# On the Origin of the Solar Nebula

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*Abstract*

A consideration of star formation from the interstellar gas shows that the primitive solar condensation was probably endowed with angular momentum very much greater than that now possessed by the Sun. It is found, however, that the initial angular momentum agrees with that possessed by the primaeval planetary material. It is suggested therefore that the angular momentum of the solar nebula was transferred to the planetary material. A similar process of transfer appears to take place generally in dwarf stars. Massive early-type stars, on the other hand, possess the expected angular momentum, and presumably do not have planetary systems. In the solar case, the process of angular momentum transfer was probably hydromagnetic—a field with intensity $\sim 1$ gauss being required to connect the Sun and the planetary material.

As the planetary material acquired angular momentum it moved farther and farther away from the solar condensation. Materials of low volatility condensed out of the gas and were left behind as the main mass of gas moved outwards. The fact that the terrestrial planets are (i) of small mass (ii) composed almost wholly of low volatility materials (iii) on the inside of the system, is attributed to this leaving behind of the low volatility materials.

Various subsidiary problems, such as the formation of satellites, and the escape of gas from the periphery of the solar system, are briefly discussed.

## 1. INTRODUCTION

At a symposium on the Origin of the Planets, held at the Moscow meeting of the I.A.U., it became clear that there is a very widespread disagreement among astronomers on this whole problem. The point of view expressed below was stated by the writer in 1954 in lectures given at California Institute of Technology and at Princeton University, and published (1) in 1955 in qualitative form. In the large spread of opinion that exists, the present ideas fall nearest to the work of Lüst and Schlüter (2). There are no large proto-planets in the sense advocated by Kuiper (3). Nor is there any crucial relation with the capture of interstellar material by the Sun, as in the work of Alfvén (4) and of Soviet astronomers following Schmidt (5). The idea of a magnetic coupling between the

Sun and planetary material is taken from Alfvén, however. In a somewhat similar way, the general outlook of the paper has been influenced by von Weizsäcker (6), even though the exposition is very different.

## 2. THE ANGULAR MOMENTUM PROBLEM

It is well known that the angular momentum per unit mass about the centre of gravity of the solar system is on the average some 50 000 times greater for planetary material than it is for solar material. This remarkable fact has usually been interpreted in the sense of the question: How did the planetary material acquire so much angular momentum? In the present discussion, however, the problem will be inverted—we ask: How did the solar material come to receive so little angular momentum?

A simple calculation shows that a star would rotate with a speed of the order of light, it it were formed by isotropic compression from the interstellar gas, and if angular momentum were conserved during the compression. The following suggestions for reducing the rotation speed have been made:

(i) the condensing volume may be filamentary rather than spherical,
(ii) condensation may take place in regions of a cloud where the local differential rotation is abnormally small,
(iii) angular momentum may not be conserved during condensation.

Of these (ii) is implausible. Dwarf stars in general rotate slowly as the Sun does, and one cannot suppose that condensation always takes place in regions of quite exceptionally small differential motion. Nor is the possibility (i) a resolution of the problem, although it could reduce the excessive angular momentum by a moderate factor, a factor say of 10.

Turning to (iii), it is generally believed that a magnetic field pervades the interstellar gas. Thus a condensing volume of gas will be magnetically connected to its surroundings. The possibility then arises that a torque is conveyed through the magnetic field from the condensation to the interstellar gas, and that angular momentum is transferred outwards through the agency of this torque. Although such a process of angular momentum transfer is very probably an essential feature of star formation, there are complications. Mestel and Spitzer (7) have concluded that condensing gas must be able to slip readily across the magnetic lines of force in order that star formation can take place. It seems clear that the magnetic field cannot influence the condensing material very greatly while slipping is going on.

A plausible way of combining these requirements is to divide star formation into three parts:

(a) an initial waiting phase when material does not slip across the lines of force, and in which angular momentum is efficiently transferred from the condensation to the surrounding interstellar medium,

(b) a subsequent rapid shrinkage of the sort considered by Mestel and Spitzer in which material slips readily across the lines of force and in which angular momentum is effectively conserved,

(c) a final slow contraction to the main sequence, the slip across the lines of force being inappreciable in this phase.

It is possible in this way to combine the requirement of angular momentum loss with the requirement that material be able to condense across the lines of force—the latter requirement being necessary in some considerable degree in order that stellar magnetic fields do not become unrealistically large. But even with these favourable assumptions it seems clear that the loss of angular momentum cannot be anything like sufficient to explain the low rotation speed of the Sun and of dwarf stars in general. The waiting phase (a) cannot remove the whole of the angular momentum of a condensation—it can only destroy the rotation of the condensation relative to its surroundings.

*In particular, the general rotation of order* $10^{-15}$ *sec*$^{-1}$ *in the Galaxy must survive (a).*

Hence the angular velocity at the end of (a) cannot be less than $\sim 10^{-15}$ sec$^{-1}$

Tentative values for the solar condensation are given in the following table:

| Phase | Mean density (gm cm$^{-3}$) | Angular velocity (sec$^{-1}$) | Magnetic intensity (gauss) | Radius (cm) |
|---|---|---|---|---|
| Beginning of (a) | $10^{-24}$ | $10^{-15}$ | $10^{-5}$ | |
| End of (a) | $10^{-18}$ | $10^{-15}$ | $10^{-1}$ | |
| End of (b) | $10^{-12}$ | $10^{-11}$ | $10^{-1}$ | $10^{15}$ |
| End of (c) | 1 | | $10^{7}$ | $10^{11}$ |

In the first row we have values for the uncompressed interstellar gas. The magnetic intensity rises during (a) but not during (b), whereas the situation is reversed for the angular velocity. Phase (b) ends when the condensation becomes opaque to its own radiation (7). This may be taken as the stage where the internal temperature becomes high enough to promote some ionization of the metals—i.e. a temperature $\sim 10^3$ °K.

This is attained at a radius of order $10^{15}$ cm. (The ratio of the present solar radius to this value $\sim 1:10^4$, which agrees, as it should, with the ratio of $10^3$ °K to the present central temperature of the Sun.) All values given for the angular velocity and magnetic intensity, apart from the first row, assume that compression is three-dimensional. The lines of force of the magnetic field do not slip in (c), so that the magnetic intensity rises again during this phase.

A magnetic intensity $\sim 10^7$ gauss at density $\sim 1$ gm cm$^{-3}$ is consistent with the values observed in sun-spots. Thus a three-dimensional expansion from $\sim 1$ gm cm$^{-3}$ down to the densities occurring in sunspots, $\sim 10^{-6}$ gm cm$^{-3}$, reduces the intensity to $\sim 10^3$ gauss. The values given in the table are consistent therefore with present-day solar data.

No values have been given for the radius of the solar condensation before phase (c) for the reason (7) that fragmentation probably takes place during (b). Likewise the angular velocity is not given at the end of (c) because it is probably during this stage that angular momentum is transferred from the solar condensation to the planetary material.

It is of great interest to write down the angular momentum at the end of (b). The central condensation of material is probably not very marked at this phase, so that the angular momentum $\sim \frac{2}{5} MR^2\omega$. With $M \cong 2 \times 10^{33}$ gm, $R \cong 10^{15}$ cm, $\omega \cong 10^{-11}$ sec$^{-1}$, we have $8 \times 10^{51}$ gm cm$^2$ sec$^{-1}$, which is precisely the order of the angular momentum of the planetary material, provided we interpret "planetary material" to include not only the present planets but also the amount of hydrogen and helium that must be added to make up the normal solar abundances of these elements.

The deficiences of hydrogen and helium are acute in the cases of Uranus and Neptune. It is clear that their densities (1·56 and 2·27 respectively) require these planets to be mainly composed of common non-metals—C, N, O, and Ne possibly. Now in solar material hydrogen and helium have $\sim 10^2$ times the concentration by mass of the common non-metals. Hence if Uranus and Neptune were derived from material of solar composition, as will be assumed, the mass of the material must originally have been about $10^2$ times greater than the present combined masses of these planets; i.e. it must have been about 3 000 Earth-masses. This overwhelms the contribution of Jupiter and Saturn; the hydrogen–helium concentration in the latter planets is not greatly different from the solar value, so that no such large correction factors have to be applied.

On this reckoning, the original planetary material had a mass close to 1 per cent of the Sun, not to the $\frac{1}{10}$ per cent possessed by the present planets.

Thus increasing the masses of Uranus and Neptune, the original angular momentum of the planetary material is easily shown by a direct calculation to be $\sim 4 \times 10^{51}$ gm cm$^2$ sec$^{-1}$. This result is in excellent agreement with the value estimated above, thereby pointing strongly to the conclusion:

*That a process occurred whereby the angular momentum of the primitive solar condensation was transferred from the Sun to the planetary material.*

In succeeding sections we shall attempt to trace the nature and operation of this process.

*It is implicit that a similar process has been operative in the great majority of dwarf stars, and hence that such stars possess planetary systems.*

It is emphasized that the present considerations have the advantage that dwarf stars and massive stars are regarded as originally having possessed very much the same angular momentum per unit mass. The two types of star evolved differently so far as the storage of angular momentum was concerned, but there was no artificial initial difference between them.

A subsidiary problem is raised by the above discussion: the large-scale escape of hydrogen and helium from the periphery of the solar system must be explained. This issue will be taken up at a later stage.

## 3. A PRELIMINARY CONSIDERATION OF THE PROBLEM OF ANGULAR MOMENTUM TRANSFER

In the following discussion the angular momentum of the solar condensation will be taken to be $\sim 4 \times 10^{51}$ gm cm$^2$ sec$^{-1}$.

We now show that the solar condensation must have become rotationally unstable before the full contraction to a main-sequence star was completed. Let $R$, $\Omega$ denote the radius and angular velocity of the condensation at the onset of instability. The angular velocity will be supposed uniform and $k^2$—the square of the radius of gyration—will be taken as $\sim 0{\cdot}1 R^2$. Then

$$0{\cdot}1 M R^2 \Omega \cong 4 \times 10^{51} \text{ gm cm}^2 \text{ sec}^{-1}. \qquad (1)$$

For rotational instability we require

$$R^3 \Omega^2 \cong GM, \qquad (2)$$

where $G$ is the gravitational constant. Eliminating $\Omega$, and putting $M = M_\odot$, gives

$$R \cong 3 \times 10^{12} \text{ cm}. \qquad (3)$$

*The contracting solar condensation developed rotational instability when the diameter became comparable with the radius of the orbit of Mercury.*

Next it will be shown that rotational instability taking place under classical conditions (the only forces being gravitation and gas pressure) does not produce a system possessing any real resemblance to the solar system as we know it.

The investigations of Jeans (8) show that, after a marked flattening at the poles has taken place, a rotationally unstable gaseous body develops a sharp edge in the equatorial plane. Material then emerges through this edge to form a disk. The resulting loss of angular momentum from the central mass has a stabilizing effect, but this is offset by the continuing contraction of the inner regions. Thus more and more material joins the disk until eventually the central star is prevented from further contraction by the onset of nuclear reactions.

Let $r$, $\omega$ refer to the radius and angular velocity during this last stage of contraction. The mass that comes to reside in the outer disk can readily be calculated, subject to two plausible hypotheses:

(i) the solar condensation is always on the verge of rotational instability, i.e.

$$r^3\omega^2 \cong GM, \quad r < R, \tag{4}$$

(ii) angular momentum is lost by material passing at the circular velocity through the sharp edge. Then we have

$$\frac{1}{10}\frac{d}{dt}(Mr^2\omega) \cong (GMr)^{1/2}\frac{dM}{dt}, \tag{5}$$

where $M$, the mass of the central body, is now a function of time $-dM/dt$ being the rate at which material passes through the sharp edge. Equation (5) is approximate on two counts, the use of $0\cdot 1r^2$ for $k^2$, and of $(GM/r)^{1/2}$ for the circular velocity—the latter because of the polar flattening of the central mass.

Eliminating $\omega$ with the aid of (4) and (5) gives

$$\frac{d(M^{3/2}r^{1/2})}{M^{3/2}r^{1/2}} = 10\frac{dM}{M} \tag{6}$$

which yields $r \propto M^{17}$. With instability starting at $r = R \cong 3\times 10^{12}$ cm and ending at $r \cong 7\times 10^{10}$ cm, this requires

$$\frac{M(r = 3\times 10^{12}\,\text{cm})}{M(r = 7\times 10^{10}\,\text{cm})} \cong 1\cdot 25 \tag{7}$$

i.e. the mass of the disk should ultimately become about $\frac{1}{4}$ of the mass of the central body. It is also easy to see from $r \propto M^{17}$ that most of the material of the disk lies near the harmonic mean of the largest and smallest values of $r$, i.e. near $r = 4\times 10^{11}$ cm.

It is clear that this picture is in gross disagreement with the situation in the solar system. The immediate difficulties are:

(1) If the material of the disk condenses into one or more bodies, the bodies will be of stellar rather than of planetary mass.

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(2) The radius of the orbit of such a stellar body will be as little as $\sim 5 \times 10^{11}$ cm, and the period about the central star will be only a couple of days or so, in contradiction with the large orbits of the major planets and with their long periods.

(3) The main central body will be left in a state of rapid rotation.

It is to be observed that an attempt to overcome the difficulty (2) by endowing the original condensation with greater angular momentum must of necessity exaggerate the difficulty (1); the larger the angular momentum the greater the mass that must be pushed into the disk. These difficulties are inherent in any purely dynamical description of the evolution of a rotationally unstable condensation. In fact, these are the objections to the Kant-Laplace hypothesis.

A suggestion has been made that it may be possible for the central body to transfer angular momentum to the disk by turbulent friction. This would deny hypothesis (ii) (preceding equation (5)). That is to say it is suggested that the transfer of angular momentum can take place without any net transference of material from the central body to the disk. Objections (1) and (2) might be overcome in this way, but as we shall see below, objection (3) still presents an unsurmountable difficulty.

Before considering this question it is worth remarking that the above argument based on equation (5) derives powerful observational support from the existence of the W Ursae Majoris stars (**9**). About one dwarf stellar system in $10^3$ is of the W Ursae Majoris type, consisting of two close companion stars, the major component being of approximately solar mass and the minor component of about $\odot/2$ (although masses less than this are observed in spite of the obvious selection effect against their detection). The binary periods average about 0·6 days. Since our argument leads to exactly this type of system, it is difficult to believe that the argument contains any really serious flaw, although the precise numerical values are of course uncertain as a consequence of uncertainties in the radius of gyration of the central body, and of the approximation used in (5) for the circular velocity.

In spite of this strong empirical evidence in favour of the general qualitative accuracy of the picture given above, it is worthwhile considering a little further the suggestion of turbulence, since the difficulties serve to point the way to a much more hopeful possibility.

For angular momentum to be transferred outward by turbulent friction the angular velocity must decrease outward. Since the material of the disk moves in essentially circular orbits, it is clear that the material in frictional contact with the Sun has an orbital velocity equal to the circular velocity, i.e. about 400 km per second (with the solar condensation shrunk to the size of the Sun at present).

The contradiction is obvious: frictional contact between the Sun and a disk could never reduce the equatorial rotational velocity of the Sun to a value less than this circular velocity. The objection (3) is therefore overriding, and we arrive at the conclusion:

*That no purely hydrodynamic process can explain the very low rotation speed of the Sun.*

## 4. A MAGNETIC TORQUE TRANSMISSION

The condition mentioned above for an outward transference of angular momentum—that there shall be an outward decrease of angular velocity—would seem to apply quite generally to any plausible mode of transmission of angular momentum from the Sun to a disk. This condition requires the angular velocity of the Sun to be greater than the angular velocity at the inner edge of the disk. It is emphasized that in the present section the inner edge of the disk does not have direct contact with the Sun (as of course it must for the frictional case considered above). In fact, it will now be shown that the Sun cannot be slowed down to its present angular velocity unless the inner radius, $a$ say, of the disk is very appreciably greater than the present radius of the Sun.

The angular velocity of material at distance $a$ from the Sun is $(GM/a^3)^{\frac{1}{2}}$, while the present angular velocity of the Sun $\sim 2 \cdot 7 \times 10^{-6}$ sec$^{-1}$. Thus to slow the Sun to its present angular velocity the value of $a$ must satisfy the inequality

$$(GM/a^3)^{\frac{1}{2}} \geqslant \sim 2 \cdot 7 \times 10^{-6} \text{sec}^{-1}. \tag{8}$$

This gives $a \geqslant \sim 3 \times 10^{12}$ cm.

Since (3) gives $\sim 3 \times 10^{12}$ cm for the radius of the condensation at the time when the disk was first formed, and since the radius of its inner edge can only increase as angular momentum is transferred to the disk, it follows that the condition (8) is satisfied, provided there is no further addition to the disk once it has been formed. Indeed the question arises as to whether significance should be attached to the very close coincidence between the lower limit for $a$ as given by (8) and the value given by (3) for the initial radius of the disk. The coincidence would be highly significant if we could argue that the radius of the inner edge of the disk remained constant in spite of angular momentum being transferred to the disk. In this case we could say that the Sun has been slowed down to precisely the lowest possible rotation speed—the present rotation speed would be explained. There appears to be no immediate physical reason, however, why the inner radius of the disk should thus remain constant; so that the above coincidence may be no more than accidental.

The proviso just mentioned, that no further material shall join the disk after it is once formed, is of paramount importance; for it ensures that the disk shall remain of comparatively small mass. This means that in overcoming difficulty (3) (of the preceding section) we also overcome difficulties (1), (2). We see, therefore, that these three points are interconnected, and all are resolved if a strong torque coupling between the disk and the Sun can be found. In this connection it may be noted that (1) is connected to (2) by angular momentum requirements: if the disk remains of small mass it cannot store the large angular momentum of the solar condensation except by a very large increase of its outer radius.

Already the logical thread of the process becomes clear. The following steps may be distinguished:

(i) The solar condensation became rotationally unstable when its radius was $\sim 3\times 10^{12}$ cm. The instability took the form discussed by Jeans in which material passed through a sharp edge to form a disk. The material of the disk moved about the central condensation in essentially circular orbits.

(ii) A strong torque coupling between the disk and the central condensation was established (in some way still to be discussed). This torque operated to maintain an equality of angular velocity between the central condensation and the inner edge of the disk. This equality was maintained throughout the remaining condensation of the central body. The ratio of gravitational to centrifugal acceleration increased rapidly within the shrinking main body; the gravitational acceleration increased as $R^{-2}$, and at constant angular velocity the centrifugal acceleration decreased proportionally to $R$. Hence the shrinking main body soon became quite stable. Consequently no further material was added to the disk.

(iii) Except possibly near its inner edge, the material of the disk spiralled outwards (since the time required for the transfer of angular momentum must have been comparable with the time required for the shrinkage of the central body, and since this was much greater than the orbital period in the disk, it is clear that at any particular epoch the motion in the disk must have been almost circular, as stated above in (i)). In this way the bulk of the material of the disk moved outwards to great distances from the Sun.

These considerations may be summarized as follows:

*There are reasons to believe that a strong torque coupling between the primitive Sun and the planetary material became established immediately after the latter separated from the Sun.*

The next step is to identify the nature of the torque coupling. Plainly the coupling must be capable of acting across the region that opened up between the shrinking central condensation and the inner edge of the disk. It is important to realize that although this region contained little material it was not a vacuum (any more than the region around the present-day Sun is a vacuum). The essential point is that material within the gap could not have been subject to any appreciable torque. It is also important to realize that this condition of zero torque within the gap does not necessarily require the whole body force acting on material within the gap to be zero (cf. below).

We shall suppose that the material of the disk was initially linked to the solar condensation by a magnetic field, and we shall suppose further that the electrical conductivity was everywhere sufficiently large to prevent ohmic decay from producing any important uncoupling of the field. This requires lines of force that initially connected the disk and the condensation to have continued to maintain the same connection throughout the shrinkage of the central condensation. The requirement on the conductivity for this to be the case will be discussed at a later stage in Section 6. The magnetic field was then capable of acting as a torque transmitter across the region that developed between the disk and the central condensation.

The qualitative features of the process can easily be understood. If at a particular moment there is an equality of angular velocity between the central condensation and the inner edge of the disk, any subsequent shrinkage of the condensation will cause the central body to tend to rotate with a greater angular velocity than the inner edge of the disk. This causes the lines of force that bridge the gap to develop windings like those of a clock spring—and this must be the case whatever the precise initial configuration of the field.

The intensity of the field as it emerges from the solar condensation will of course be greater than the intensity of the field as it reaches the inner edge of the disk. This again is a general feature that does not depend on the precise configuration of the field. Thus let $b$ be the radius of the central body and $a$ the radius of the inner edge of the disk. The lines of force from the central body emerge over a surface with area of order $b^2$, while the same lines of force enter the disk over an area of order $a^2$. It follows that as $b$ becomes appreciably smaller than $a$ the field intensity at the central body must become considerably greater than the intensity at the disk. This effect will become more and more marked as the gap opens up between the disk and the solar condensation.

Because of its greater intensity, the field is stiffer near the solar condensation than it is at the disk—the stress being dependent on quadratic terms in the field components. Hence the field is easier to wind at the disk than it is at the Sun. We therefore expect that to begin with the windings of the field must mainly accumulate at the disk, not at the Sun. This can be visualized as follows: The field near the Sun rotates with the Sun, but as the field spreads out it is dragged back by the disk.

The accumulation of windings at and near the inner edge of the disk transmits a forward torque to the disk, in the sense that it tends to increase the angular momentum of the disk. Still working on general principles, there must be a reaction of equal magnitude on the material within the gap. But since there has to be zero net moment on diffuse material within the gap, an inner forward torque from the Sun must balance the outer reaction from the disk. This means that the winding of the lines of force at the outside must twist the whole magnetic configuration within the gap to a degree that distorts the field near the Sun, the latter distortion being such that a balance of moments for the material within the gap is always maintained.

A plausible magnetic configuration is shown in Figures 1 and 2. It is particularly to be noticed that a sheet of material must be present in the

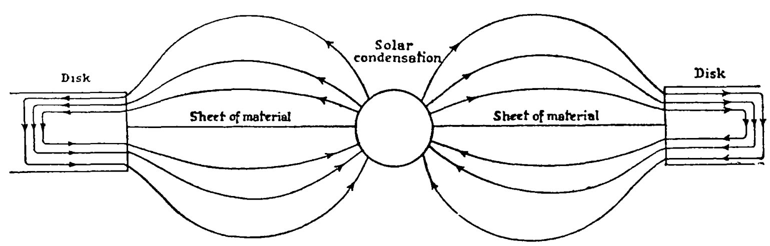


FIG. 1.—Schematic side view, showing the solar condensation and the inner edge of the disk.

equatorial plane in order to withstand the compressive force of the field normal to the plane. This does not require the material to bear any torque, however; and hence does not violate the essential conditions. It will also be noticed that the lines of force seem to end in Figure 2. This is because only the half of the configuration above the equatorial plane has been shown. The lines of force eventually cross the equatorial plane and return inwards along similar spiral paths.

There is no certain requirement that the lines of force must continue to extend outwards in the disk from its inner to its outer edge. While it is quite possible that the field extends itself in this way, it can return to the Sun from regions near the inner edge, as in Figure 1. Our requirement is for angular momentum to be transmitted across the gap from the Sun to the disk. Once the angular momentum reaches the disk it might be transferred outwards by turbulent friction (which can more properly be invoked in this case).

## 5. THE MAGNITUDE OF THE TORQUE

Our aim in this section is to obtain an order of magnitude estimate for the torque coupling between the disk and the solar condensation. For this purpose it is sufficient to take the field at the disk and at the Sun to be essentially parallel to the equatorial plane, i.e. to consider the problem in two dimensions. We employ the symbols shown in Figure 3, which

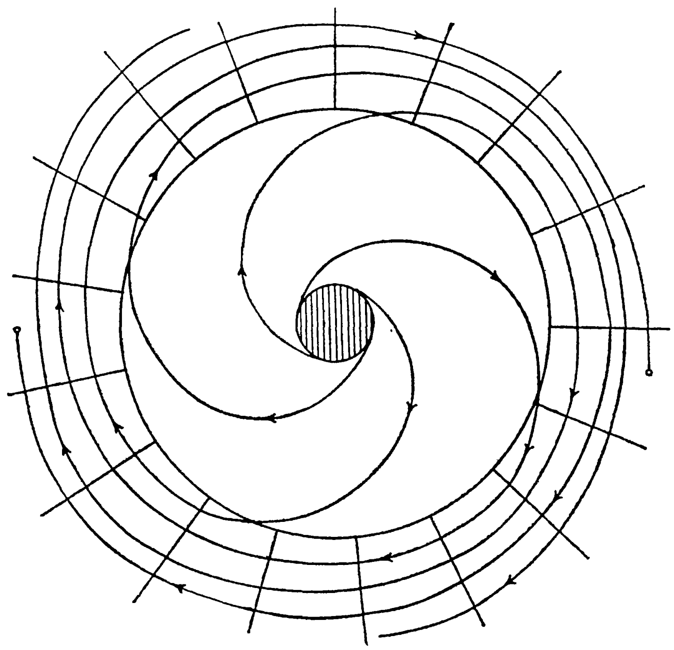

FIG. 2.—Schematic plan view, showing only the upper half of Fig. 1; the lower half is similar, but with the lines of force directed in the opposite sense.

is again a plan view of the hemisphere that lies above the equatorial plane (cf. Figure 1). Immediately we have

$$H_t = H_r \tan\theta, \qquad h_t = h_r \tan\phi. \tag{9}$$

The total magnetic flux emerging from the solar condensation (the *emergent* flux must be taken over the hemisphere shown in Figure 2) is of order $2\pi b^2 H_r$. The total flux entering the inner edge of the disk $\sim 2\pi a^2 \xi h_r$, where $2a\xi$ represents the total thickness of the disk. These fluxes must be equal so that

$$b^2 H_r \simeq \xi a^2 h_r. \tag{10}$$

The tangential stress per unit area opposing the rotation of the solar condensation is $H_r H_t/4\pi$. Thus the integrated stress over the hemisphere shown in Figure 2 is of order $\frac{1}{2} b^2 H_r H_t$, and the total integrated stress is $b^2 H_r H_t$. The moment of this stress about the solar centre has magnitude $\sim b^3 H_r H_t$.

Similarly, the tangential stress per unit area tending to increase the angular momentum of the disk is $h_r h_t/4\pi$. Thus the total integrated stress on the disk $\sim \xi a^2 h_r h_t$. This stress has moment $\sim \xi a^3 h_r h_t$ about the centre. Since these moments must have equal magnitude

$$b^3 H_r H_t \simeq \xi a^3 h_r h_t. \tag{11}$$

We regard the radii $a$, $b$ as being given. The total flux emerging from the solar condensation is determined by the disposition of the magnetic field at the time of formation of the disk itself. In order to be able to compute the torque this also must be given—which implies that $H_r$ is given. Even so, there are 5 unknowns ($H_t$, $h_r$, $h_t$, $\theta$, $\phi$) appearing in the above 4 equations. Hence an additional condition is required. In fact the degree of winding of the field still remains to be specified.

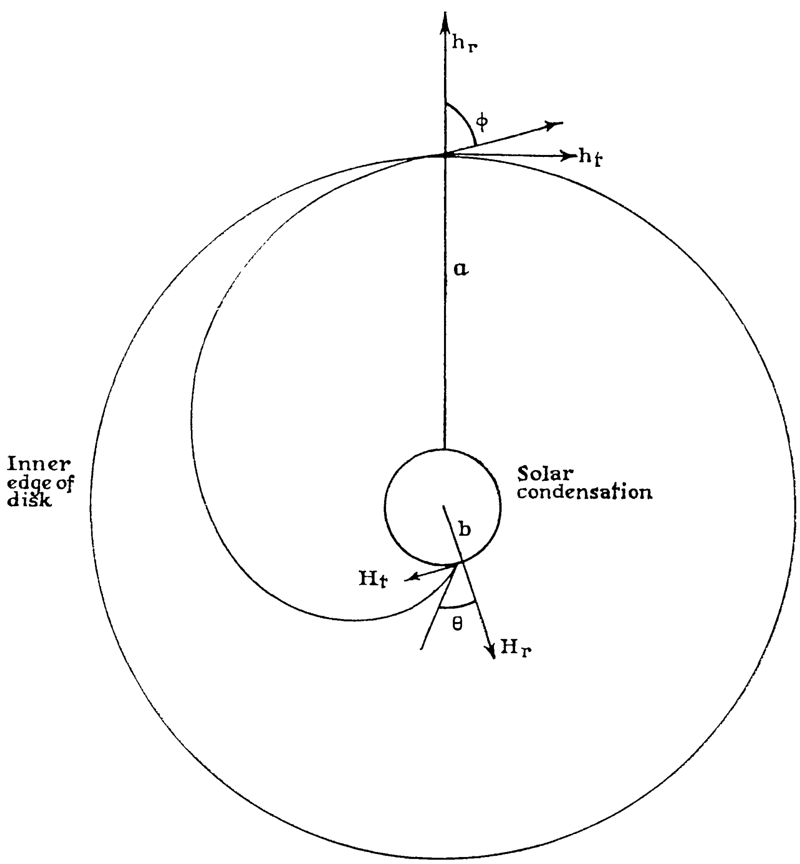


FIG. 3.

At first sight the degree of winding of the field would appear to be so variable with time that little is to be gained from a consideration of one particular case. A closer consideration shows, however, that this is not so, and that we can adopt

$$\theta = \pi/4 \tag{12}$$

as the last determining condition. We proceed now to consider this question.

Eliminating $H_t$, $h_r$, $h_t$ between equations (9), (10) and (11) gives

$$\tan\theta = \frac{b}{\xi a}\tan\phi. \tag{13}$$

Increased winding in the disk brings $\phi$ closer to $\pi/2$ and hence increases $\theta$ towards $\pi/2$. Now windings must accumulate somewhere so long as a difference of angular velocity exists between the solar condensation and the disk. Because of the torque-free condition, comparatively few windings can be stored in the gap: the windings must mainly be stored either in the disk or in the solar condensation itself. We expect that windings are stored in the disk if $\theta < \pi/4$ (so that $\phi$ and $\theta$ increase), but that the windings are stored inside the Sun when $\theta$ rises to $\pi/4$.

To see this, we note that for windings to be stored inside the Sun the magnetic pressure at the solar surface must tend to press the lines of force inwards. The inward normal component of the magnetic stress at the surface is

$$\frac{1}{8\pi}(H_t^2 - H_r^2) \text{ per unit area,}$$

which can immediately be written as

$$\frac{1}{8\pi}H_r^2(\tan^2\theta - 1).$$

Thus the inward magnetic pressure is positive for $\theta > \pi/4$ and negative for $\theta < \pi/4$.

This in itself is not sufficient, however, for us to be able to assert that the lines of force must sink into the solar condensation when $\theta > \pi/4$. It is also necessary that the surface material be free to move inwards, since the electrical conductivity is so high that the lines of force cannot diffuse relative to the material with sufficient rapidity. The latter condition is satisfied in the solar case, and in the case of all dwarf stars, but is not satisfied for main-sequence stars of large mass. Dwarf stars are known to possess convection zones that start immediately below the surface layers and that extend downwards to great depths. In such stars the surface material is therefore already at the verge of mechanical instability (consider the granulation of the present-day Sun). Hence it is extremely plausible to suppose that a downward motion of the lines of magnetic force must take place whenever the inward magnetic pressure becomes positive—i.e. when $\theta$ rises above $\pi/4$: In this case, lines of force simply wind themselves on to the central star, like thread on a bobbin of cotton.

*The present convective criterion provides a basis for separating dwarf stars from massive stars, since massive main-sequence stars do not possess deep convection zones.*

These considerations require the first windings of the field to accumulate in the disk. Quickly $\phi$, and hence $\theta$ also, increases. When $\theta$ reaches $\pi/4$ no further turns of the field are stored in the disk, only in the central condensation. The half-thickness $\xi a$ at the inner edge of the disk is not likely to be much greater than the radius $b$ of the central body. It follows therefore from (13) that $\phi$ and $\theta$ are comparable angles, from which it is clear that not many turns need be stored in the disk before $\theta$ rises to $\pi/4$—not more than $\sim 10$ relative rotations between the central body and the disk are required.

With $\theta = \pi/4$, (9) gives $H_t = H_r$, so that the total torque has magnitude $\sim b^3 H_r^2$. In a time interval $t$ this torque transfers angular momentum $\sim tb^3 H_r^2$ from the Sun to the disk.

Throughout the shrinkage of the solar condensation the emergent magnetic flux remains constant. This requires $b^2 H_r$ to remain constant; from which it follows that the torque increases as the condensation shrinks, the increase being proportional to $b^{-1}$. Thus we see that the process of angular momentum transfer is least effective when $b$ is largest—i.e. when $b$ is comparable with the radius of the inner edge of the disk. From the previous section we saw that $b$ is then of order $3 \times 10^{12}$ cm. Moreover the amount of angular momentum that must be transferred to the disk is greatest when $b$ is largest. Hence from every point of view the critical phase of the whole process is the one that immediately follows the separation of the Sun and disk. In fact we require

$$\begin{aligned} tb^3 H_r^2 &\simeq 4 \times 10^{51} \text{ gm cm}^2 \text{ sec}^{-1}, \\ b &\simeq 3 \times 10^{12} \text{ cm}, \\ t &\simeq 10^{14} \text{ sec}, \end{aligned} \qquad (14)$$

where the necessary angular momentum transfer is taken to be $\sim 4 \times 10^{51}$ gm cm$^2$ sec$^{-1}$, and the value chosen for $t$ is the Kelvin–Helmholtz contraction time for the solar condensation.

Equations (14) give $H_r = 1$ gauss, an entirely reasonable result. It is to be observed that if objection be raised to the above argument leading to $\theta = \pi/4$, if $\theta$ were taken to increase above $\pi/4$, *the torque coupling between the solar condensation and the disk would be even more efficient than is calculated from* (14).

## 6. THE CONDUCTIVITY OF THE PLANETARY GAS

It was assumed throughout the above argument that no significant ohmic decay of the magnetic field takes place over a time interval of the order of the contraction period of the Sun, i.e. over a time interval $\sim 10^{14}$ seconds. It is important now to consider the conductivity condition implied by this requirement.

The time of decay of a field of linear dimension $L$ is of order $\sigma L^2/c^2$, where $\sigma$ is the conductivity and $c$ is the velocity of light (cf. Cowling **10**, page 5 for example). Hence we require

$$\sigma > \sim 10^{14} c^2 L^{-2}.$$

In the solar system problem $L \cong 10^{12}$ cm, so that the condition becomes

$$\sigma > \sim 10^{11}\,\text{sec}^{-1}. \tag{15}$$

For a wholly ionized gas the conductivity parallel to the magnetic field is only slightly dependent on the density of the gas. At particle densities of the general order occurring in the solar system problem—$10^{16}$ atoms per $\text{cm}^3$, or less, the value of $\sigma$ is of order $10^{13}\,\text{sec}^{-1}$. (cf. Spitzer **11**, page 81), so that (15) would be satisfied in this case with a factor $\sim 10^2$ to spare.

But of course we are not concerned with a wholly ionized gas; and we are not concerned with electric currents wholly parallel to the magnetic field. We take these complications in turn.

(i) *The conductivity parallel to the field for a partially ionized gas*

The effects of partial ionization have been considered by Piddington (**12**) and later by Cowling (**10**, page 99). The following formulae of this section are taken from Cowling's treatment of the problem.

We neglect negative-ion attachment. In view of the flux of dissociating radiation emitted from the surface of the solar condensation this would seem correct. Negative-hydrogen is likely to be the most important ion, and this has an attachment energy of 0·75 eV, which falls close to the maximum of the Planck radiation curve for a body with surface temperature $\sim$ 1 000–2 000 deg K; and this is exactly the order that the surface temperature of the primitive Sun may be expected to have had at a time when its radius was $\sim 10^{12}$ cm.

For the present purpose it is therefore sufficient to consider the planetary material to be a simple mixture of neutral hydrogen atoms and of ionized hydrogen atoms. We write $n$ for the number of neutral hydrogen atoms per $\text{cm}^3$, and $xn$ for the number of ionized atoms.

It turns out (**10**) that the conductivity is almost wholly unaffected by the presence of the neutral atoms so long as

$$x\Sigma > \Sigma_e \tag{16}$$

where $\Sigma$ is the collision cross-section between electrons and protons, and $\Sigma_e$ is the collision cross-section between electrons and neutral atoms. For material at temperatures of order 1 000°K, the value of $\Sigma$ is of order $10^{-10}\,\text{cm}^{-2}$, while $\Sigma_e$ is of order $10^{-15}\,\text{cm}^{-2}$. Hence the neutral atoms produce no important modification in the conductivity provided $x > 10^{-5}$. As we have already seen, condition (15) is then satisfied with a factor $\sim 10^2$ to spare. If, on the other hand, the fraction of atoms that are ionized falls below $10^{-5}$, the conductivity is reduced; and it simply falls linearly with $x$. *It follows immediately therefore that condition* (15) *remains satisfied provided $x$ does not fall below* $\sim 10^{-7}$.

This deals with the situation for currents parallel to the field.

(ii) *The conductivity perpendicular to the field for a partially ionized gas.*

The situation for the conductivity perpendicular to the field depends on the following rather complicated quantity

$$m\bar{u} \times M\bar{U}\left(\frac{nc}{eH}\right)^2 \Sigma_p(\Sigma_e + x\Sigma), \tag{17}$$

where $m\bar{u}$, $M\bar{U}$ are the mean electron and proton momenta respectively, $\Sigma_p$ is the collision cross-section for protons and neutral atoms, $e$ is the electron charge, and $H$ is the magnetic intensity. If the expression (17) is $> 1$ the conductivity perpendicular to the field is essentially unchanged from its value parallel to the field. If (17) is $< 1$, on the other hand, the conductivity perpendicular to the field is reduced by just this factor.

We now show that (17) exceeds unity in the solar system problem, so that we must again have $x > \sim 10^{-7}$. (It may be noted in this connection that if the magnetic field does not extend far outwards in the disk—and we have seen that there is no essential requirement that it should extend very much beyond the inner edge—it is of course unnecessary that ionization shall extend throughout the whole disk. It is sufficient if an adequate ionization be maintained near the inner edge of the disk.)

With $10^{40}$ cm$^3$ as a reasonable estimate for the total volume of the disk, and with $2 \times 10^{31}$ gm for the mass, the density $n \cong 10^{15}$ atoms per cm$^3$. With a temperature $\sim 1\,000$ deg K near the inside of the disk, we also have $m\bar{u} \cong 2 \times 10^{-20}$ gm cm sec$^{-1}$, $M\bar{U} \cong 7 \times 10^{-19}$ gm cm sec$^{-1}$. Setting $\Sigma_e \cong \Sigma_p \cong 10^{-15}$ cm$^{-2}$, and $H \cong 1$ gauss, as calculated in the previous section, we find that (17) is of order $50(1 + x\Sigma/\Sigma_e) > 50$.

*Hence we conclude that, provided at least one in $\sim 10^7$ of the hydrogen atoms near the inner edge of the disk is ionized, the conductivity is high enough to prevent any serious ohmic decay of the currents that maintain the torque coupling between the Sun and the disk.*

Although the required degree of ionization is small, it is not immediately clear that it would in fact be maintained. Under thermodynamic conditions at temperatures of order $1\,000\,^\circ$K the fraction of hydrogen atoms that are ionized falls considerably below the necessary proportion, i.e. $\ll 10^{-7}$. But in view of the non-thermodynamic activity of the present-day Sun it would be rash to argue on these grounds that the requisite ionization could not be maintained. Further discussion of this question will be postponed until the end of the following section.

## 7. ENERGY CONSIDERATIONS

We have so far paid attention only to the conservation of angular momentum. Energy must also be conserved. The question immediately arises as to fate of the energy of rotation of the primitive solar condensation. When the rotation of the Sun is reduced by the magnetic torque, in what new form does this energy appear?

First we notice that the initial energy of rotation is not small. Returning to the notation of Section 3, the energy of rotation at the onset of instability is

$$\tfrac{1}{2}Mk^2\Omega^2 \cong \frac{1}{20}R^2M\Omega^2.$$

Using (2), this may be written as $\sim \frac{1}{20}(GM^2/R)$, and with $R \cong 3\times 10^{12}$ cm,

$$\frac{1}{20}\frac{GM^2}{R} \cong 5\times 10^{45}\,\text{erg}. \tag{18}$$

A portion of this energy is stored in the disk, since energy is needed to move the material of the disk outward from the Sun. The latter energy $\sim GMm/R$, where $m$ is the mass of the disk. Since $m \sim 10^{-2}M$, this is less than the energy available by a factor $\sim 5$. *Hence only a modest proportion of the energy is stored in the disk.*

The rotation does work against the magnetic field, so that the rotational energy appears first as magnetic field energy—it appears in the windings of the field.

Now the total magnetic energy of a field of intensity $\sim 1$ gauss, distributed throughout a region of linear dimension $\sim 3\times 10^{12}$ cm is only of order $10^{36}$ erg. Evidently then, the magnetic intensity must be enhanced by a factor $\sim 10^5$ in order that the magnetic energy shall rise to a value $\sim 5\times 10^{45}$ erg. The enhancement must come from the winding of the field, the necessary number of windings being also of order $10^5$. That is to say, the number of relative rotations between the solar consideration and the disk must be $\sim 10^5$. Since a time interval $\sim 10^{14}$ seconds is available, this implies only one relative rotation every 30 years or so, which is entirely plausible.

According to the discussion of Section 5 the great majority of these windings are stored inside the solar condensation. We saw that the tendency is for the lines of force to be wrapped on to the Sun as soon as the angle between the field and the radial direction exceeds $\pi/4$. To accommodate the large magnetic energy, it is clear that storage must take place throughout a considerable fraction of the whole volume of the central body. In fact the energy involved, $5 \times 10^{45}$ erg, raises a considerable storage problem, quite regardless of whether the energy is present as rotation or as a magnetic field—in either case the energy is sufficient to produce a considerable geometrical distortion of the whole solar condensation, the magnetic field having much the same effect in this respect as the rotation itself. Moreover, the presence of a large internal magnetic energy raises shrinkage problems of much the same sort as a rapid rotation does; namely that the magnetic energy increases with shrinkage more rapidly than the gravitational or the thermal energy—geometrical distortion becoming more severe as the field is compressed.

At first sight it might therefore seem as if little had been gained by the conversion of rotational energy into magnetic energy. But this is not so. With angular momentum conserved by transference to the disk, there is no conservation law to prevent the magnetic energy of the wrapped field from being dissipated. In fact the wrappings inside the central condensation can simply disappear provided the internal material moves in a suitable way. This would not be the case if these were a rigid shaft along the axis of rotation but in the absence of such a shaft the wrappings can simply be deformed away at the axis, in which case the magnetic energy is converted first into mechanical energy, then into heat, and ultimately into radiation that is lost from the system by emission from the surface of the solar condensation.

It is also highly probable that a mode of dissipation analogous to that which takes place at the surface of the present-day Sun, in sun-spots, flares, etc., would be set up. This view receives observational support in the abnormally intense non-thermodynamic activity that seems to take place at the surfaces of T Tauri stars. Such stars are known to be of recent formation and may well be in the process of dissipating large reservoirs of internally stored magnetic energy.

The possibility of a large scale non-thermodynamic activity occurring at the surface of the shrinking solar condensation adds plausibility to the ionization requirement discussed in the previous section. This requirement amounted to a maintained electron density $> \sim 10^8$ per cm$^3$ at the inner edge of the disk. When we consider that the present-day Sun readily maintains an electron density around itself that does not fall much below this lower limit, it is plainly reasonable to assume that under the much more violent conditions surrounding the solar condensation the conductivity requirement was satisfied with ease.

## 8. MASSIVE STARS OF HIGH SURFACE TEMPERATURE

Stars of high surface temperature do not possess deep convection zones. Consequently there is no mechanism for carrying the windings of a magnetic field into the deep interior of such a star. The magnetic pressure must therefore build up in the immediate sub-photospheric layers until the inward magnetic pressure at the surface is balanced by the outward pressure from the sub-photospheric layers. When this occurs no further storage of the windings of the field can take place within the star itself. But we have already seen that a disk of small mass also cannot accommodate the whole of the magnetic energy that is forthcoming from the winding process: it seems that the windings of the field must build up in such a disk until the energy becomes sufficient to drive the material of the disk to infinity, rather as a flywheel explodes if its rotation is steadily increased.

This suggests the following picture for stars of high surface temperature:

(i) rotational instability leads to the formation of a disk,
(ii) the disk must not only store the angular momentum of the central condensation, but also its rotational energy (through the winding of a magnetic field),
(iii) the rotational energy soon explodes the initial disk, thereby destroying the effectiveness of the process of transference of angular momentum.
(iv) further shrinkage of the central condensation again leads to rotational instability, and the whole process is repeated.
(v) after many repetitions, the central star is ultimately left at the verge of instability—i.e. in a state of rapid rotation.

This process appears actually to be taking place in the Be stars, and in shell stars such as Pleione (cf. **9**). It is also not out of the question that some such process may be responsible for the nova phenomenon (these are stars of high surface temperature and comparatively small mass at an advanced stage of evolution, not massive stars of main-sequence type like the Be stars). The nova phenomenon is certainly a surface effect. Moreover the ejection of material would seem to be an axially symmetric process, indicative of the importance of rotation.

One complication must be mentioned. A massive condensing star, even if it ultimately becomes a main-sequence star of high surface temperature, will not be at high surface temperature throughout the whole of its condensation. For example, a B star that condenses to the main-sequence with surface temperature 25 000 °K will have the much lower temperature of 5 000 °K at the stage of condensation when its radius $\sim 3 \times 10^{12}$ cm. This means that at precisely the stage where the first rotational instability may be expected to occur the surface temperature will be low, and there is every likelihood that the primitive condensation then possesses a deep convection zone. This implies that the transference of angular momentum may well be effective at first. But with continued shrinkage the surface temperature must rise, the convection zone will disappear, and the disk will ultimately be blown away from the star in the manner described above. From what has been said it will be realized, however, that no absolutely sharp line of demarcation exists between dwarfs and massive stars. In particular, an intermediate situation occurs for main-sequence condensations of spectral types A and F.

## 9. CONDENSATION IN THE PLANETARY MATERIAL NEAR THE INNER EDGE OF THE DISK

The surface temperature of the solar condensation cannot have been much in excess of 1 000 °K at the time of rotational instability (radius $\sim 3 \times 10^{12}$ cm). At this temperature all materials of high boiling point—the common metals, silicates, etc., must already have been condensed into a solid form, probably as fine smoke particles. It follows that all materials of high boiling point must already have been condensed when the disk of planetary material was first formed. The important question now arises: would solid particles be carried outward by the gaseous component of the material as it gradually took up the angular momentum of the solar condensation?

In this connection we note that the magnetic field itself cannot have an appreciable effect on solid particles, so that such particles will not move outwards unless they are swept along by a purely hydrodynamic interaction with the gas.

It will be shown below that if their diameters remain less than $\sim 10^2$ cm solid particles are indeed swept along by the gas, but that if aggregation to a size greater than this takes place the particles will be left behind in the inner regions of the disk; i.e. at a distance from the Sun of order $10^{13}$ cm (whereas the bulk of the gas moves outwards to distances of $10^{14}$ $-10^{15}$ cm).

The gaseous material of the disk must experience a steady transverse acceleration ($f$ say) in order that it shall gradually increase its distance $r$ from the Sun, the value of the acceleration being given by (cf. **13**, page 237)

$$f = \left(\frac{GM}{4r^3}\right)^{\frac{1}{2}} \frac{dr}{dt}, \tag{19}$$

where $t$ is the time. This acceleration causes the orbital velocity of the gas to be slightly greater than the circular speed, the excess being determined by the velocity increment that the acceleration $f$ produces in a time interval of the order of the orbital period, i.e. in $\sim (r^3/GM)^{\frac{1}{2}}$. This means that the orbital velocity of the gas exceeds the circular speed by an amount of order $dr/dt$—as would be expected on general grounds.

Suppose that a solid particle is *not* swept along by the gas, but that it continues to move around the Sun in a more or less fixed circular orbit. Then there will be a transverse relative velocity $\sim dr/dt$ between the particle and the gas. This produces a transverse acceleration on the particle tending to carry the particle along with the gas. In order that the particle shall *not* be carried by the gas, the acceleration must evidently be less than $f$. (It may be noted that there is also a radial acceleration of similar magnitude acting on the particle, but this does not produce a systematic increase in the mean distance from the Sun (cf. **13**). In physical language, the effect produced in one part of the orbit by a radial acceleration is cancelled in the diametrically opposite part of the orbit. In a similar way, no systematic effect arises from a non-circular motion of the particle. For these reasons we consider only the transverse acceleration for the case of nearly circular motion.)

We may consider the particle to be a sphere of diameter $d$. The acceleration on such a sphere is given to sufficient accuracy by the well-known Stokes formula

$$3\pi \frac{\eta d}{\mu} \frac{dr}{dt}, \tag{20}$$

where $\eta$ is the viscosity, and $\mu$ is the mass of the particle. (The value of $d$ obtained below is greater by $\sim 10^2$ than the mean free path in the gas—hence it is appropriate to use the Stokes formula here.) Writing $\mu = \pi\sigma d^3/6$, $\sigma$ being the density of the material of which the particle is constructed, the condition that (20) be less than (19) yields

$$d > 6\left(\frac{\eta}{\sigma}\right)^{\frac{1}{2}}\left(\frac{r^3}{GM}\right)^{\frac{1}{4}}. \tag{21}$$

Putting $\eta = 8{\cdot}71 \times 10^{-5}$ gm cm$^{-1}$ sec$^{-1}$ for a gas composed mainly of hydrogen, $\sigma = 3$ gm cm$^{-3}$, $r = 10^{13}$ cm, $M = 2 \times 10^{33}$ gm, gives

$$d > \sim 10^2 \text{ cm}. \tag{22}$$

This is the condition that the particle shall not be swept along by the gas.

Since it is to be expected that the high boiling point materials would initially condense into a fine smoke, it is clear that the solid particles must be carried along by the gas unless a considerable measure of aggregation takes place. It is possible to take two quite opposite points of view about the probability of aggregation. If the smoke possessed the properties of finely pulverized rock it is difficult to imagine much aggregation taking place. It can, however, be argued that the planets did in fact form, and that the first condensations presumably grew through the aggregation of solids, not through gravitation. And if it was possible for a planetary condensation to aggregate, how much more easily might bodies with diameters of only a few metres be aggregated?

The whole question of aggregation would seem to turn on the precise nature of the smoke particles, on whether the different elements and compounds were separated into different particles, or whether each particle was a mixture of materials. The former case would give a stronger chemical binding of the atoms and molecules within the individual particles, and would seem less likely to favour an aggregation process. But the second case in which the first small smoke particles consist of more or less incoherent mixtures could well lead to aggregation, since chemical energy released by the sorting of materials would then be available to promote the aggregation.

To follow this question a little further, it may be pointed out that the sudden cooling of a high temperature gas would be far more likely to produce solid particles of incoherent composition than would be the case for a slow cooling. Because of the convective instability of the outer regions of the solar condensation, individual smoke particles could not remain permanently at the surface, but would soon be carried inwards by the convection. Their place at the surface would be taken by a steady supply of new particles condensing from cooled gas rising from the inner regions. Hence we must think of a dynamic circulation: hot gas rising to the surface is rapidly cooled and smoke condenses; then, after remaining at the surface for a little while, the gas and smoke particles fall inwards to regions of much higher temperature where the particles evaporate.

## 10. THE TERRESTRIAL PLANETS

The terrestrial planets are characterized by three crucial properties:

(1) their masses are small compared with the total mass of the planetary material,
(2) they are composed very largely of low volatility materials,
(3) their orbits lie within the main distribution of the planetary material.

These properties are immediately explained if we suppose that aggregation to bodies with diameters $> \sim 10^2$ cm did in fact take place for a considerable proportion of the low volatility materials.

In an earlier section it was seen that the total mass of the planetary material must have been ~ 3 000 Earth-masses, If the composition was of the normal "cosmic" form, the low volatility materials would contribute about 5 Earth-masses to this total. To see this, we note that the main contributions come from magnesium, silicon, and iron. According to Suess and Urey (**14**) the ratio of the abundances by mass of these elements to the total abundance (essentially hydrogen plus helium) are $4{\cdot}10\times 10^{-4}$ : $5{\cdot}2\times 10^{-4}$ : $6{\cdot}3\times 10^{-4}$ : 1 respectively. The combined abundance is thus $\sim 1{\cdot}6\times 10^{-3}$ : 1. It follows immediately that, if the total mass of material ~ 3 000 Earth-masses, the low volatility materials contribute ~ 5 Earth-masses (allowance for oxygen present in metallic oxides would increase this estimate somewhat).

The materials of the terrestrial planets are thus provided for, if about half of the low volatility materials become aggregated into particles of appreciable size that were left behind in the inner regions of the planetary disk.

## II. THE MAJOR PLANETS

It is to be expected that water and ammonia condensed from the planetary material when the latter reached a distance from the Sun of order $10^{14}$ cm (cf. Urey **15**, page 120). This means that, with the exception of H, He, Ne, and C (present as $CH_4$), the common non-metals condensed from the gas at distances ranging from the radius of the orbit of Jupiter to that of Saturn. Solid (or liquid) particles with diameters $< \sim 10$ metres would tend to be swept still farther outward with the gas until distances $\sim 5\times 10^{14}$ cm were reached. Particles with diameters $> \sim 10$ metres would be left behind, however, in the same way as the materials of the terrestrial planets were left behind. This can be seen from (21), using appropriate values for $\sigma$, $r$.

The aggregation of ice particles may well have constituted the first stage in the formation of all four major planets. The total mass of ice must have been of the order of 30 Earth-masses (in "cosmic material" oxygen has an abundance by mass of about 1 per cent), which constitutes such a large quantity of material that the first condensations would readily have grown to the stage where the gravitational accretion of gas became important. It is here that a crucial difference between the pair Jupiter and Saturn, and the pair Uranus and Neptune, emerges:

*By the time the first aggregations become large enough for the gravitational accretion of gas to be a possibly important process, the bulk of the gas must already have escaped from the periphery of the solar system* (*i.e. from the region of Uranus and Neptune*).

This is precisely to be expected so far as hydrogen is concerned, for hydrogen will escape from the periphery of the solar system in a time scale $\sim 10^{14}$ seconds if the temperature of the gas $\sim 75\,^\circ$K and this is just the order of the "black-body" temperature of the gas (cf. Urey **15**, page 118).

The time scale for the thermal evaporation of gas can be obtained from a calculation similar to the calculation of the loss of gas from planetary atmospheres (cf. **16**, for example). The relevant formula is

$$\frac{\sqrt{(6\pi)}}{3g}\left(\frac{3kT}{m}\right)^{\frac{1}{2}}\frac{e^Y}{Y}, \tag{23}$$

where

$$Y = \frac{mv_\infty{}^2}{2kT},$$

and $m$ is the atomic or molecular mass under consideration.

$g$ is the solar gravitational acceleration,

$k$ is Boltzmann's constant,

$T$ is the temperature,

$v_\infty$ is the thermal velocity required for escape to infinity from the solar system.

We set $m = 3{\cdot}3 \times 10^{-24}$ gm for molecular hydrogen, and $g = 10^{-3}$ cm sec$^{-2}$ at the periphery of the solar system. Care is needed in assessing $v_\infty$. The total velocity of escape to infinity from the position of Uranus $\sim 10$ km per sec. We must remember, however, that owing to their orbital velocity in the disk hydrogen molecules already possess some 7 km per sec. Thus an additional 3 km per sec contributed thermally will promote escape, provided the thermal contribution is parallel and in the same sense as the orbital velocity. In assessing the rate of escape of hydrogen, it is this latter favourable case that must be used, so that $\sim 3$ km per sec represents the correct choice for $v_\infty$. Finally by equating (23) to the time scale $\sim 10^{14}$ sec, we obtain $T \cong 65\,^\circ$K. This value is a slight underestimate since (23) exaggerates the escape rate because only molecules that move nearly parallel to the orbital velocity can escape. Allowance for this effect raises the temperature to $\sim 75\,^\circ$K.

The present result, that hydrogen escapes thermally from the periphery, applies *a fortiori* for the reason that the "black-body" temperature almost certainly is a lower-limit to the actual temperature. Once most of the water, ammonia, etc., is condensed out, the gas becomes a very poor radiator in the infra-red. It remains, however, an efficient absorber in the ultra-violet. Under such circumstances, and with an input of ultra-violet radiation from the Sun, the kinetic temperature of the gas can rise appreciably above the black-body value. Indeed once $H_2$ has escaped, radiation in the infra-red must depend mainly on $CH_4$, in which case it is entirely possible that the temperature of the gas rises as high as 150°K, when helium would also escape into space.

## 12. THE SATELLITE SYSTEMS OF JUPITER AND SATURN

The satellites of Jupiter fall into two groups: those that lie beyond Callisto appear to be captured bodies of small mass, their orbits possessing elements with marked irregularities such as would be expected to arise from a process of capture; in contrast, the four Galilean satellites possess such highly regular orbits that it is difficult to avoid the impression that their origin (and possibly that of the innermost satellite also) must have been associated with the origin of Jupiter itself.

A similar situation plainly exists for the satellites of Saturn, and probably also for Uranus and Neptune, although the situation is less clear in the latter cases since the available observational information on the existence of small irregular satellites is less complete.

It is tempting to argue that the regular satellites were formed in a manner similar to the process by which the planets themselves were formed; that the original planetary condensations were rotationally unstable; that the planets grew disks; that solids condensed within the disks, and that the satellites were formed by the aggregation of these solids; that the gaseous component of the disks escaped from the parent planets, leaving only the satellites. The first of these points can be checked by a direct calculation. The original angular momenta of Jupiter and Saturn can be estimated. This was done by the writer some years ago (**17**, Section 4). It was shown that indeed these planets were initially unstable: their original angular momenta exceeded present-day values by a factor $\sim 3$.

It is clear, however, that the process of disk formation must have differed from the planetary case in one crucial respect: the satellite disks must have carried proportionately much more mass. This can be seen very simply from a composition argument.

Calculating for the case of Jupiter, the combined mass of the Galilean satellites amounts to a fraction $\sim 2 \times 10^{-4}$ of the mass of Jupiter itself. These satellites must be composed to a large extent of low volatility materials which cannot originally have comprised more than about 0·1 per cent of the total mass of the disk. Evidently then, the original total mass of the Jovian disk must have been as much as $\frac{1}{5}$ of the primary.

This rather surprising result is consistent with angular momentum requirements. If most of the material of the disk existed at, or was pushed out to, the distance of Callisto—the most distant of the Galilean group—a mass $\sim \frac{1}{5}$ for the disk is required in order that the disk be able to store an amount of angular momentum that would make Jupiter rotationally stable. (If the disk were less extensive than the orbit of Callisto its mass would of course need to be still greater.)

Similar results can be obtained for Saturn, although in this case the composition argument is less secure because of uncertainty in the density (and hence in the composition) of Titan. If we accept a density $\sim 2{\cdot}0$ for Titan, an appreciable fraction of this satellite—the only important one in the case of Saturn—may well be composed of low volatility materials, in which case the composition calculation yields a closely similar result to that obtained for Jupiter.

The reason for this crucial difference between the planetary and the satellite cases is highly significant and is not far to seek. It is most unlikely that the conductivity was adequate to maintain a magnetic torque coupling between the planetary condensations and their surrounding disks.

*The process of shedding of material into the disks should therefore have followed the purely hydrodynamic lines discussed in Section* 3. Thus if we suppose the Jovian disk to have been first formed at the radius of Callisto's orbit, equation (6) may be applied to give the ratio of the mass of the disk to that of the planet itself. The result is

$$\sim(26{\cdot}4^{1/17} - 1) \simeq 0{\cdot}212,$$

in excellent agreement with the above estimate of $\frac{1}{5}$ from the composition argument.

In Section 3 we saw that the purely hydrodynamic formation of a disk leads to the following three properties (1), (2) and (3), and that these properties are not obeyed by the planets:

(1) the mass of the disk is of the same order, but less than, the mass of the central condensation,
(2) the system is comparatively compact,
(3) the central condensation is left in a state of rapid rotation.

These three properties are all satisfied by the systems of regular satellites. Indeed (2) has often been remarked as a crucial difference between planets and satellites.

The essential difference would seem to be that, whereas the formation of satellites was hydrodynamic, the formation of planets was hydromagnetic.

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