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EVIDENCE FOR A WORLD CIRCULATION PROVIDED  
BY THE MEASUREMENTS OF HELIUM AND WATER  
VAPOUR DISTRIBUTION IN THE STRATOSPHERE

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SUMMARY

Information is now available regarding the vertical distribution of water vapour and helium in the lower stratosphere over southern England. The helium content of the air is found to be remarkably constant up to 20 km but the water content is found to fall very rapidly just above the tropopause, and in the lowest 1 km of the stratosphere the humidity mixing ratio falls through a ratio of 10—1.

The helium distribution is not compatible with the view of a quiescent stratosphere free from turbulence or vertical motions. The water-vapour distribution is incompatible with a turbulent stratosphere unless some dynamic process maintains the dryness of the stratosphere. In view of the large wind shear which is normally found just above the tropopause it is unlikely that this region is free from turbulence.

The observed distributions can be explained by the existence of a circulation in which air enters the stratosphere at the equator, where it is dried by condensation, travels in the stratosphere to temperate and polar regions, and sinks into the troposphere. The sinking, however, will warm the air unless it is being cooled by radiation and the idea of a stratosphere in radiative equilibrium must be abandoned. The cooling rate must lie between about 0.1 and 1.1°C per day but a value near 0.5°C per day seems most probable. At the equator the ascending air must be subject to heating by radiation.

The circulation is quite reasonable on energy considerations. It is consistent with the existence of lower temperatures in the equatorial stratosphere than in polar and temperate regions, and if the flow can carry ozone from the equator to the poles then it gives a reasonable explanation of the high ozone values observed at high latitudes. The dynamic consequences of the circulation are not considered. It should however be noted that there is considerable difficulty to account for the smallness of the westerly winds in the stratosphere, as the rotation of the earth should convert the slow poleward movement into strong westerly winds.

I. INTRODUCTION

Between 1943 and 1945 the writer made some 16 ascents into the stratosphere over Southern England during which humidity measurements were made by means of a frost-point hygrometer. On some of the ascents the carbon dioxide (CO<sub>2</sub>) content of the air of the stratosphere was measured by using the hygrometer at the CO<sub>2</sub> point. The hygrometer has been described by Brewer, Dobson and

Cwilong (1947) and will not be described here. The results obtained on four typical ascents are shown in Figs. 1, 2, 3 and 4 in which the temperature and frost-point are plotted against height for the upper parts of the ascents. The  $\text{CO}_2$  measurements gave the result that the  $\text{CO}_2$  content of the atmosphere (0.3 per cent) is substantially independent of height. Following the measurements of Gluckauf and Paneth (1945), who found only very small increases in the helium content of the air up to 25 km, the constancy of the carbon dioxide content was expected.

As regards the water content, all the ascents show that immediately above the tropopause there is a very rapid fall of frost-point, and above 1 km above the tropopause the air is very dry, with a frost-point of the order of  $195^\circ$ — $200^\circ\text{A}$ . The mean frost-point observed at the summit of those ascents which penetrated more than 1 km into the stratosphere is  $194^\circ\text{A}$ , and the mean relative humidity at the summits is about 3 per cent. The mean frost-point at the tropopause is  $215^\circ\text{A}$  and the ratio between the corresponding humidity mixing ratios at the tropopause and the summits of the ascents is about 20—1. Since every ascent shows the effect, and since the air travels great distances in short times, there seems every reason to presume that the dryness of the stratosphere is to be found in all temperate latitudes.

If we consider the way in which temperature, humidity mixing ratio and carbon dioxide or helium content vary with height in the lower stratosphere the differences are striking. The constancy of the  $\text{CO}_2$  and helium content suggests considerable turbulence which maintains uniform concentration in spite of gravitational settling out, but the temperature and humidity distributions are not consistent with significant turbulence. The temperature distribution may be explained by the effects of radiation but the dryness of the stratosphere can only be maintained against the effects of turbulence if some mechanism is continuously drying the stratosphere air. The purpose of this note is to examine the probable cause of these apparent contradictions and to consider how the facts may be reconciled.

## 2. THE TURBULENCE OF THE LOWER STRATOSPHERE

The constancy of the  $\text{CO}_2$  or helium content of the air up to about 20 km is generally accepted as evidence of the existence of turbulence in the stratosphere in spite of its stable temperature distribution, but unfortunately there is very little direct evidence of the extent to which turbulence does in fact occur. To an observer in an aircraft the stratosphere normally seems smooth but quite severe bumps are *occasionally* felt. The air in the immediate vicinity of the tropopause is often quite rough but this is by no means the rule. Also there is no evidence that the smoke from shell bursts in the stratosphere behaves significantly differently from that from bursts in the troposphere.

If we turn to theoretical evidence we may follow L. F. Richardson (1920) and regard the turbulence as the balance between forces due to the shearing of the wind, which generate turbulence, and the stability of the air, which suppresses it. In the region just

above the tropopause where we find the sharp fall in water content, the lapse rate is always stable and there is often a temperature inversion which gives great stability. But also there is often a very strong wind shear, for just above the tropopause the wind speed usually decreases rapidly with height. As a result, as L. F. Richardson remarks in his paper, the lower stratosphere is probably a source of atmospheric eddies, second only in importance to the layers near the ground. Unfortunately there is no modern analysis of the application of Richardson's criterion to the lower stratosphere though Petterssen and Swinbank (1947) have discussed its application to the troposphere.

We may also use the observed variations in the helium content of the air to obtain a qualitative value for the diffusion coefficient in the stratosphere, though in interpreting the results we must remember that the mechanism which maintains the dryness of the stratosphere, and which certainly is not turbulence, may also help to keep the proportion of the non-condensable vapours and gases constant.

Neglecting entirely the effects of horizontal or vertical advection we may write the diffusion equation in the form

$$\frac{\partial p'}{\partial t} = \frac{\partial}{\partial z} \left\{ k \left( \frac{\partial p'}{\partial z} - \gamma \right) \right\} \quad (1)$$

where  $p'$  is the helium partial pressure,  $k$  is the diffusion coefficient and  $\gamma$  is the lapse rate obtained for infinitely large values of  $k$ . If we consider molecular diffusion then  $k$  becomes the molecular diffusion coefficient  $D$ , and  $\gamma$  is the lapse rate of partial pressure given by Dalton's law  $=gp'/R'T$  where  $R'$  is the gas constant for helium.

If we consider turbulent diffusion then  $k$  becomes the coefficient of eddy diffusion  $K$  and  $\gamma$  is the lapse rate of partial pressure of helium under complete turbulent mixing  $=gp'/RT$  where  $R$  is the gas constant for air.

In the steady state the two processes will both be occurring and will balance and we have

$$\frac{\partial p'}{\partial t} = \frac{\partial}{\partial z} D \left( \frac{\partial p'}{\partial z} - \frac{gp'}{R'T} \right) + \frac{\partial}{\partial z} K \left( \frac{\partial p'}{\partial z} - \frac{gp'}{RT} \right) = 0 \quad (2)$$

in the steady state.

Integrating we have

$$D \left( \frac{\partial p'}{\partial z} - \frac{gp'}{R'T} \right) + K \left( \frac{\partial p'}{\partial z} - \frac{gp'}{RT} \right) = 0 \quad (3)$$

$$\text{since } p \rightarrow 0 \text{ and } \frac{\partial p}{\partial z} \rightarrow 0 \text{ as } z \rightarrow \infty$$

or rearranging

$$\frac{dp'}{p'} = \frac{\frac{D}{R'} + \frac{K}{R}}{D + K} \frac{g}{T} dz = \frac{D \frac{R}{R'} + K}{D + K} \frac{dp}{p} \quad (4)$$

where  $p$  = air pressure

$$\text{since } \frac{gp}{RT} dz = dp$$

If for simplicity, to see the magnitude of the effects, we consider an isothermal stratosphere in which  $D$  and  $K$  are constant, then on integrating, we have

$$\frac{\log p'_1/p'_2}{\log p_1/p_2} = \frac{D \frac{R}{R'} + K}{D + K} \quad (5)$$

where the suffixes refer to the heights  $z_1$  and  $z_2$ . But

$$p'_1/p'_2 = p_1/p_2 (1+f) \quad (6)$$

where  $f$  = the fractional change in composition. Hence

$$\frac{\log (1+f)}{\log p_1/p_2} = \frac{D \left( \frac{R}{R'} - 1 \right)}{D + K} \quad (7)$$

Now the diffusion coefficient  $D$  for helium in air in the stratosphere will increase from about  $1.0 \text{ cm}^2 \text{ sec}^{-1}$  at the tropopause to about  $3.0$  at  $20 \text{ km}$ , and Gluckauf and Paneth find that at about  $20 \text{ km}$  there is a just detectable increase in the helium content of the air of about  $0.5$  per cent for which  $\log_{10} (1+f) = 2 \times 10^{-3}$ . If we assume that all of this settling out occurs in the stratosphere between  $10 \text{ km}$  and  $20 \text{ km}$  then  $\log_{10} p_1/p_2 = 0.47$ . Hence, substituting in (7) we get  $K \approx 400 \text{ cm}^2 \text{ sec}^{-1}$ .

Now this value of  $K = 400$  is about the least value of  $K$  in the stratosphere which, if there is no horizontal or vertical advection, can be considered reasonable. If it is assumed that separation is also occurring equally below  $10 \text{ km}$  then the value of  $K$  must be increased somewhat. Further the value of  $400$  is obtained on the assumption of constant  $K$  independent of height up to  $20 \text{ km}$ .

Now Gluckauf and Paneth's results show, in general, a significant increase in the helium contents above  $20 \text{ km}$  and it seems most likely that the upper stratosphere is rather less turbulent than the lower stratosphere and in this case a  $K$  rather larger than  $400$  must be expected in the lower stratosphere. This is quite reasonable since it is in the region just above the tropopause where the strong wind shear is normally found.

The evidence of the turbulence of the stratosphere is therefore consistent with the view that just above the tropopause the air is turbulent with a  $K$  of the order of  $10^3$  but at higher levels the air is probably less turbulent, with a  $K$  of the order of  $10^2$ . In what follows we shall assume that  $K$  just above the tropopause is about  $10^3$ . A significantly lower value seems hardly tenable, in view of the large wind shears which are present.

### 3. THE DRYNESS OF THE STRATOSPHERE

If  $K$  for the lower stratosphere is as large as  $10^3$  how is the great dryness of the air only one or two km above the tropopause of southern England maintained? Two methods can be suggested (a) there is photo-chemical destruction of water or (b) by the advection of stratosphere air from the equator where the tropopause temperature is sufficiently low to condense the water out (Fig. 5).

The idea that photo-chemical destruction of water occurs in the stratosphere is not attractive and no serious suggestions have been

put forward which could account for the large scale destruction of water at any level. It is particularly difficult to see how photolysis of water could occur so low in the atmosphere as the lower stratosphere. If the destruction occurs at high levels then significant and widespread downflow in the stratosphere is necessary to bring the dry air down, and this would require an upward flow elsewhere to supply it. Since the upward flow would most probably be at the equator, this becomes equivalent to a circulation via the equatorial tropopause which by itself can account for the dryness of the stratosphere. Photolysis has been discussed by Bamford (1943) and will not be considered further here. It seems most improbable that this effect is significant.

The lowest frost point which has been measured in the stratosphere is  $189^{\circ}\text{A}$  and this is subject to an uncertainty of two or three degrees. If the air is never significantly drier than this then the dryness can be explained by assuming that the air has been dried at the equatorial tropopause where temperatures of  $190^{\circ}\text{A}$  are commonly found. The excess water will be condensed out and will fall as snow. If allowance is made for the compression between the equatorial tropopause which is at a pressure level of about 100 mb, and the highest level of our ascents at about 170 mb, then the lowest frost points which would be expected would be about  $190\text{--}195^{\circ}\text{A}$ .

#### 4. CAN THE DRYNESS BE MAINTAINED BY MOVEMENTS IN THE STRATOSPHERE PARALLEL TO THE TROPOPAUSE?

If the dryness is maintained by advection from the equator by movement of air parallel to the tropopause, without any interchange of air whatever between the troposphere and stratosphere, and if we assume  $K$  the coefficient of turbulent diffusion constant with height, then we may follow Taylor (1915) to calculate the water vapour distribution after diffusion has been effective for a time  $t$ , that is, roughly, since the air was last dried at the equator. Taylor's treatment shows that if  $z'$  is the height at which the water content has fallen to  $1/10$  of the value of the tropopause then  $z'/\sqrt{4Kt}=1.2$ . For our results  $z'$  is about 1 km so that if  $K=10^3$  then  $t\approx 20$  days and if  $K=2\times 10^3$  then  $t\approx 10$  days.

Thus to maintain the observed dryness of the stratosphere by motions parallel to the tropopause would require that the air be returned to the equator to be redried every 10 or 20 days, corresponding to advection speeds of the order of 10 or 20 km per hour. In almost all our measurements the air track could be traced for four or five days previously and in most cases the origin was in Canada and there was never any evidence of the air having recently been at the equator. Also, the potential temperature of the equatorial tropopause is some  $30^{\circ}\text{C}$  higher than the potential temperature where the dry air was observed, so that radiative cooling of some  $1\frac{1}{2}\text{--}3^{\circ}\text{A}$  per day would also be required and this is rather high. It is difficult to see the dynamic reasons why the air should move parallel to the sloping tropopause. Horizontal movement would bring moist air into the stratosphere, and this does not occur.

5. MAINTENANCE OF THE DRYNESS BY A CIRCULATION OF AIR

Alternatively we may consider that the dryness is maintained by a slow circulation of the air in which air rises at the equator moves poleward in the stratosphere and then descends into the troposphere in temperate and polar regions as shown in Fig 5. In this way, as air is moistened by diffusion upwards, so the sinking process will continually carry the moist lower layers into the troposphere and replenish the dry air from above. The descent of air will, in this way, confine the transition from the moist air of the troposphere to the dry air of the stratosphere into a shallow layer just above the tropopause.

We may consider this process in detail. The usual diffusion equation is

$$\frac{\partial r}{\partial t} - u \frac{\partial r}{\partial x} - v \frac{\partial r}{\partial y} - w \frac{\partial r}{\partial z} = \frac{\partial}{\partial z} K \frac{\partial r}{\partial z} \quad (8)$$

$r$  = humidity mixing ratio.  $u, v, w$  are the air velocities in directions  $x, y, z$ .  $z$  is vertical,  $K$  is the eddy diffusion coefficient. We now make the fairly reasonable assumption that  $K$  in the lowest one or two km of the stratosphere is constant, and if we ignore for the present the effects of horizontal advection the equation becomes:—

$$\frac{\partial r}{\partial t} - w \frac{\partial r}{\partial z} = K \frac{\partial^2 r}{\partial z^2} \quad (9)$$

In the steady state, when air is continuously descending into the tropopause, we have  $\partial r / \partial t = 0$  and the equation reduces to:—

$$K \frac{\partial^2 r}{\partial z^2} + w \frac{\partial r}{\partial z} = 0 \quad (10)$$

the solution to which is

$$\frac{r - r_0}{r_t - r_0} = e^{-\frac{w}{K} z} \quad (11)$$

where  $r_0$  is the humidity mixing ratio of the dry air supplied from above,  $r_t$  is the humidity mixing ratio at the tropopause and  $z$  is measured above the tropopause. This expression fits the observed water vapour height curves quite well the value of  $w/K$  varying on different occasions from  $1.8 \times 10^{-5}$  to  $5 \times 10^{-5}$  cgs units with a mean value of about  $3 \times 10^{-5}$ .

In Figs. 1-4 a curve according to Eq. (11) is plotted as a heavy line using a suitable value of  $w/K$  to give a reasonable fit. In the calculated curves  $r_0$  is taken to be equivalent to a frost point of  $190^\circ\text{A}$  and  $r_t$  is taken equivalent to the observed frost point at the tropopause. One of two of the observed frost points at the highest level of observation correspond to slightly drier air than might be expected on this theory. If this effect is real and is not due to errors of observation then drying must be occurring by some process other than condensation at the equatorial tropopause.

Now if we consider the value of  $w/K$  as  $3 \times 10^{-5}$  then if  $K = 10^8$ ,  $w$  becomes about 25 m/day and if  $K = 2 \times 10^8$  then  $w = 50$  m/day. This continuous sinking of the air in the stratosphere will of course warm the stratosphere by adiabatic compression and if the temperature is to be maintained at a constant value then there must be radiative cooling of 0.25 C/day for 25 m/day or 0.5°C for

50 m/day. There is no reasonable theory of diffusion which could account for the removal of such large quantities of heat from every layer of the lower stratosphere with a diffusion coefficient of the order of 1 or  $2 \times 10^3$ . Priestley and Swinbank (1947) suggest that on account of the effect of buoyancy forces the transfer of heat by

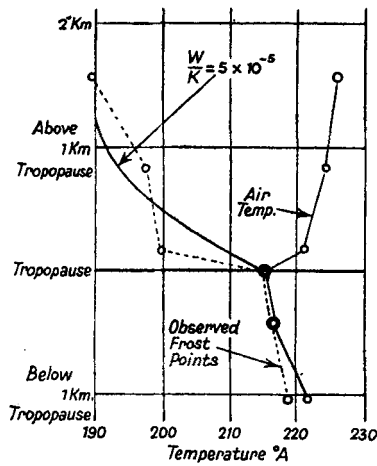


FIG. 1. 30 May 1945. Tropopause ht 33,400 ft.

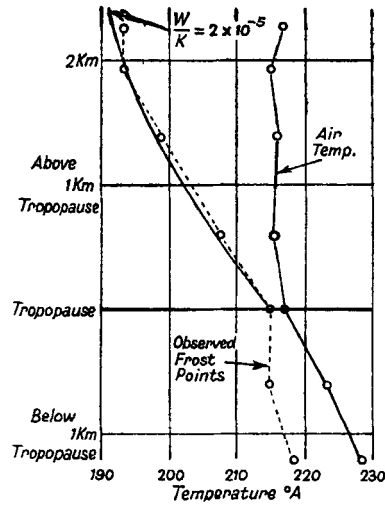


FIG. 2. 25 April 1945. Tropopause ht 34,000 ft.

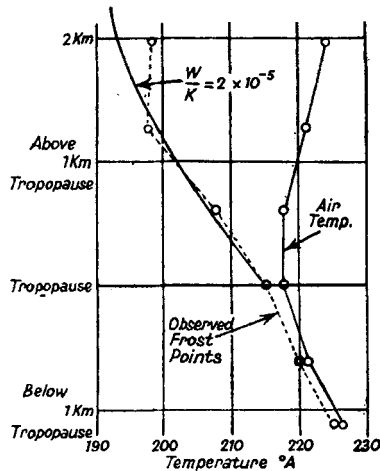


FIG. 3. 22 April 1945. Tropopause ht 34,000 ft.

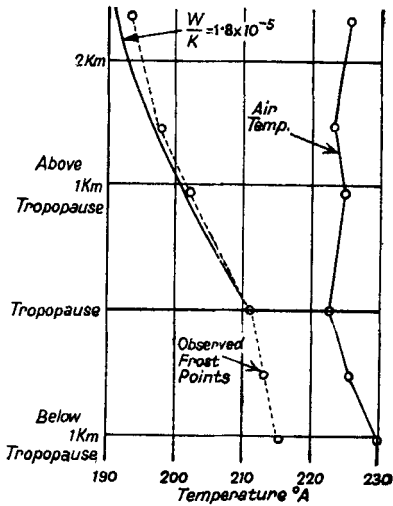


FIG. 4. 7 Feb. 1945. Tropopause ht 30,000 ft.

FIGS. 1-4. Distribution of temperature and frost point near the tropopause on four ordinary occasions, compared with the calculated frost point curves for the lower stratosphere.

turbulence in the stratosphere may be extremely small in spite of turbulence and the isothermal conditions.

On the other hand radiative cooling of the stratosphere by this amount is not unreasonable though a radical change is necessary

from the idea of a stratosphere which is in true radiative equilibrium. Elsasser (1942) suggests a possible rate of cooling of the stratosphere by outgoing radiation of  $1.1^\circ\text{C}$  per day. This would seem to be about the largest reasonable value for the rate of cooling and the temperature would be maintained by sinking at 110 m/day and the corresponding value of the diffusion coefficient to account for the water vapour profiles would be about  $4 \times 10^9$ .

The need for better understanding of the problems of the radiative interchanges of the atmosphere is clear. In particular the question of whether the stratosphere is in radiative equilibrium could be determined by measuring the variation in the vertical of the upward and downward total flux of radiation. These measurements, for all levels at which the measurements could be made, would be of great interest—and the results may well prove rather surprising.

#### 6. THE EFFECTS OF THE ADVECTION NECESSARY TO SUPPLY THE DOWN FLOW

If there is sinking of the air of the stratosphere over all temperate and polar regions and the air is rising in equatorial regions then there must be substantial mean horizontal poleward flow in the stratosphere of middle latitudes to supply the air which sinks into the troposphere in polar regions. Calculation of the horizontal velocity of the mean poleward flow enables us (a) to show that the effect of the horizontal advection of dry air is small compared with the effects of the vertical motion and (b) to calculate the approximate time taken by air in its journey from the equator to high latitudes.

We may calculate the horizontal velocity at latitude  $50^\circ\text{N}$ . if air is subsiding at velocity  $w$  out of the stratosphere in all areas N. of latitude  $50^\circ$ . The volume of air lost in unit time is  $w2\pi R^2(1 - \sin 50^\circ)$  where  $R$ =radius of the earth. If  $u$  is the mean velocity of poleward flow of the air in the stratosphere at latitudes  $50^\circ$  then the volume of air advected is  $ud2\pi R \cos 50^\circ$ , where  $d$  is the depth of horizontal flow. Equating the two volumes and writing  $d=3\text{km}=3 \times 10^5$  cm, which is half the approximate depth of the stratosphere if it were all at the pressure of the tropopause, gives us  $u/w \approx 600$ .

If  $w=25$  m/day then  $u=20$  km/day [say 0.5 kt], if  $w=50$  m/day then  $u=40$  km/day [1 kt]. Since the distance from equatorial to temperate latitudes is about 5,000 km then the average time taken by air will be of the order of 200 or 100 days according at  $K=10^9$  or  $K=2 \times 10^9$ .

We saw in considering whether the observed dryness of the stratosphere could be explained by the effects of advection alone that it would be necessary for air to have been dried at the equator within 10 or 20 days of the observation. If therefore the air spends on an average some 100 or 200 days in its journey from the equator then it may be deduced that the effects of the advection is only about  $\frac{1}{10}$  of the effect of the sinking and this may be confirmed by a more elaborate analysis.

#### 7. EFFECTS OF THE CIRCULATION OF AIR ON THE HELIUM CONTENT

The steady circulation of air into the stratosphere at the equatorial tropopause will, of course, carry air of normal helium



content and prevent settling out in accordance with Dalton's Law. In fact large-scale persistent movements of this kind are most powerful in opposing the effects of turbulent or molecular diffusion and a circulation so slow as to take many years for a complete change of stratosphere air would be sufficient to prevent gravitational settling out of the various constituents of the air.

For example, for a large volume of air which enters a region free from turbulence in a fully mixed state, we may calculate the time taken for the helium concentration to increase a small amount due to gravitational settling by evaluating the terms in Eq. (2) as follows

$$\frac{\partial p'}{\partial t} = \frac{\partial}{\partial z} D \left( \frac{\partial p'}{\partial z} - \frac{g p'}{R'T} \right) + \frac{\partial}{\partial z} K \left( \frac{\partial p'}{\partial z} - \frac{g p'}{RT} \right) \quad (2)$$

if there is no turbulence  $K=0$  and if the air is fully mixed

$$\frac{\partial p'}{\partial z} = \frac{g p'}{RT}$$

and we therefore obtain

$$\frac{\partial p'}{\partial t} = \frac{\partial}{\partial z} D \left( \frac{g p'}{RT} - \frac{g p'}{R'T} \right) = \frac{Dg}{T} \left( \frac{1}{R} - \frac{1}{R'} \right) \frac{\partial p'}{\partial z} \quad (12)$$

if  $D$  is constant with height, and substituting again for  $\partial p'/\partial z$  we obtain

$$\frac{1}{p'} \frac{\partial p'}{\partial t} = -D \left( \frac{g}{RT} \right)^2 \left( \frac{R'}{R} - 1 \right) \quad (13)$$

Writing  $\frac{R'}{R} = \cdot 13$  and  $\left( \frac{g}{RT} \right) = 1 \cdot 5 \times 10^{-6}$  and  $D = 3 \cdot 0$ , for conditions at 20 km we get

$$\frac{1}{p'} \frac{\partial p'}{\partial t} \approx 8 \times 10^{-12} \text{ sec}^{-1}.$$

For  $\frac{1}{p'} \frac{\partial p'}{\partial t} = \cdot 005$  the time taken is about 30 years.

At levels above 20 km Paneth and Gluckauf find increases of several per cent, for this degree of separation the time taken is two or three hundred years. This treatment is obviously not a "solution" of the problem since it ignores the effects of the boundaries but it should suffice to indicate the sort of speed at which changes can occur.

We may say therefore that if settling out is in fact occurring at levels above 20 km to the extent of a 3 or 5 per cent change in helium content then there is no systematic interchange of air between these levels and the troposphere at a rate faster than would correspond to a complete change of air every few centuries!\*

#### 8. PROBABILITY OF THE CIRCULATION AND THE ENERGY RELEASED BY IT

A circulation of air of this kind is only likely to occur if energy is released by it and the greater the amount of energy released, then the more likely the circulation becomes.

Shaw (1923) has considered a very similar circulation though he assumed that the air would travel poleward just beneath the

\* Note added in proof. There is probably no settling out; a recent measurement, made in the U.S.A. from a V2 rocket, suggests that there is no change in composition up to 80 km.

tropopause rather than in the stratosphere as is suggested here. Shaw calculates the energy released by the circulation to be  $23.7 \times 10^7$  ergs per gram and the thermodynamic efficiency of the cycle is 0.23. The large amount of energy liberated and the high efficiency makes the circulation very probable. Shaw suggests that this circulation may provide the energy necessary to maintain the general circulation of the atmosphere, but he notes that "The evidence for a direct descent of air through the layers beneath is not very strong". It is suggested that evidence is now available.

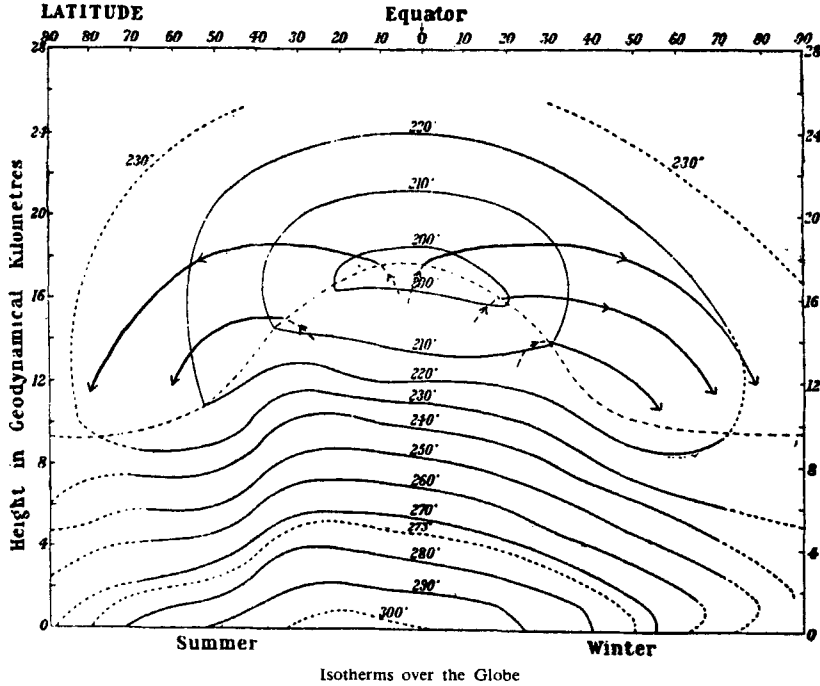


FIG. 5. A supply of dry air is maintained by a slow mean circulation from the equatorial tropopause.

We may use this to calculate the total energy released by the circulation. Let  $R$  be the radius of the earth and  $w$  the mean rate of sinking through the tropopause which is presumed to occur in all latitudes higher than  $45^\circ$ . Let  $\rho$  be the density of the air at the tropopause, then the weight of air circulated in both hemispheres is  $4\pi R^2 \rho w (1 - \sin 45^\circ)$  writing  $R = 6,000$  km and  $w = 50$  m/day we obtain  $2.5 \times 10^{18}$  gm per day as the weight of air circulated. This, by Shaw's data, would release  $23.7 \times 10^7$  ergs per gm or  $6 \times 10^{26}$  ergs per day. This would be increased to about  $10^{27}$  if  $w = 100$  m/day.

The energy required to maintain the general circulation has been variously estimated by different workers. Sverdrup (1918) estimates the requirement at 2 per cent of the solar energy which equals  $1.4 \times 10^{27}$  ergs/day, Brunt (1926) estimates the requirements at 5 watts/m<sup>2</sup> which is equal to  $2 \times 10^{27}$  ergs/day. It will be seen

therefore that if the subsidence of the stratosphere is as high as 100 m/day the circulation of air through the stratosphere could make a useful contribution to the energy of the general circulation.

#### 9. ANNUAL VARIATION OF THE RATE OF CIRCULATION

The energy released by the circulation will be increased in winter owing to the lower tropopause height and lower air temperatures at high latitudes so that the rate of circulation is probably increased in the winter. On the other hand the general activity at the tropopause will also increase in the winter which would result in a higher rate of upward diffusion of water vapour. On the balance therefore the water vapour height profiles may show little or no annual variation.

#### 10. EFFECT OF THE CIRCULATION ON THE STRATOSPHERE TEMPERATURE DISTRIBUTION

If air is rising in the equatorial stratosphere and sinking in the stratosphere of temperate and polar regions then for the isothermal state the air must be below its radiation equilibrium temperature at the equator, and above its equilibrium temperatures in regions where the air is subsiding. For cooling or heating as fast as 0.3 to 0.6°C per day quite significant departures from equilibrium temperature will be necessary, and this is quite possibly the reason for the latitude variation of stratosphere temperature.

To say that the stratosphere is not in radiative equilibrium does not necessarily imply that radiation is not in substantial control of the temperature but rather that the temperature will be a balance between radiative effects and the effects due to rising or sinking. Thus the temperature will be changed either if the radiation conditions change (say by variation of the ozone concentrations) or if the vertical movements change.

#### 11. EFFECT OF THE CIRCULATION ON THE LATITUDE VARIATION OF ATMOSPHERIC OZONE

This has been considered by Dobson, Harrison and Lawrence (1929) who then said "The only way in which we can reconcile the observed high ozone concentration in the Arctic in spring and the low concentration in the tropics, with the hypothesis that the ozone is formed by the action of the sunlight, would be to suppose a general slow poleward drift in the highest atmosphere with a slow descent of air near the poles. Such a current would carry the ozone formed in low latitudes to the poles and concentrate it there. If this were the case the ozone at the poles would be distributed through a moderate depth of atmosphere while that in low latitudes would all be high up".

At that time adequate data of the vertical distribution of ozone was not available. Modern data does, in fact, show that the mean height of the ozone is greater at the equator than at high latitudes though the effect seems to be rather less than might be expected. The spring maximum of ozone at the pole may be due to the combined effect of a slow decay of the winter strength of the circulation, which carries ozone to the pole, and increasing local formation due to the increasing insolation.

## 12. MOST PROBABLE RATE OF CIRCULATION

If a circulation is the cause of the various effects which we have considered, the rate of circulation, which may be expressed in terms of the sinking rate which occurs over southern England, may vary within wide limits. It seems probable that the constancy of the helium (to within  $\frac{1}{2}$  per cent up to 20 km) could be maintained by a circulation too slow to be significant, as has been discussed in section 7. The lowest limit to the circulation rate is determined by the molecular diffusion of water vapour and the sharp fall of water content above the tropopause. But no one would suggest that the air just above the tropopause is entirely free from turbulence and the lower limit to the circulation rate is determined by the lowest value of the diffusion coefficient which is likely to be envisaged there. A value of  $K=300$  might be suggested for which the water distribution could be maintained by a sinking rate of only 8 m/day and a radiation cooling rate of  $0.08^\circ\text{C}$  per day. But a value of  $K$  as low as 300 just above the tropopause seems most unlikely in view of the very large wind shear which is usually found there. Radiative cooling by amounts larger than  $0.08^\circ\text{C}$  per day must be possible in the stratosphere, as otherwise the stratosphere temperature could not adjust itself with the speed with which it often does do so, when disturbed by large N. or S. movements of air.

On the other hand radiative cooling of the stratosphere significantly faster than about  $1^\circ\text{C}$  per day, corresponding to a sinking rate of 100 m/day, hardly seems possible. This would correspond to a value of the coefficient of diffusion just above the tropopause over southern England of about  $4 \times 10^3$ . For the conditions this is a much more probable value for the diffusion coefficient.

On the balance a value of  $K=2 \times 10^3$ , and  $w=50$  m/day corresponding to a radiative cooling at a rate of  $0.50^\circ\text{C}/\text{day}$  seems a probable set of values which is reasonable in every way.

Since the value of  $w/K$  is relatively closely fixed the rate of circulation could be determined either by measuring the diffusion coefficient just above the tropopause, say by observing the dispersal of shell bursts, or by radiation measurements as suggested in section 5.

## CONCLUSIONS

A static stratosphere in radiative equilibrium and without turbulence or vertical motions is untenable on account of the observed constancy of the helium content up to about 20 km.

A turbulent stratosphere in radiative equilibrium but without steady vertical movements is untenable on account of the observed dryness of the stratosphere and the sharp transition zone between the moist troposphere and the dry stratosphere.

All the observed phenomena can be explained if it is assumed that air circulates by a slow mean motion into the stratosphere at the equator, moves poleward in the stratosphere and sinks into the troposphere in temperate and polar regions. On energy considerations, this is quite reasonable and a similar circulation has been suggested by Shaw. It is, however, then necessary to abandon the idea that the stratosphere is in radiative equilibrium as a steady

loss of heat is necessary in the polar and temperate stratosphere if subsidence occurs without disturbing the isothermal state.

The ratio of the mean subsidence rate to the mean value of the diffusion constant just above the tropopause can be fixed by the water vapour profiles fairly closely to  $3 \times 10^{-5}$  cgs units. In the absence of data of the rate of radiative cooling or of the degree of turbulence of the lower stratosphere actual values for  $w$  and  $K$  cannot be fixed. The values can probably be said to lie within the limits 300 and 4,000 cgs units and 8 and 100 m/day.

The matter can only be decided by measurements of  $K$  or of the radiative conditions of the stratosphere and both are possible.

The writer considers that  $K$  will prove to be of the order of 1 or  $2 \times 10^3/\text{cm}^2 \text{ sec}^{-1}$  and  $w$  about 50 m/day. If the circulation is as rapid as this it will make a significant contribution to the energy of the general circulation.

The dynamic consequences of the circulation have not been discussed. There are considerable difficulties in this respect.

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