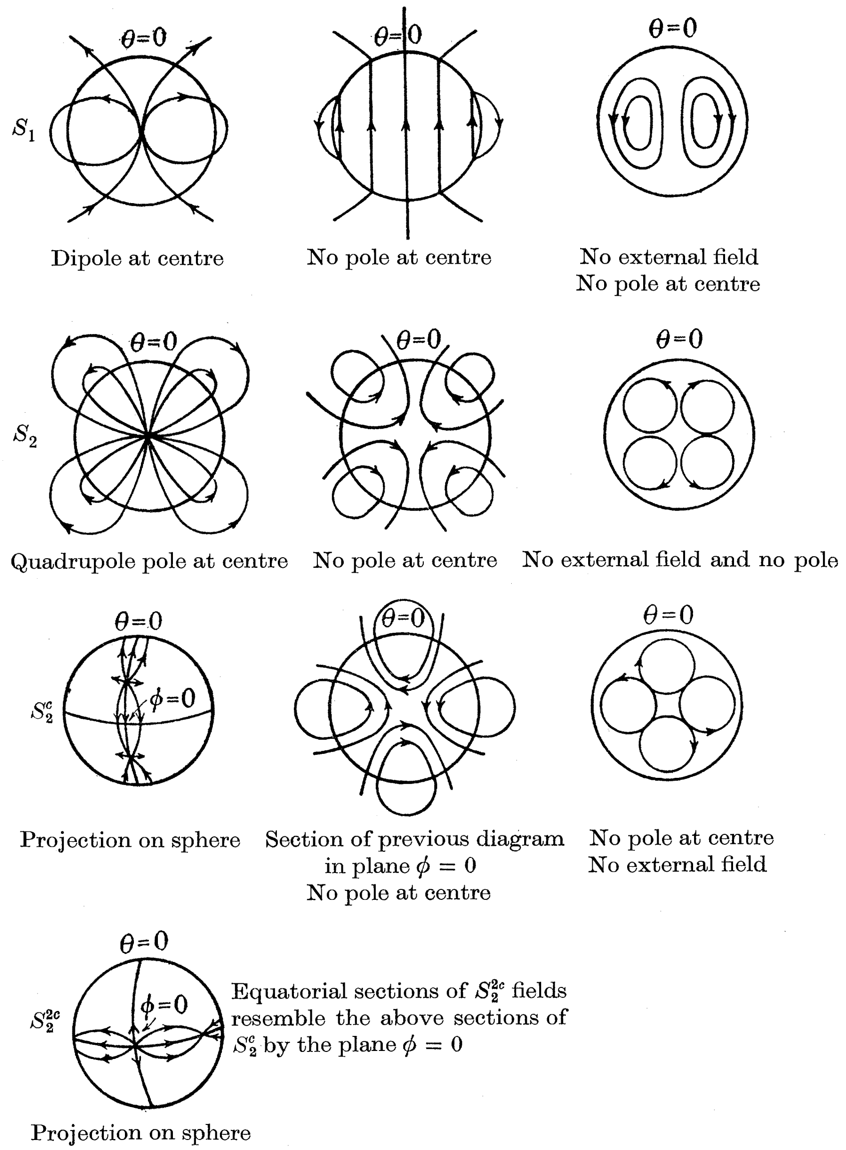


FIGURE 4*b*. Classification of poloidal fields. The diagrams for S_1 and S_2 are sections in any meridian plane. These fields are symmetrical about the axis $\theta = 0$, and are the same in all meridian planes. The first diagram for S_1 represents a dipole at the centre, the second represents a uniformly magnetized sphere, and the third a field confined within the sphere. A field confined within the sphere with a dipole at the centre is also possible. The S_1^c and S_1^s are similar to S_1 but have their axes along $\theta = \frac{1}{2}\pi$, $\phi = 0$ and $\phi = \frac{1}{2}\pi$. S_2^s is similar to S_2^c turned through $\frac{1}{2}\pi$ about the axis $\theta = 0$. S_2^{2s} is similar to S_2^{2c} turned through $\frac{1}{4}\pi$ about the axis $\theta = 0$. The radial variation shown is diagrammatic only.

process has been thoroughly discussed in §§2 and 6, where it is shown that the T_2 field may be many times the dipole field. The only experimental evidence for its magnitude is the westward drift of the centres of secular variation. This suggests about 50 gauss.

In the S_2^{2c} motion material emerges in the neighbourhood of two diametrically opposite points on the equator at $\phi = 0$ and π and sinks near the points on the equator half-way between them. A projection of the motion on the outside of the core is shown in figure 6*c*, and an equatorial section in figure 6*d*. The T_2 field interacts with this motion to give an S_2^{2c} current. The origin of this current may be seen by considering the interaction of the northward component of the motion along the meridian $\phi = 0$ (figure 6*c*) with the westward T_2 field in the northern hemisphere.

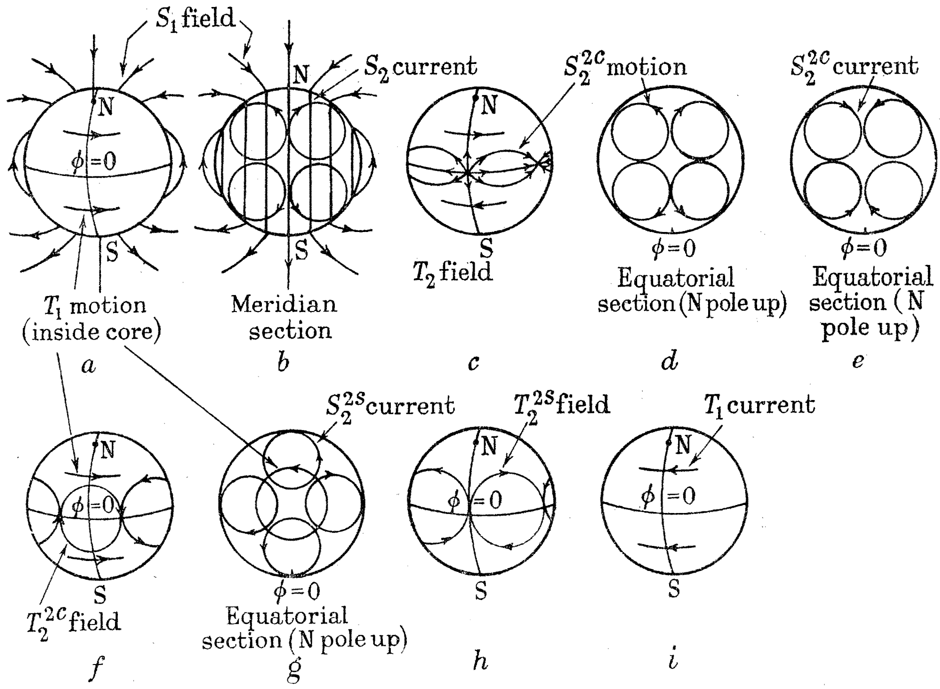


FIGURE 6. Motions, currents and fields required to maintain an external dipole field by the process of table 1 and figure 5.

This produces an electromotive force which has a component inwards. In the southern hemisphere the motion is southward and the field westward; the electromotive force is therefore also inwards. Similar arguments give an inward electric field at $\phi = \pi$ and an outward field at $\phi = \pm \frac{1}{2}\pi$. Completing the circuits of currents produced by these fields gives an S_2^{2c} current (figure 6*e*). This current cannot be geometrically similar to the S_2^{2c} motion that produces it. Their angular variation must be the same, but they will differ in a radial direction, that is, the $R(r)$ of (7) will differ. As the radial variation of neither is known this difference is not represented in figures 6*d* and *e*.

The S_2^{2c} current gives a T_2^{2c} field (figure 6*f*). The interaction of this with the T_1 rotation gives a similar current system and field, with the centres of the circular lines of force still on the equator but no longer at $\phi = 0$ and $\frac{1}{2}\pi$ (figures 6*g* and *h*); that is, some S_2^{2s} current and T_2^{2s} field are present as well as the S_2^{2c} and T_2^{2c}. This interaction is similar to that discussed in § 8 of III, except that the radial variation there assumed for the T_2^{2c} field is not the correct one for the present problem. The interaction will be strong and the T_2^{2s} field comparable with the T_2^{2c} one if $4\pi\kappa av$ is greater than 1, where v is the maximum velocity due to the T_1 motion. With a velocity of 0·03 cm./sec. $4\pi\kappa av$ is about 400. The T_2^{2c} and T_2^{2s} fields are therefore strongly coupled.

Finally, the T_2^{2s} field interacts with the S_2^{2c} motion to give a T_1 current (figure 6*i*) which reinforces the original S_1 field.

The directions of the fields and currents are given in figure 6, which demonstrates that the final field is in the correct direction to reinforce the initial field. This is the natural result of the chain and not the consequence of an arbitrary choice. The same result is obtained if the S_2^{2c} motion is reversed. The direction of the T_1 motion is fixed by the dynamical arguments of § 6. The argument does not, however, fix the direction of S_1; reinforcement could occur in either direction. If the earth were unmagnetized but possessed the T_1 and S_2^2 motions, a small field along the axis in either direction could start the process of regeneration, and cause the field to increase till the electromagnetic forces controlled the motions.

The external field produced by this process is that due to a dipole directed along the earth's axis. The internal field consists of three parts; an approximately uniform field of about 4 gauss associated with the external dipole, the large T_2 field discussed above and a T_2^2 field, probably of an intermediate magnitude. The T_2^2 field has a northerly or southerly direction near the equator, and is of the kind required to remove the difficulties of § 7 concerning the direction of the field producing the secular variation. A properly disposed T_2^2 field can have a component to the south-east in South Africa and to the south-west in South America.

The possibility of the process outlined depends essentially on whether the large gain in the step from the S_1 field to the T_2 is sufficient to balance possible losses in the steps from T_2 to T_2^{2c} and from T_2^{2s} back to S_1. It is important that this problem should be solved in detail. That is to say that a solution of Maxwell's equations should be found that is a combination of the S_1, T_2 and T_2^2 fields (and perhaps fields with $n > 2$), which satisfies the boundary conditions and which does not decay to zero.

Until such a solution is found we must rely on general arguments and on analogy with the problems solved in III. It is shown in III, § 2, that Maxwell's equations without the displacement current term may be written in a non-dimensional form so that the only parameter involved is $4\pi\kappa va$, where v is the maximum velocity occurring in the motion. The ratio of the induced to the inducing field in any interaction will be a function of this parameter only. If the interaction will only work in one direction, as in the production of a T_2 field from an S_1 field by a T_1 rotation, the induced field increases indefinitely as v is increased. If there is an interaction in both directions, as in the interaction of an S_1^c field with a T_1 rotation, the induced field may tend to a limit of the same order as the inducing field (see figure 1 of III). It seems likely that the two interactions with which we are concerned are of the latter type, and that if a large factor is not to be lost in each $4\pi\kappa va$ must not be much below unity. With the velocity of $1{\cdot}4\times10^{-4}$ cm./sec. found in § 6 $4\pi\kappa va$ is 1·8. There is thus a good prospect of only a moderate loss in the two steps, and that it may be counterbalanced by the large gain in the first step. The scheme is more complicated

30-2

than Elsasser's, but the motions are simple and physically plausible. The complication is the inevitable result of Maxwell's equations.

The schemes with $T_1 S_2^c$ and $T_1 S_1^c$ motions are more complicated than that of table 1. They have several closed chains of interactions, and without a detailed solution it is not possible to know if the resultant of them all is in a direction to reinforce the dipole field. The three schemes are not mutually exclusive, and an actual system of convection currents may well involve all three. Only the S_1 and S_2 part of the motion, that is, the part that is symmetrical about the axis, does not enter into any of the schemes.

The scheme with T_1 and S_2^c motions automatically produces a dipole inclined to the earth's axis. Like that of table 1 and figure 5 it produces an external field with no second harmonics.

Cowling (1934) has shown that a motion that is purely in meridian planes cannot maintain a field, and Elsasser has shown that the same is true of a purely toroidal motion. Our schemes do not violate these theorems, as the motion is of a more complex type.

The theory, like all theories that ascribe a deep internal origin to the field, is inconsistent with the experiments of Hales & Gough (1947) and of Runcorn (Chapman 1948) on the variation of field with depth. It is not impossible that their results are due to local anomalies, and it is of great importance that the doubt should be removed by measurements at other places and especially at sea.

Before the theory can be regarded as giving a satisfactory explanation of the origin of the main field two things are necessary. First the convective motion occurring in a rotating sphere must be calculated and shown to be of a type capable of maintaining a field. The electromagnetic problem presented by this scheme must then be solved and the values of the fields calculated in terms of the velocities. The velocities found must be reasonable judged by the thermodynamic arguments of § 6.

The proposed calculations are formidable and the construction of a model is therefore worth consideration. Such a model might consist of a solid sphere of copper carrying inside it four copper cylinders which could be rotated and which would represent the four circulations of the convective motion. Outside the sphere would be a rotating copper shell representing the T_1 motion. The condition for similarity is that $\kappa a v$ should be the same in the model as in the earth. For copper κ is about 200 times greater than we have assumed for the earth. Thus if a is 20 cm., which is about the largest practicable size, v must be $0{\cdot}8 \times 10^5$ times that in the earth. For the convective motion this will be only a few cm./sec., and is easily attained. For a T_1 rotation of $0{\cdot}1$ cm./sec. in the earth, it would give about 10^4 cm./sec. or 5000 r.p.m. While this is not absolutely unattainable it is so high as to make the construction of a model a considerable undertaking and not justifiable till the calculations are further advanced.

9. Conclusion

The arguments of this paper are believed to demonstrate that a very moderate radioactivity of the material of the core will have profound effects on the magnetic field within the core. This field is larger and more complicated than appeared at first sight to be likely. There seems now no serious difficulty in accepting the induction theory of the origin of the secular variation and of the lower harmonics of the non-dipole field. The main field itself also finds a natural and unforced explanation, though the arguments used still fall short of a proof that the mechanism postulated will produce a field.

The explanation of the westward drift of the centres of secular variation is some confirmation of the general correctness of the ideas employed.

The only arbitrary factor in the calculation is the rate of generation of the heat in the core by radioactivity. This has been taken from studies of meteorites. It proves to be much greater than is necessary to supply the energy required, though somewhat smaller than is necessary to produce the adiabatic gradient.

In this paper I have differed in several matters from Professor Elsasser, and have to some extent built a theory with ideas which he has discarded for reasons that I consider inadequate. Without some further remark it might be thought that our views differed more than they do. In fact, the development of the ideas in this paper has been very greatly influenced by his work.

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