On the Role of Interior Mixing and Air–Sea Fluxes in Determining the Stratification and Circulation of the Oceans

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ABSTRACT

The problem of determining the (eastern boundary) basic stratification and the buoyancy-driven circulation of the oceans is addressed. A global integral constraint relating the interior stratification and the air–sea heat fluxes is derived, based on the condition that the total mass of water of given density is constant in a steady state ocean. This constraint is then applied to two simple analytic models: The first is a continuous nonlinear diffusive model of the lower mid-depth and bottom circulation below the influence of the wind-driven circulation. It shows the tendency of the vertical density profile to look like an exponential profile in the presence of mixing. The integral constraint is used to relate the stratification and circulation of the bottom and mid-depth waters to the air–sea heat fluxes at the surface, where the deep densities outcrop. The second model is a layered one of the wind-driven circulation, mid-depth, and bottom water. The eastern boundary stratification of the model is determined from the air–sea heat fluxes, using the integral constraint and a parameterization of the mixing processes in layer models. A two gyre mid-depth circulation is found, driven by the cross-isopycnal diffusive velocities and affected by the variations in the depth of the main thermocline above it. The bottom circulation in both models is similar to that of the Stommel–Arons model.

Air–sea heat fluxes affect the deep buoyancy-driven flows not by direct cooling or heating, but through the formation of water masses that sink and spread in the deep ocean. Thermal boundary conditions for the interior thermocline problem seem to require specification of the air–sea heat fluxes in addition to the specification of the density distribution at the base of the Ekman layer. Cross-isopycnal mixing processes are a crucial part of the dynamics, although numerically small. Together with the air–sea heat fluxes, they determine the basic vertical stratification of the wind-driven and deep circulations.

1. Introduction

Theories of the oceanic general circulation are aimed at explaining the time–mean density and velocity fields in terms of the forcing by wind stress and heat fluxes at the upper surface of the ocean. However, the problem posed this way is very complicated, and researchers have usually tried to simplify the boundary conditions as well as the dynamics as much as possible. Among the first things to be sacrificed were the details of the thermodynamical processes. These simplifications allowed significant progress to be made, but also revealed several gaps in the theories that did not explicitly include the thermodynamics as part of the physics. These gaps hint that the density-changing processes, although weak in the ocean's interior, may nevertheless be a crucial part of the physics of the general circulation.

In this paper we try to explore several areas of the theoretical study of the oceanic general circulation in which some understanding may be gained by including the physics of the density changing processes.

In *thermocline theories,* air–sea heat fluxes were usually not considered explicitly, and one specified the density at the base of the mixed layer in order to account for the effect of these fluxes. The role of diffusion in classical thermocline theories also seems to be unclear: In some similarity solutions (Needler, 1967) the diffusion contributed a constant deep upwelling of no major dynamical importance, while ideal fluid thermocline theories (Welander, 1971) had at least as much success in explaining the structure of the thermocline as the diffusive ones. (Reviews of these efforts and earlier ones can be found in Veronis, 1969, 1981.)

This seeming unimportance of the diffusion, and its being smaller than the advection terms in the density equation, had led to two recent theories of the wind-driven thermocline circulation (Rhines and Young, 1982a; Luyten et al., 1983), which are both density conserving. These models were able to reproduce the horizontal variations in the depth of the thermocline and to demonstrate the importance of several physical processes, but had to specify the basic vertical density stratification on the eastern boundary (in addition to the outcrop positions). In particular, the thickness of the lower layers in the ventilated thermocline model has to be specified on the eastern boundary, and the thickness of the upper layers vanishes there. When more physics is added to allow nonzero thickness for these layers on the eastern boundary (Pedlosky, 1983), the stratification must still be specified there.

We show here that however small the diffusion is, it is still a crucial part of the thermocline dynamics, and has to be included in the physics in order to determine the basic stratification. Air–sea heat fluxes must also be considered, and specifying the surface density does not account for their full effect on the interior circulation.

The physical principle guiding us throughout this investigation is simple: Air–sea interaction may result in a net production of water of some density type that sinks and spreads in the ocean interior. To keep the total mass of this density type constant, interior mixing must act to change the density of this water to other density ranges. The mixing effects are assumed to depend on the density stratification, so that the condition of constant mass of water of given density can be used to link the air–sea heat fluxes to the interior stratification!

The *mid-depth circulation* below the main thermocline is not very well understood observationally (see Reid, 1981, for a review), nor theoretically. It is probably not primarily wind driven like the upper circulation, but buoyancy driven by the mixing processes. We try below to construct a theory for this water range, in which the driving force is the diffusive vertical velocity, and both wind and air–sea heat fluxes have indirect but important effects.

Finally, we note that the only model existing for the *bottom water circulation* is one for the vertically integrated transport (Stommel, 1958; Stommel and Arons, 1959a,b), driven by a uniform upwelling at the top of the bottom water. Air–sea interaction enters only implicitly as a source of bottom water that replaces the upwelling water. By explicitly considering the air–sea fluxes and using the condition of constant mass of water of given density, we are able to develop a simple diffusive, continuous, nonlinear model of the bottom and lower mid-depth circulation, and to relate its stratification to the air–sea fluxes.

The development in the rest of this paper is as follows. In section 2 we derive an integral constraint relating the air–sea fluxes to the interior stratification. This constraint is based on the condition of constant total mass of water of given density in a steady state ocean. In section 3 the constraint is applied to a continuous nonlinear diffusive model of the deep circulation below the influence of the wind driven circulation. The basic stratification of the model tends to *look like* an exponential profile, but the small deviations from exponential are crucial to the dynamics.

In section 4, which is independent of section 3, a three layer *diffusive* model of the deep, mid-depth and upper ocean is examined. The upper layer is a wind-driven two-gyre ventilated thermocline, and the lower layers are driven by diffusive cross-interfacial velocities. Air–sea heat fluxes are specified as part of the thermal boundary conditions of the model, and the stratification on the eastern boundary is calculated in terms of these fluxes using the constraint from section 2. A two gyre mid-depth circulation is found, while the bottom circulation is similar to that of the Stommel–Arons model.

2. Derivation of a constraint on the basic stratification

In this section we derive an integral relation between the air–sea fluxes of heat and fresh water and the interior stratification, in the presence of small scale mixing. The relation is based on the condition that the total mass of water of given density is constant in a steady state ocean. Before going into the details of the derivation, it is useful to examine the physics behind it, and in particular to see what "net production" means.

There are two processes acting to change the density of a given water particle in the ocean—air–sea exchanges that affect the surface water and small scale mixing in the ocean interior. Consider now the schematic, zonally averaged picture in Fig. 1, and concentrate on the water between two isopycnals ρ_1 and ρ_2.

We first examine the effects of the air–sea fluxes. Suppose that the density surface ρ_1 outcrops where the ocean is losing heat to the atmosphere. As a result, some mass of water of density $\rho < \rho_1$ is cooled per unit time, and its density becomes $\rho_1 < \rho < \rho_2$. (We are not interested now in the question of whether this water sinks or is advected horizontally to an area where the surface density is larger than ρ_1, but only in the density change itself.) Water of density $\rho_1 < \rho < \rho_2$ is also exposed to the atmosphere, and suppose it also loses heat to the atmosphere, but less than the water in the density range just smaller than ρ_1. As a result, a mass of water of density $\rho_1 < \rho < \rho_2$ is cooled and its density becomes $\rho > \rho_2$. However, this time less water is involved in the process, because the mass of water whose density is changed is proportional to the heat loss experienced by this water. In the situation described here there is more water *entering* the density range $\rho_1 < \rho < \rho_2$ than *leaving* it, and therefore there is a *net production* of water of this density per unit time. Air–sea

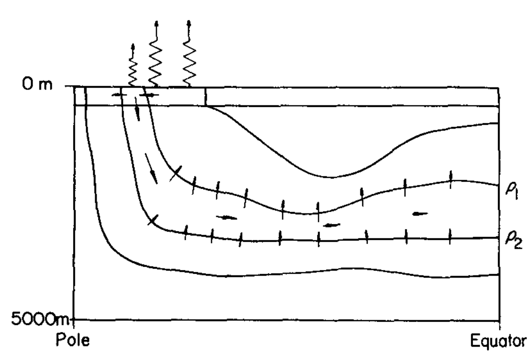

FIG. 1. A schematic north–south vertical section showing the production of water of density between ρ_1 and ρ_2 by air–sea heat fluxes, and dissipation of this water type by interior mixing.

heat fluxes act in this case as a *source* of water of density between ρ_1 and ρ_2.

Next, consider the effects of small scale mixing. The mixing processes act to change the density of water particles, and therefore force cross-isopycnal velocities. These velocities depend on the interior stratification through the density equation $\mathbf{U}\cdot\nabla\rho = \nabla(\lambda\nabla\rho)$. Consider again the schematic picture in Fig. 1. Suppose that the interior stratification is such that there is net upwelling across the ρ_2 surface. This means that the mixing processes act to reduce the density of some mass of water heavier than ρ_2 and this mass upwells to the density range between ρ_1 and ρ_2. If there is also upwelling across the ρ_1 surface, but of larger magnitude, then more water leaves the density range $\rho_1 < \rho < \rho_2$ than enters it, and the interior mixing acts as a *sink* of this water type.

In a steady state the sinks and sources of any density type must balance to give *no net production.* This constraint of zero net production relates the air–sea fluxes to the interior stratification.

Two derivations of the constraint are given below. The first is more heuristic, the net production by each of the possible processes is derived separately, while ignoring the effects of the others. The second derivation is more formal, and includes all of the processes together, including the effects of seasonal variability of the air–sea interaction.

Small scale processes are modeled throughout this paper in the simplest possible way, with a constant eddy-diffusivity coefficient in the density equation. Still, the procedures and physical principles we use do not depend on the particular parameterization chosen, and can be used with any other parameterization relating the small scale mixing to the mean fields. Given such a parameterization one can derive the constraint presented in this section, and then use it to find the effects of the mixing processes on the general circulation, as shown in the following sections. It is not clear to what extent the more specific results of the models developed below depend on the parameterization used.

a. *An intuitive derivation*

In this subsection, the expressions for the net production of water of given density by interior mixing, by air–sea heat fluxes, and by evaporation and precipitation are derived separately. This derivation should give an intuitive understanding of the physics involved, and will allow interpretation of the more formal results later in b.

1) PRODUCTION OF WATER OF GIVEN DENSITY BY INTERIOR DIFFUSION

Ignoring air–sea fluxes, see Fig. 2. The mass flux across an isopycnal surface ρ is, for a Boussinesq fluid,

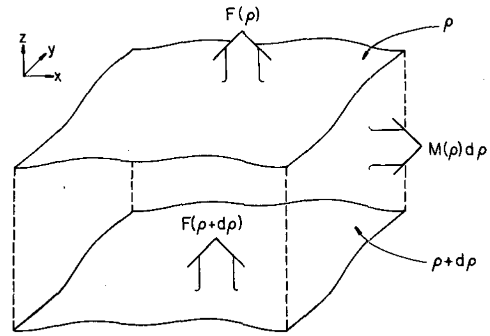

FIG. 2. A perspective view of surface elements of two isopycnal surfaces, showing the local cross-isopycnal mass fluxes [$F(x, y, \rho)$] and the local production [$M(x, y, \rho)$] of water of density (ρ, $\rho + d\rho$) by the mixing processes.

the component of the velocity normal to a density surface, multiplied by a reference density ρ_0:

$$F(x, y, \rho) = \rho_0\mathbf{U}\cdot\mathbf{n}, \tag{2.1}$$

where $\mathbf{n}$ is a unit vector perpendicular to a surface of constant density. Using $\mathbf{n} = -\nabla\rho/|\nabla\rho|$, and the density equation

$$\mathbf{U}\cdot\nabla\rho = u\rho_x + v\rho_y + w\rho_z = \lambda\nabla^2\rho, \tag{2.2}$$

we find

$$F(x, y, \rho) = -\lambda\rho_0\nabla^2\rho/|\nabla\rho|,$$

and for almost horizontal density surfaces,

$$\approx -\lambda\rho_0\rho_{zz}/\rho_z. \tag{2.3}$$

The mass of water within the density range (ρ, $\rho + d\rho$) that is produced per unit area, per unit time is the difference between the mass flux across the ρ-density surface and the $\rho + d\rho$ surface,

$$M_{\mathrm{diff}}(x, y, \rho)d\rho = F(x, y, \rho + d\rho) - F(x, y, \rho),$$

so that

$$M_{\mathrm{diff}}(x, y, \rho) = \partial F(x, y, \rho)/\partial\rho. \tag{2.4}$$

Using $\partial\{\ \}/\partial z = (\partial z/\partial\rho)^{-1}\partial\{\ \}/\partial\rho$, the cross-isopycnal mass flux (2.1) can be written in (x, y, ρ) coordinates instead of (x, y, z) coordinates:

$$F(x, y, \rho) \approx -\lambda\rho_0\rho_{zz}/\rho_z$$

$$= -\lambda\rho_0\frac{\partial}{\partial\rho}\left\{\frac{1}{\partial z/\partial\rho}\right\}, \tag{2.5}$$

and we finally have

$$M_{\mathrm{diff}}(x, y, \rho) = -\lambda\rho_0\frac{\partial^2}{\partial\rho^2}\left\{\frac{1}{\partial z/\partial\rho}\right\},$$

or in the (x, y, z) coordinate system:

$$M_{\mathrm{diff}}(x, y, \rho) = -\lambda\rho_0\frac{\partial}{\partial\rho}\left\{\frac{\nabla^2\rho}{|\nabla\rho|}\right\}$$

$$= (1/\rho_z)\partial_z\{-\lambda\rho_0\nabla^2\rho/|\nabla\rho|\}$$

$$\approx \lambda(\rho_0/\rho_z)\partial_{zz}\{\ln\rho_z\}. \tag{2.6}$$

2) Production of water of given density by air–sea heat fluxes

Ignoring salinity variations of the surface water and effects of evaporation-precipitation, assuming $\rho = \rho_0 - \alpha(T - T_0)$, *see Fig. 3.* Suppose that an amount of heat $H(\rho)d\rho$ is lost per unit time to the atmosphere by surface water of density ρ all over the basin. As a result, a volume dv_1 of water of (temperature, density) = (T, ρ) is cooled per unit time, to $(T - dT)$ and its density becomes $\rho + d\rho$. The volume dv_1 can be found by calculating the heat budget,

$$dv_1\rho C_pT + H(\rho)d\rho = dv_1(\rho + d\rho)C_p(T - dT). \quad (2.7)$$

At the same time, a volume dv_2 of $(\rho - d\rho)$-water loses an amount of heat $H(\rho - d\rho)d\rho$ and becomes ρ-water. The net production of ρ-water is $M_{\text{heat-fluxes}}(\rho)d\rho = \rho(dv_1 - dv_2)$. Substituting the values of dv_1 and dv_2

$$M_{\text{heat-fluxes}}(\rho) = (\alpha/C_p)\partial H(\rho)/\partial\rho. \quad (2.8)$$

3) Production of water of given density by evaporation-precipitation

Ignoring temperature variations of the surface water and effects of air–sea heat fluxes, assuming $\rho = \rho_0 + \beta(S - S_0)$, *see Fig. 4.* Let the evaporation–precipitation as a function of the surface water density be $Q(\rho)$. This means that all over the basin, a volume $Q(\rho)d\rho$ of fresh water is added per unit time to surface water of density ρ. As a result, a volume dv_1 of water of (salinity, density) = (S, ρ) is joined by a volume $Q(\rho)d\rho$ of fresh water, and becomes a volume $d\mathcal{V}_1$ of $(S - dS, \rho - d\rho)$-water.

The volume elements dv_1, $d\mathcal{V}_1$ can be found by calculating the mass and salt balances,

$$\text{salt:} \quad dv_1\rho S = d\mathcal{V}_1(\rho - d\rho)(S - dS), \quad (2.9)$$

$$\text{mass:} \quad dv_1\rho = d\mathcal{V}_1(\rho - d\rho) + Q(\rho)d\rho. \quad (2.10)$$

At the same time, a volume dv_2 of $(\rho + d\rho, S + dS)$-water is joined by a volume $Q(\rho + d\rho)d\rho$ of fresh water, and becomes a volume $d\mathcal{V}_2$ of (ρ, S)-water. The net production of water of density ρ is

$$M_{\text{fresh water}}(\rho)d\rho = \rho(d\mathcal{V}_2 - dv_1). \quad (2.11)$$

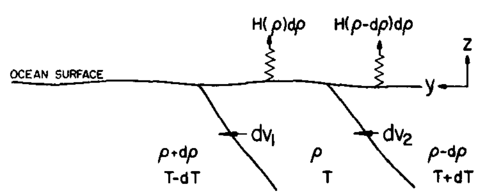

FIG. 3. A schematic north–south vertical section through an outcropping region, showing the air–sea heat fluxes and the resulting cross-isopycnal mass fluxes.

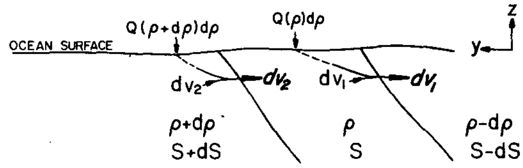

FIG. 4. As in Fig. 3 but with fresh water fluxes forcing the cross-isopycnal fluxes.

By substituting the values of dv_1, $d\mathcal{V}_2$ we have

$$M_{\text{fresh water}}(\rho) = 2Q(\rho) + \beta S\partial Q(\rho)/\partial\rho$$
$$= \partial\{\beta SQ(\rho)\}/\partial\rho + Q(\rho). \quad (2.12)$$

Because the total production of water of density ρ should vanish, we can combine (2.6), (2.8) and (2.12) into

$$\frac{\partial}{\partial\rho}\iint \{\lambda\rho_0\nabla^2\rho/|\nabla\rho|\}$$
$$= \partial\{\beta SQ\}/\partial\rho + Q(\rho) + \partial\{\alpha H/C_p\}/\partial\rho, \quad (2.13)$$

where the double integral is over the entire area of a density surface. The rhs of (2.13) is an expression for the mass of water of density ρ which is produced by the given air–sea fluxes of heat (H) and fresh water (Q). This expression does not change much when we allow for seasonal variability in the air–sea interaction [see (2.23)]. The water mass production by the air–sea fluxes is balanced by the production by the mixing processes which, for our choice of the parameterization of the mixing processes, is given by the lhs. Equation (2.13) can be integrated over ρ from the highest surface density (ρ_b) to ρ, to give

$$\iint \{-\lambda\rho\nabla^2\rho/|\nabla\rho|\} + \beta SQ(\rho) + \alpha H(\rho)/C_p$$
$$= -\int_{\rho b}^{\rho} Q(\rho')d\rho'. \quad (2.14)$$

Each of the terms on the lhs is a contribution to the mass flux across the density surface ρ from one of the processes described in subsections 1, 2 and 3. The physical statement in (2.14) is that the total mass flux across the isopycnal surface ρ (the lhs) is equal to the flux of fresh water from the atmosphere into surface water of density greater than ρ (rhs). A similar result was discussed by Walin (1982) who considered the mass and heat balances for a volume of fluid bounded by an isothermal surface. He related the air-sea heat fluxes to interior cross-isothermal diffusive fluxes, and used this relation as an observational diagnostic tool to deduce the diffusive fluxes in the ocean interior.

b. A more formal derivation

We now want to see if and how the results of the preceding heuristic derivation change when all the density-changing processes act together. We also allow seasonal time variations in the air–sea interaction and derive the time averaged constraint of zero net production. The results turn out to be essentially the same as in the previous subsection, and the reader may skip this subsection on first reading.

The derivation is divided into three parts. We first derive the density and continuity equations in the presence of heat and fresh water sources. Then these equations are written in density coordinates, and relations are found between several quantities in (x, y, z) coordinates and in (x, y, ρ) coordinates. Finally the continuity equation is integrated over an isopycnal surface and averaged in time to obtain the constraint of zero net production of water of given density.

In this derivation, the air–sea heat and fresh water fluxes are represented by distributed sources of heat $[\mathscr{H}(x, y, z, t)]$ and fresh water $[Q(x, y, z, t)]$. (The sources are different from zero only near the surface.)

The mass, heat, and salt budgets for a fixed volume of Boussinesq fluid are:

$$0 = -\iint d^2x\{\rho_0\mathbf{U}\cdot\mathbf{n}\} + \iiint d^3x[Q(x, y, z, t)\rho_0], \tag{2.15}$$

$$\partial_t \iiint d^3x\{\rho_0 C_p T\} = -\iint d^2x\{\rho_0 C_p T\mathbf{U}\cdot\mathbf{n}\} + \iiint d^3x[\mathscr{H}(x, y, z, t)\rho_0] + \iiint d^3x[Q(x, y, z, t)\rho_0 C_p T] + \text{heat diffusion}, \tag{2.16}$$

$$\partial_t \iiint d^3x\{\rho_0 S\} = -\iint d^2x\{\rho_0 S\mathbf{U}\cdot\mathbf{n}\} + \text{salt diffusion}. \tag{2.17}$$

Using these relations and the equation of state $\rho = \rho_0 - \alpha(T - T_0) + \beta(S - S_0)$, we obtain the incompressibility equation

$$\nabla\cdot\mathbf{U} = Q(x, y, z, t), \tag{2.18}$$

and the density equation

$$d\rho/dt = \rho_t + \mathbf{U}\cdot\nabla\rho = -\beta S(x, y, z, t)Q(x, y, z, t) - (\alpha/C_p)\mathscr{H}(x, y, z, t) + \lambda\nabla^2\rho. \tag{2.19}$$

The gradient of the density field, represented in density coordinates, is

$$\nabla\rho = (\rho_x, \rho_y, \rho_z) = (\rho_0/h)(z_x, z_y, 1) \tag{2.20}$$

and its Laplacian,

$$\nabla^2\rho = (\rho_0/h)\left\{z_{xx} + z_{yy} + (z_x^2 + z_y^2 + 1)\frac{\partial}{\partial\rho}(\rho_0/h)\right\}, \tag{2.21}$$

$$\text{where}\quad h \equiv -\rho_0\partial z/\partial\rho. \tag{2.22}$$

With these relations, we can rewrite the density equation (2.19) as

$$d\rho/dt = \dot{\rho} = -\beta SQ - (\alpha/C_p)\mathscr{H} + \lambda(\rho_0/h) \times \left\{z_{xx} + z_{yy} + (z_x^2 + z_y^2 + 1)\frac{\partial}{\partial\rho}\left(\frac{\rho_0}{h}\right)\right\}, \tag{2.19a}$$

while the continuity equation (2.18) in density coordinates is

$$h_t + [\partial_x(uh) + \partial_y(vh)]|_{\rho=\text{constant}} + \frac{\partial}{\partial\rho}(h\dot{\rho}) = hQ. \tag{2.18a}$$

For the constraint of no net production of water of a given density, substitute (2.19a) in (2.18a), average the continuity equation (2.18a) over time, and integrate over the entire area of an isopycnal surface. Using the periodicity to eliminate the term

$$\iint dxdy \int h_t dt,$$

and noting that the along-isopycnal transport $(uh)_x + (vh)_y$ vanishes when integrated over the entire density surface, we have

$$\begin{aligned}\partial/\partial\rho \int dt \iint_{\rho=\text{constant}} dxdy\{\lambda\rho[z_{xx} + z_{yy} + (z_x^2 + z_y^2 + 1)\partial(\rho_0/h)/\partial\rho]\} \\ - \partial/\partial\rho \int dt \iint_{\rho=\text{constant}} dxdy\{(\alpha/C_p)h\mathscr{H}(x, y, \rho, t)\} \\ - \partial/\partial\rho \int dt \iint_{\rho=\text{constant}} dxdy\{\beta ShQ(x, y, \rho, t)\} \\ = \int dt \iint dxdyhQ(x, y, \rho, t).\end{aligned} \tag{2.23}$$

To see that (2.23) is a generalization of (2.13), we now derive the relation between the distributed sources $Q(x, y, z, t)$, $\mathscr{H}(x, y, z, t)$, and the fluxes $H(\rho)$, $Q(\rho)$ that were used in the heuristic derivation in section 2a.

The total heat gained by the ocean during one year in the derivation in section 2a was

$$(1\ \text{yr}) \times \int H(\rho)d\rho.$$

In terms of the distributed sources it is

$$\int dt \iiint \mathscr{H}(x, y, z, t)\rho_0 d^3x.$$

Now,

$$(1\text{ yr}) \times \int H(\rho)d\rho$$

$$= \int dt \iiint \mathscr{H}(x, y, z, t)\rho_0 dxdydz$$

$$= \int dt \iiint \mathscr{H}(x, y, \rho, t)\rho_0 dxdy(\partial z/\partial\rho)d\rho$$

$$= \int d\rho\left\{\int dt \iint (-h)\mathscr{H}(x, y, \rho, t)dxdy\right\}$$

$$\Rightarrow H(\rho) = (1\text{ yr})^{-1} \int dt \iint (-h)\mathscr{H}(x, y, z, t)dxdy. \tag{2.24}$$

The same applies for the mass sources (evaporation–precipitation):

$$Q(\rho) = (1\text{ yr})^{-1} \int dt \iint (-h)Q(x, y, z, t)dxdy. \tag{2.25}$$

With the identities (2.24) and (2.25), the similarity between (2.23) and the result derived in a more heuristic way in section 2a is clearly seen.

3. A continuous model of the deep and mid-depth circulation

In section 2 we have not considered explicitly the dynamics of the circulation and the velocity field, but only assumed implicitly the existence of a velocity field connecting sources and sinks of a given density. In this section, a continuous model of the deep and mid-depth circulation below the influence of the wind-driven circulation is described. The model is a simple application of the constraint derived in the previous section: A steady state density stratification is maintained by a balance between the production of water by air–sea heat fluxes and by interior small scale mixing.

Figure 5 is a schematic north–south vertical section, showing to what part of the oceanic circulation the model applies. The thermocline circulation (shown vertically hatched) is probably mostly wind driven. The upper mid-depth water (diagonally hatched) is probably buoyancy driven, but is certainly influenced by the distortion of the isopycnals just above it by the thermocline circulation. These two upper regions and the interaction of the wind-driven circulation with the diffusive processes are not considered here. They are part of the model presented in section 4, which is independent of this one. The lower mid-depth and the bottom waters (unshaded in Fig. 5), where the isopycnals are

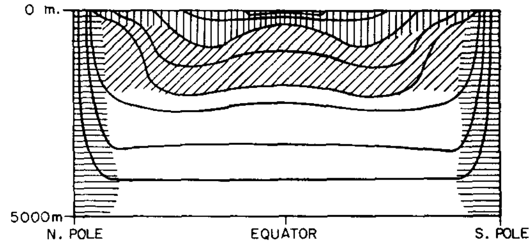

FIG. 5. A north–south schematic vertical section, showing the regions of outcropping, of the upper circulation (shaded), and of the deeper circulation (unshaded) that is modeled in section 3.

nearly flat, are those addressed by the model in this section.

The basic dynamics of the model are geostrophic, hydrostatic and diffusive and it is nonlinear in the sense that there is no linearization about some specified basic stratification. Instead, the heating of the ocean by the atmosphere is specified as a function of the density of the surface water where the deep water outcrops (horizontally hatched region in Fig. 5). Then the constraint which was derived in section 2 is used to calculate the stratification and the cross-isopycnal velocities in the interior. Finally, the geostrophic equations are used to calculate the deep horizontal velocity field.

We start by nondimensionalizing the equations, then a perturbation expansion in powers of the nondimensionalized diffusion coefficient is used to obtain the solution. The equations are

$$\begin{aligned} f_* u &= -(1/\rho_0)p_y \\ f_* v &= (1/\rho_0)p_x \\ p_z &= -g\rho \\ u_x + v_y + w_z &= 0 \\ u\rho_x + v\rho_y + w\rho_z &= \lambda_* \nabla^2\rho, \end{aligned} \tag{3.1}$$

where $f_* = 2\Omega\,\sin(\text{latitude})$ is the Coriolis parameter, x, y, z and u, v, w are the (east, north, vertical) coordinates and velocity components, and we also use the notation $\beta = df_*/dy$; p, ρ and ρ_0 are the pressure, density and a constant reference density. The boundary conditions are:

- The air–sea heat fluxes as function of the surface density in areas where the deep and mid-depth water outcrops are specified: $H(\rho)$.
- No zonal flow into the eastern boundary: $u(x_e, y) = 0$.

We introduce the scaling

$$\begin{aligned} x, y \sim L, \quad & z \sim H \\ u, v \sim V, \quad & w \sim W = VH/L \\ \rho \sim B, \quad & p \sim P, \end{aligned} \tag{3.2}$$

and get the nondimensional equations

$$\begin{aligned} fu &= -p_y \\ fv &= p_x \\ p_z &= -\rho \\ u_x + v_y + w_z &= 0 \\ u\rho_x + v\rho_y + w\rho_z &= \lambda\nabla^2\rho, \end{aligned} \tag{3.3}$$

where

$$f = \sin(y)/\sin(45°); \quad \lambda = \lambda_* L/VH^2; \quad \delta = H/L$$
$$\nabla^2 = \delta^2(\partial_{xx} + \partial_{yy}) + \partial_{zz} \quad P = \rho_0 L f_*(45°)V;$$
$$B = \rho_0 L f_*(45°)V/\text{Hg}. \tag{3.4}$$

Substituting reasonable values for the scales:

$$L = 3000 \text{ km}; \quad H = 1 \text{ km}; \quad V = 0.1 \text{ cm s}^{-1};$$
$$\lambda_* = 1 \text{ cm}^2 \text{ s}^{-1}; \quad g = 10^3 \text{ cm s}^{-2}, \tag{3.5}$$

we obtain $\lambda = 0.3$, so that we can treat λ as a small parameter. Expanding the variables in a perturbation series

$$u = u_0 + \lambda u_1 + \lambda^2 u_2 + \cdots, \quad v = \cdots, \tag{3.6}$$

the order-one equations are

$$\begin{aligned} fu_0 &= -p_{0y} \\ fv_0 &= p_{0x} \\ p_{0z} &= -\rho_0 \\ u_{0x} + v_{0y} + w_{0z} &= 0 \\ u_0\rho_{0x} + v_0\rho_{0y} + w_0\rho_{0z} &= 0. \end{aligned} \tag{3.7}$$

Because to this order there are no cross-isopycnal fluxes, and because we are below the influence of the wind driven circulation, there is no forcing, and a solution is:

$$u_0 = v_0 = w_0 = 0, \quad \rho_0 = \rho_0(z), \quad p_0 = p_0(z). \tag{3.8}$$

To determine $p_0(z)$, $\rho_0(z)$ we must go to the $O(\lambda)$ equations and use the constraint from section 2. But before doing this, a comment on the $O(1)$ solution is needed.

It is clear that this solution of horizontal isopycnals cannot hold everywhere, because we expect the deep isopycnals to outcrop and to be influenced by the atmosphere in polar regions. Figure 5 shows schematically the large region (unshaded) in which the isopycnals are almost horizontal, and the small polar region where they are supposed to outcrop (horizontally hatched).

The $O(\lambda)$ equations are

$$\begin{aligned} fu_1 &= -p_{1y} \\ fv_1 &= p_{1x} \\ p_{1z} &= -\rho_1 \\ u_{1x} + v_{1y} + w_{1z} &= 0 \\ w_1\rho_{0z} &= \rho_{0zz}. \end{aligned} \tag{3.9}$$

The last equation can be written as

$$w_1 = w_1(z) = \rho_{0zz}/\rho_{0z} = \frac{\partial}{\partial\rho_0}\left(\frac{1}{\partial z/\partial\rho_0}\right) = w_1(\rho_0), \tag{3.10}$$

where ρ_0 is the order-one density field, and it is used as the vertical coordinate instead of z.

We saw in section 2 that if the effects of evaporation–precipitation in the outcropping region are ignored, then the net mass flux across an isopycnal surface due to air–sea heat exchanges and due to interior mixing is zero. Because the outcropping region is small, we assume that the total cross-isopycnal mass flux there *due to small scale mixing* is small compared to the cross-isopycnal fluxes integrated all over the density surface in the larger part of the ocean, where our $O(1)$ solution is valid. (We also ignore the boundary mixing; see comment in section 4.) These assumptions allow us to write the constraint of zero net mass flux across an isopycnal surface as

$$\iint dxdy\mathbf{U}\cdot\mathbf{n} \approx \iint dxdyW\lambda w_1(z) = (\alpha/C_p)H(\rho), \tag{3.11}$$

where $H(\rho)$ is the heat flux from surface water of density ρ to the atmosphere. Since w_1 is only a function of z, (or equivalently of ρ_0) we have

$$W\lambda w_1 \approx (\alpha/C_p\text{Area})H(\rho). \tag{3.12}$$

Using (3.10), we obtain an equation for the horizontally uniform $O(1)$ density stratification. In dimensional form:

$$\lambda_*\frac{\partial}{\partial\rho}\left(\frac{1}{\partial z/\partial\rho}\right) = \frac{\alpha}{C_p\text{Area}}H(\rho), \tag{3.13}$$

and the solution for $z(\rho_0)$ is (C_1 and C_2 are two integration constants):

$$z(\rho_0) = \int^{\rho_0} d\rho' \times \left\{\int^{\rho'} d\rho''\left[\left(\frac{\alpha}{\lambda_* C_p \times \text{Area}}\right)H(\rho'')\right] + C_1\right\}^{-1} + C_2. \tag{3.14}$$

The solution (3.14) for $z(\rho_0)$ can be inverted to obtain $\rho_0(z)$. This procedure will be demonstrated by considering a specific heating function $H(\rho)$, but first it is possible to obtain the important $O(\lambda)$ and $O(\lambda^2)$ corrections to the velocity and density fields, and in particular the deviations from horizontally uniform stratification and upwelling.

Knowing $\rho_0(z)$ from (3.14), and $w_1(\rho_0)$ from (3.12), we can find $w_1(z)$. From the $O(\lambda)$ equations we have the other fields in terms of w_1

$$
\begin{aligned}
u_1 &= \int_x^{x_e} \frac{1}{f}\frac{\partial}{\partial y}\left(\frac{f^2}{\beta}\right)\frac{\partial w_1}{\partial z}\,dx' \\
v_1 &= \frac{f}{\beta}\frac{\partial w_1}{\partial z} \\
p_1 &= \int_{x_e}^{x} \left(\frac{f^2}{\beta}\right)\frac{\partial w_1}{\partial z}\,dx' + G_1(z) \\
\rho_1 &= \int_x^{x_e} \left(\frac{f^2}{\beta}\right)\frac{\partial^2 w_1}{\partial z^2}\,dx' - \frac{\partial G_1}{\partial z},
\end{aligned}
\tag{3.15}
$$

where $G_1(z)$ is a function of z only. To determine $G_1(z)$ we have to reapply the constraint on the density field, this time with the $O(\lambda^2)$ corrections to the cross-isopycnal velocity and to the density field. But because G_1 is only a small correction to the basic vertical stratification, and we are interested in the horizontal deviations from this uniform state, we may ignore $G_1(z)$.

To find the dynamically important corrections to the uniform upwelling w_1, we consider the $O(\lambda^2)$ density equation:

$$(u_1\partial_x + v_1\partial_x + w_1\partial_z)\rho_1 + w_2\rho_{0z} = \nabla^2\rho_1,$$

and we get w_2 in terms of the already known $O(\lambda)$ fields,

$$w_2(x, y, z) = (\nabla^2\rho_1 - \mathbf{U}_1\cdot\nabla\rho_1)/\rho_{0z}. \tag{3.16}$$

An example with a specific heating function. The zonally integrated heat fluxes from the ocean to the atmosphere as a function of latitude, have the schematic shape shown in Fig. 6a. Because surface density is roughly monotonically increasing with latitude, we can assume that the heat fluxes as a function of the density of the surface water which is losing that heat, have the shape shown in Fig. 6b. We are only interested now in $H(\rho)$ for the bottom and mid-depth densities, so that a reasonable choice for that density range is the one shown in Fig. 6c.

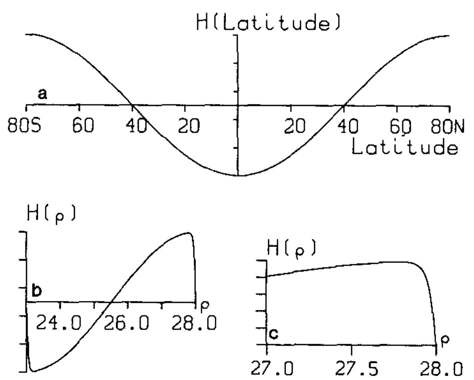

FIG. 6. Schematic pictures of (a) zonally averaged air–sea heat fluxes as function of latitude, (b) air–sea fluxes as function of the surface density, and (c) the heating (as function of the surface density in outcropping regions) used as forcing in the bottom and lower mid-depth circulation model in section 3.

A convenient analytic form is chosen for this heating:

$$H(\rho) = D\left[\cos\left(\frac{\rho-\rho_b}{\rho_b-\rho_s}\pi\right) - \exp\left(\frac{\rho-\rho_b}{\Delta}\right)\right]$$

(ρ_s, Δ and D are constants, ρ_b is the bottom density) and is substituted in the solutions derived before for the density and velocity fields. Because w_1 is known from (3.12) as an explicit function of the basic density stratification ρ_0, it is convenient to evaluate the solutions (3.15) and (3.16) with ρ_0 as the vertical coordinate. This is done by using $\partial w/\partial z = \{1/(\partial z/\partial\rho_0)\}\partial w/\partial\rho_0$, with $\partial z/\partial\rho_0$ taken from (3.14). The explicit analytic solutions (3.15) and (3.16) in terms of the specific heating are quite long expressions, and were found by using the MACSYMA symbolic-manipulation computer program (Mathlab Group MIT, 1983).

A few words should be said about the boundary conditions for the density and vertical velocity profiles. The constants C_1 and C_2 in (3.14) were chosen to satisfy $z(\rho = 1.027) = -2$ km, $z(\rho = 1.028) = -5$ km. The vertical velocity is zero at the bottom [by (3.12)] because the air–sea heat fluxes vanish for the highest surface density, ρ_b, which is also the density at the bottom of the model (Fig. 6c). If $H(\rho)$ were not zero for $\rho = \rho_b$, then a finite mass $(\alpha/C_p)H(\rho_b)$ of a *single* density $\rho = \rho_b$ would form at the surface. This would mean a completely homogeneous unstratified layer at the bottom of our model, where the physics we use does not apply.

Figure 7 shows the solutions for the basic density stratification $\rho_0(z)$ and for the lowest order upwelling velocity $w_1(z)$. Note that the solution for $\rho_0(z)$ resembles an exponential profile, which is (Munk, 1966) a solution of $w\rho_z = \lambda\rho_{zz}$, with constant upwelling w. But note that w in our solution is a strongly varying function of depth! The apparent insensitivity of the exponential profile to variations in the vertical velocity is a result of the smoothing effect of the vertical diffusion. Two integrations are needed to solve the vertical density balance $w\rho_z = \lambda\rho_{zz}$ for ρ in terms of w, so that even a relatively large variation in $w(z)$ is smoothed and is not seen in $\rho(z)$. The tendency toward an exponential density profile will still, of course, be present when λ is varying with depth. [If $\lambda = \lambda_0 f(z)$, we can write the vertical density balance as $\{w(z)/f(z)\}\rho_z = \lambda_0\rho_{zz}$, and $\rho(z)$ would again tend to look like an exponential profile for different forms of $w(z)$ and $f(z)$.] This insensitivity is unfortunate when we want to calculate $w(z)$ from observations of $\rho(z)$. It is probably unsafe to calculate $w(z)$ by substituting $\rho(z) = \exp(z/H)$ in $w\rho_z = \lambda\rho_{zz}$ with a constant λ (Munk, 1966), or even with a more realistic structure for $\lambda(z)$ (Gargett, 1984). A very small deviation of the actual density profile from an exponential shape, perhaps even below the noise level, may lead to a large change in the calculated structure of $w(z)$.

The variation in $w_1(z)$ is responsible for driving the horizontal circulation in the solution showed here,

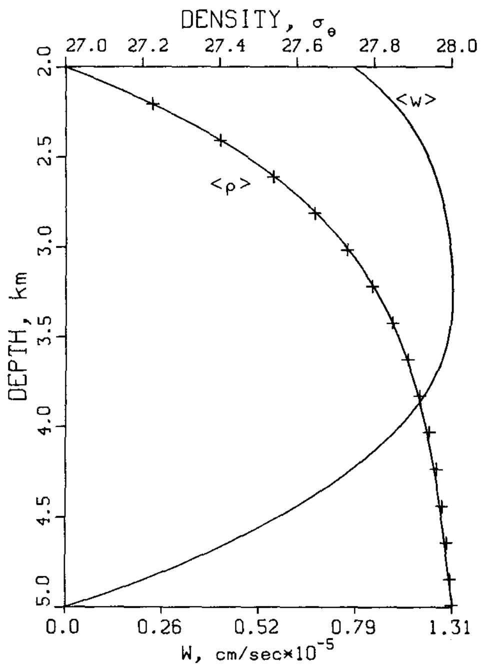

FIG. 7. An example of the bottom and mid-depth circulation model of section 3. The solution for the basic density profile $\rho_0(z)$ (full line) together with a matched exponential profile (crosses) and the vertical velocity $w_1(z)$ for the same solution.

through the linear vorticity equation $\beta v = fw_z$. The bottom water circulation, which occupies the upper density range where $H(\rho)$ is rapidly decreasing, is driven poleward in the interior by an upward increasing $w(z)$. The mid-depth circulation in this example is driven *equatorward* in the interior by an upward decreasing upwelling, as determined by the structure of $H(\rho)$ for the mid-depth densities.

Figure 8a–c shows the deviations of the pressure, density and upwelling from the horizontally uniform lowest order fields, at a depth of the bottom water circulation. The density field has the expected gyre shape, with the strongest signal at the northwest corner. This shape is induced by the deep circulation itself, through the distortion of the isopycnals by the velocity field. Note that boundary currents are needed to close the circulation and to connect the interior flows to the outcropping region.

4. The interaction of the wind driven circulation with thermal processes

In this section some aspects of the interaction of the wind driven circulation with the cross-isopycnal processes are investigated using a simple layer model. We demonstrate how the principle of no net production of water of given density can be used to determine the basic stratification (stratification on the eastern boundary) of the wind driven circulation. We also discuss the way the wind and buoyancy forcings combine to determine the circulation of the mid-depth water just below the main thermocline, and we try to obtain some insight into the problem of formulating the right thermal boundary conditions for the thermocline problem.

First, in section 4a, a parameterization of density diffusion in layer models is derived, and then, in section 4b, a simple three-layer diffusive model of the wind and thermohaline oceanic circulation is described. The reader is advised to start by reading the model description in 4b, and to refer to 4a for the details of the parameterization only when it is actually used in the model.

a. A parameterization of density diffusion in layer models

The parameterization we suggest here is based on the similarity between layer models and density coordinates, and is actually a finite difference approxi-

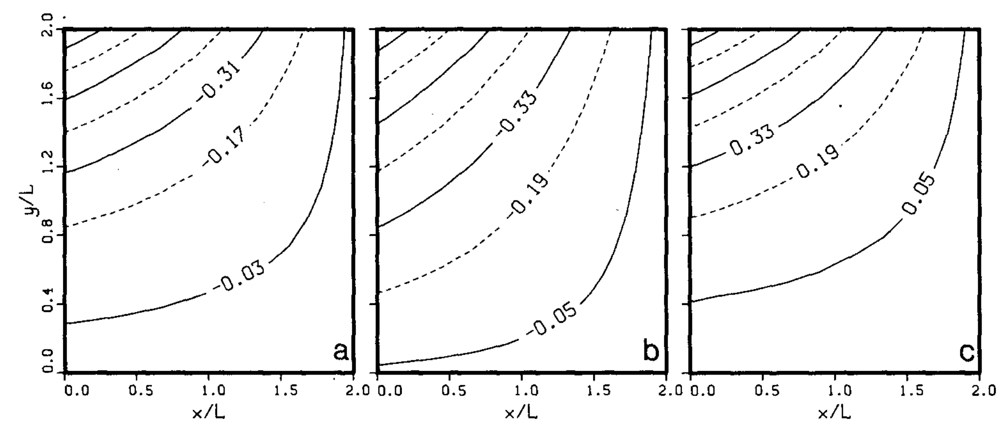

FIG. 8. Corrections to the horizontally uniform basic stratification, pressure field and upwelling velocity, on a constant depth surface at a depth of the bottom circulation [all nondimensional, see (3.2), (3.4).]: (a) ρ_1 normalized by its maximum absolute value $|\rho_1|_{max} = 0.132$; (b) p_1/f normalized by $|p_1/f|_{max} = 0.267$; and (c) w_2 normalized by $|w_2|_{max} = 0.00513$.

mation to the equations in density coordinates. It enables one to enjoy the mathematical simplification of layers versus continuous stratification, while not ignoring the cross-isopycnal processes. The equations reduce to the usual equations for an immiscible layer model if the coefficient of diffusion is set to zero. The diffusive processes are modeled as before with a constant eddy coefficient in the density equation.

In order to establish some necessary notation, consider a continuously stratified ocean which is to be modeled by a finite number of discrete layers of uniform density: Suppose we choose to represent the density range between the two densities ρ_a and ρ_b by the nth layer. Then the density of this layer is $\rho_n = (\rho_a + \rho_b)/2$, its thickness is $h_n(x, y) = z(x, y, \rho_a) - z(x, y, \rho_b)$, where z is the height of a density surface, and we define $\Delta_n\rho$ to be the density range represented by this layer: $\Delta_n\rho = \rho_a - \rho_b$.

With this notation we can now proceed to calculate the velocity across an interface between two layers. In continuous stratification, the cross-isopycnal velocity in the direction normal to a constant density surface is $(\mathbf{U}\cdot\mathbf{n})\mathbf{n}$, where $\mathbf{n}$ is the unit vector normal to a density surface: $\mathbf{n} = \nabla\rho/|\nabla\rho|$. Using the density equation (2.2), we have:

$$\mathbf{U}\cdot\mathbf{n} = \frac{1}{|\nabla\rho|}\mathbf{U}\cdot\nabla\rho = \frac{1}{|\nabla\rho|}\lambda\nabla^2\rho. \qquad (4.1)$$

If the density surfaces are horizontal or very nearly so, then:

$$\mathbf{n} \approx \mathbf{k}, \quad (\mathbf{U}\cdot\mathbf{n})\mathbf{n} \approx \mathbf{k}\lambda\rho_{zz}/\rho_z, \qquad (4.2)$$

where $\mathbf{k}$ is a unit vector in the vertical (z) direction.

Writing this in density coordinates, we have

$$(\mathbf{U}\cdot\mathbf{n})\mathbf{n} \approx \mathbf{k}\lambda\frac{\partial}{\partial\rho}\left(\frac{1}{\partial z/\partial\rho}\right). \qquad (4.3)$$

Now, $\partial z/\partial\rho$ can be approximated for the nth layer in a layer model by $\Delta z/\Delta\rho = h_n/\Delta_n\rho$ and the derivative wrt ρ of some quantity B evaluated at the interface between the n, $n+1$ layers, can be replaced by

$$\partial B/\partial\rho \approx (B_{n+1} - B_n)/(\rho_{n+1} - \rho_n), \qquad (4.4)$$

where B_n is the value of B in the nth layer. Combining (4.3) and (4.4), the cross-interfacial velocity, w_n^*, is

$$w_n^* \approx \lambda(\Delta_{n+1}\rho/h_{n+1} - \Delta_n\rho/h_n)/(\rho_{n+1} - \rho_n) \qquad (4.5)$$

(this expression is all we need for the layer model presented in 4b).

The approximation in (4.2) is not necessary, and the parameterization can be extended to nonhorizontal isopycnals. It is also possible to extend the parameterization to the case of outcropping layers, while avoiding the singularity in $\Delta_n\rho/h_n$ where $h_n \to 0$ in the outcropping region.

As another example of the parameterization, we briefly derive the gyre-scale potential vorticity equation for layer models, including the diffusive effects. In a continuously stratified ocean, for planetary scale motions, small Rossby number and in the presence of vertical diffusion, the potential vorticity equation is (Pedlosky, 1979)

$$\mathbf{U}\cdot\nabla(f\rho_z) = (u\partial_x + v\partial_y + w\partial_z)(f\rho_z) = \lambda\frac{\partial^2}{\partial z^2}(f\rho_z). \qquad (4.6)$$

In density coordinates this is

$$(u\partial_x + v\partial_y)\frac{f}{h}\bigg|_{\rho=\text{const}} = \frac{\lambda\rho}{h^2}\frac{\partial^2}{\partial\rho^2}\left(\frac{f\rho}{h}\right), \qquad (4.7)$$

where $h = -\rho\partial z/\partial\rho$.

For the nth layer h can be approximated by $h \approx h_n(\rho_n/\Delta_n\rho) \approx h_n(\rho_0/\Delta_n\rho)$, and (4.7) becomes

$$(u_n\partial_x + v_n\partial_y)\frac{f}{h_n} = \frac{\lambda f}{h_n^2}\left\{\frac{\Delta_{n+1}\rho/h_{n+1} - \Delta_n\rho/h_n}{\rho_{n+1} - \rho_n} - \frac{\Delta_n\rho/h_n - \Delta_{n-1}\rho/h_{n-1}}{\rho_n - \rho_{n-1}}\right\}. \qquad (4.8)$$

Note that when $\lambda = 0$, (4.8) reduces to the usual potential vorticity equation in layer models for this type of motions.

b. The model

The three-layer model we use is shown schematically in Fig. 9. The upper layer represents the wind-driven circulation above the main thermocline (vertically hatched in Fig. 5), and is driven by a wind curl that forces a two-gyre circulation. The second layer represents the upper mid-depth water, below the main thermocline and above about two kilometers depth (diagonally hatched in Fig. 5). This layer is buoyancy driven, by the cross-isopycnal velocities due to the mixing processes. The wind affects this circulation by changing the local vertical stratification (and therefore the local diffusive vertical velocities), through the changes in the depth of the main thermocline. The air–sea fluxes affect the circulation in this layer not by direct cooling or heating, but through the production of water that sinks and joins the mid-depth water. The bottom layer represents the vertically integrated transport of the lower mid-depth and bottom circulations (unshaded in Fig. 5), that were described in more details

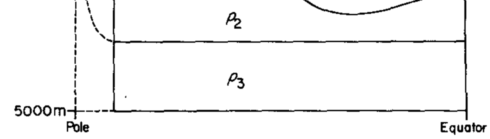

FIG. 9. The three layer model described in section 4.

in the continuous model of section 3. The circulation in this layer is similar to that of the Stommel–Arons model.

The deeper layers do not outcrop within the two gyres, (this restriction is not necessary, and is made only to keep the model as simple as possible) but it is assumed that they do outcrop somewhere, and interact with the atmosphere. This outcropping region is not explicitly a part of the model; we only specify the air–sea fluxes there as a function of the surface density, and assume that the water produced by these fluxes is carried toward the ocean interior. One can think of the Norwegian Sea as an example of such a polar outcropping region, as shown by the broken lines in Fig. 9, but the outcropping region, where the production of water types is taking place, is not necessarily northward of the subpolar gyre. The outcrop may be in the western boundary region or within the gyres, if we allow outcropping there.

The model equations for the nth layer are [see (3.1)]

$$fu_n = (-1/\rho_0)p_{ny}$$

$$fv_n = (1/\rho_0)p_{nx}$$

$$p_{nz} = -g\rho_n$$

$$u_{nx} + v_{ny} + w_{nz} = 0. \quad (4.9)$$

The boundary conditions are:

- The wind-forced Ekman pumping at the base of the mixed layer is given: $w_e(x, y)$.
- The air–sea heat fluxes as function of the surface density in the outcropping region is also assumed known: $H(\rho)$.
- No zonal flow into the eastern boundary: $u(x_e, y) = 0$.

Vertical diffusion is permitted, and we will use the parameterization derived in section 4a to calculate the small cross-interfacial velocities resulting from this diffusion.

To solve for the layer thicknesses and velocities, an approach similar to that of section 3 is used. Assume that the diffusive effects are small, so that to lowest order density is conserved, and the two deeper layers are motionless. (By assumption, the wind-driven circulation is confined to the upper layer, and the other layers are driven only by the cross-isopycnal velocities.)

With these assumptions the upper layer is a one-layer ventilated thermocline, (Luyten et al., 1983) with thickness given by

$$h_1(x, y) = (D_0^2(x, y) + H_1^2)^{1/2} \quad (4.10)$$

where

$$D_0^2(x, y) = -\frac{2f^2}{\beta\gamma}\int_x^{x_e} W_e(x', y)dx',$$

$$\gamma = g(\rho_2 - \rho_1)/\rho_0; \; H_1 = h_1(x_e, y) = \text{constant}. \quad (4.11)$$

For the second and third layers:

$$h_2(x, y) = (H_1 + H_2) - h_1(x, y)$$

$$h_3(x, y) = H_3, \quad (4.12)$$

where, again $H_n = h_n(x_e, y) =$ constant.

To find the basic stratification parameters H_1, H_2 and H_3, and the buoyancy driven circulation in the deeper layers, we *must* consider the thermal boundary conditions and the diffusive processes. Given the air–sea heat fluxes as function of the density of the surface water that is losing/gaining this heat, $H(\rho)$, we first calculate the net production of water of given density [see (2.8)],

$$M(\rho) = (\alpha/C_p)\partial H/\partial\rho.$$

The net production of water of the density ranges represented by layers 2 and 3 is

$$M_{n,\,\text{heat fluxes}} = \int_{\rho_n-\Delta_n\rho/2}^{\rho_n+\Delta_n\rho/2} M(\rho)d\rho = \frac{\alpha}{C_p}H(\rho)\Big|_{\rho_n-\Delta_n\rho/2}^{\rho_n+\Delta_n\rho/2},$$

$$n = 2, 3. \quad (4.13)$$

In terms of layers, $M_{n,\,\text{heat fluxes}}$ is the mass of water of density ρ_n which enters the nth layer, per unit time, after being formed at the surface.

Next, the net production/dissipation of water types represented by the nth layer by the diffusive processes, is found in terms of the stratification parameters H_n. This production is simply equal to the difference between the total cross-isopycnal mass flux into and out of the nth layer:

$$M_{n,\,\text{diffusion}} = \iint dxdy(w_n^* - w_{n+1}^*), \quad (4.14)$$

where w_n^* is the local velocity across the interface of the n, $n + 1$ layers. Using the parameterization (4.5) for w_n^*, we have,

$$M_{n,\,\text{diffusion}} = \lambda \iint dxdy\left\{\frac{\Delta_{n+1}\rho/h_{n+1} - \Delta_n\rho/h_n}{\rho_{n+1} - \rho_n} - \frac{\Delta_n\rho/h_n - \Delta_{n-1}\rho/h_{n-1}}{\rho_n - \rho_{n-1}}\right\} \quad (4.15)$$

and in particular,

$$M_{2,\,\text{diffusion}} = \lambda \iint dxdy \times \left\{\frac{\Delta_3\rho/h_3 - \Delta_2\rho/h_2}{\rho_3 - \rho_2} - \frac{\Delta_2\rho/h_2 - \Delta_1\rho/h_1}{\rho_2 - \rho_1}\right\} \quad (4.16a)$$

$$M_{3,\,\text{diffusion}} = \lambda \iint dxdy\left\{\frac{\Delta_3\rho/h_3 - \Delta_2\rho/h_2}{\rho_3 - \rho_2}\right\}. \quad (4.16b)$$

For the total mass of fluid represented by the nth layer to remain constant, we must have

$$M_{n,\,\text{diffusion}} + M_{n,\,\text{heat fluxes}} = 0. \quad (4.17)$$

The production $M_{n,\,\text{diffusion}}$ depends on the eastern boundary stratification parameters through (4.15) and (4.10–12), so that we can now write three equations for the three unknowns H_1, H_2 and H_3:

$$H_1 + H_2 + H_3 = H = 5\ \text{km}$$

$$M_{2,\,\text{diffusion}} + M_{2,\,\text{heat fluxes}} = 0$$

$$M_{3,\,\text{diffusion}} + M_{3,\,\text{heat fluxes}} = 0, \tag{4.18}$$

and we can solve for the stratification in terms of the air–sea fluxes as represented by the M_ns. Before showing a few examples with specific M_ns, we calculate the buoyancy-driven circulation in layers 2 and 3.

With H_1, H_2 and H_3 now known, we can find the local values of the vertical velocities across the interfaces, and then use

$$\beta h_n v_n = f(w_n^* - w_{n+1}^*), \quad n = 2, 3 \tag{4.19}$$

to find the horizontal velocities in these layers:

$$h_2 v_2 = \frac{f}{\beta}(w_2^* - w_3^*)$$

$$h_3 v_3 = \frac{f}{\beta} w_3^*. \tag{4.20}$$

A short comment on the role of western boundary currents is relevant here. Much of the heat loss from the ocean to the atmosphere probably occurs in the western boundary currents of the wind-driven circulation (Bunker, 1976), and this is considered implicitly as part of the specified heat flux $H(\rho)$. We did not, however, consider the effect of boundary mixing (Wunsch, 1970). If it is believed to be nonnegligible (although the area involved is small), it can be incorporated into (4.17) by specifying $M_{n,\,\text{boundary mixing}}$ (or calculating it by matching boundary currents to the model), and then constraining the interior by

$$M_{n,\,\text{diffusion}} + M_{n,\,\text{heat fluxes}} + M_{n,\,\text{boundary mixing}} = 0,$$

instead of by (4.17).

Examples and discussion. In the following examples we specify $M_{2,\,\text{heat fluxes}}$ and $M_{3,\,\text{heat fluxes}}$, and determine the stratification parameters H_1, H_2, H_3, and buoyancy-driven circulation in layers 2 and 3. This is done by integrating (4.16a, b) numerically for different values of H_n, until the values satisfying (4.18) are found. Three different cases are examined, and the results are shown in Figs. 10–12 and are summarized in Table 1.

Consider first the case shown in Fig. 10. The circulation in the bottom layer (Fig. 10c) is basically the same as in the Stommel–Arons model, except that the vertical velocity at the top of this layer is not uniform and is determined as part of the solution instead of being specified.

The circulation in layer 2 (Fig. 10b) shows some interesting features. In this example $M_{2,\,\text{heat fluxes}}$ is zero, so that the total upwelling across the interface between layers 2, 3 is equal to the total upwelling across the interface between layers 1, 2. Still, locally the difference $w_2^* - w_3^*$ does not vanish everywhere. The horizontal variations in the depths of layers 1 and 2 (Fig. 10a) induce variations in w_2^* and w_3^*. These variations tend to make the difference $w_2^* - w_3^*$ positive under the subpolar gyre, and negative under the subtropical gyre, therefore driving the mid-depth circulation in the same direction as that of the wind-driven circulation! Note that we do not impose heating of layer 2 in the subtropical gyre and cooling in the subpolar gyre, and that there is no momentum transfer from the upper layer to the middle one. The circulation in layer 2 is driven only by the diffusive processes, and the only effect of the upper wind-driven circulation on the second layer is through the variations in the thickness of layer 1.

Unfortunately, it is not possible to deduce the buoyancy-driven corrections to the velocities in the upper

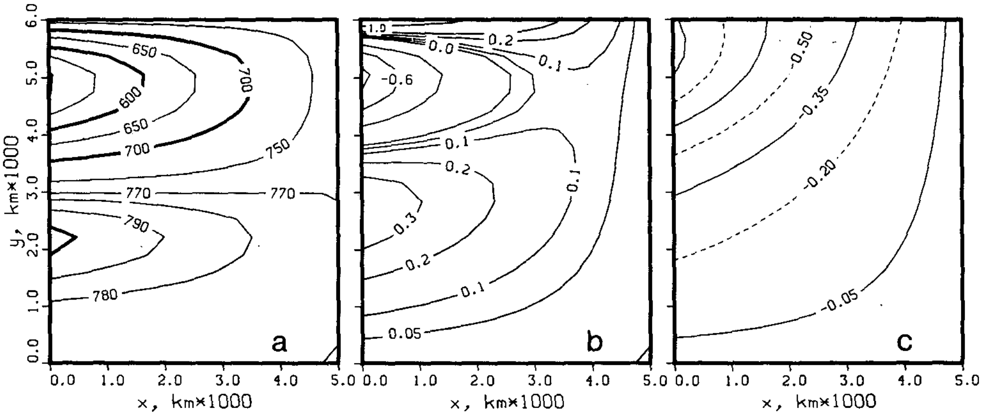

FIG. 10. Results of the layer model of section 4b, for the thermal boundary conditions $M_2 = 0$ Sv, $M_3 = 5$ Sv: (a) The thickness of layer 1, in meters; (b) Transport streamfunction ($\Psi_n = \int_{xe}^{x} \{v_n h_n\} dx$) for layer 2, normalized by its maximum absolute value $|\Psi_2|_{\max} = 1.28$ Sv; and (c) As in (b) for layer 3 $|\Psi_3|_{\max} = 6.55$ Sv.

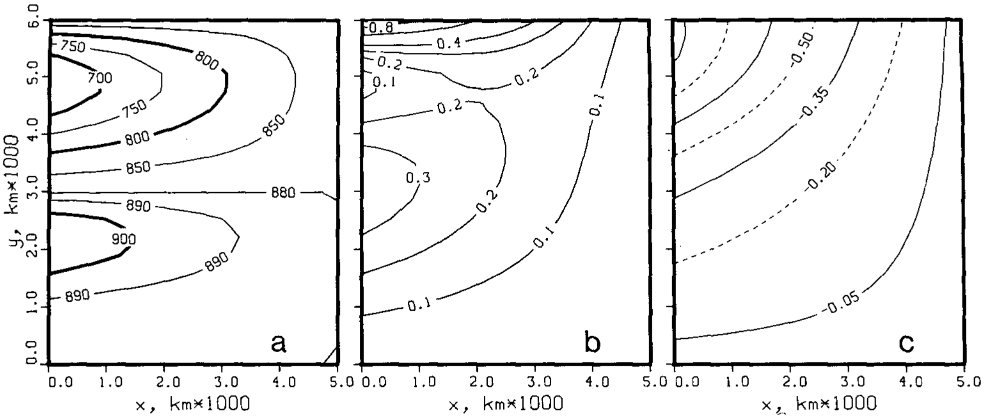

FIG. 11. As in Fig. 10 with: $M_2 = -1$ Sv, $M_3 = 5$ Sv. $|\Psi_2|_{max} = 2.34$ Sv, $|\Psi_3|_{max} = 5.32$ Sv.

layer because we do not know the diffusive corrections to the vertical velocity at the top of this layer. It is not clear how to model the diffusion between the upper layer and the Ekman layer on top of it, but we can increase the resolution of the wind driven circulation by adding more layers, and then it will be possible to find the diffusive effects on each of the layers except for the uppermost one.

In the second example considered here (Fig. 11a–c) $M_{2,\text{ heat fluxes}} = -1$ Sv (abbreviation for 10^6 m^3 s^{-1}) so that there is a net vortex compression in the middle layer: $\int\int (w_2^* - w_1^*)dxdy < 0$. This tends to induce a southward flow in layer 2 [see (4.20)], but the structure of the flow is still dominated by the variations in the thickness of the upper layer.

In the last example (Fig. 12a–c) $M_{2,\text{ heat fluxes}} = +1$ Sv, so that there is a net vortex stretching in layer 2. This enforces the northward flow under the wind-driven subpolar gyre, and weakens the southward flow under the subtropical gyre. The circulation in layer 2 is still similar to the two-gyre wind driven circulation in layer 1.

The above examples demonstrate the physics of the mid-depth circulation: It is driven by the cross-isopycnal diffusive velocities, and is affected by the air–sea heat fluxes through the formation of water masses, and by the wind driven circulation that causes the variations in the depth of the main thermocline. The two-gyre mid-depth circulation seems to be quite robust to changes in the amount of water injected into it from the outcropping region. It is not clear how a more realistic parameterization of the mixing processes will affect it. What we want to emphasize here, however, is not the specific result of two-gyre mid-depth circulation, but the mechanisms by which heat fluxes and wind affect this circulation.

Perhaps the most important conclusion of this section concerns the formulation of the correct thermal boundary conditions for the thermocline problem: The usual approach is to replace the physical boundary

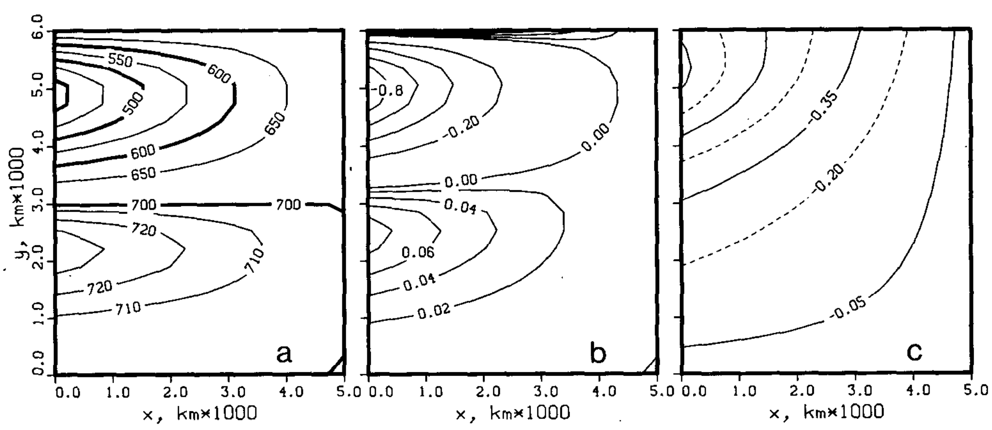

FIG. 12. As in Fig. 10 with: $M_2 = +1$ Sv, $M_3 = 5$ Sv. $|\Psi_2|_{max} = 2.71$ Sv, $|\Psi_3|_{max} = 7.82$ Sv.

TABLE 1. Parameters used in the examples of section 4.

Ekman pumping: $w_e(x, y) = -10^{-4} \sin(\pi y/3000 \text{ km})$ cm s^{-1}
Densities and density ranges of the layers (see section 4a):

$\rho_1 = 1.02575$; $\Delta_1\rho = -0.00250$
$\rho_2 = 1.02740$; $\Delta_2\rho = -0.00080$
$\rho_3 = 1.02786$; $\Delta_3\rho = -0.00012$

Summary of the results: (1 Sv = 10^6 m^3 s^{-1}.)

$M_{2,\text{heat-fluxes}}$ (Sv)	$M_{3,\text{heat-fluxes}}$ (Sv)	H_1 (m)	H_2 (m)	H_3 (m)	See Figs.
0	5	770	950	3280	10a–c
−1	5	880	950	3170	11a–c
1	5	700	950	3350	12a–c

conditions of heat fluxes with a specification of the density at the base of the mixed layer. It should be clear from the model here, that the heat fluxes have another, independent effect—the production of water masses. Information on this formation (or equivalently, on the air–sea heat fluxes) is necessary for the determination of the basic stratification and the buoyancy driven flows, and has to be specified as part of the thermal boundary conditions. The only way to avoid having to specify both surface density and heat fluxes is to include the physics of the mixed layer within the model.

5. Conclusions

We have tried in this paper to examine the importance of the thermodynamical processes to the dynamics of the general circulation. Two simple models were presented and used to understand the role of interior small scale mixing and of air–sea exchanges: A continuous model of the deep circulation and a three layer model of the deep and wind-driven circulations.

The results seem to lead to two main conclusions.

1) The mixing processes are essential not only for driving the deep thermohaline circulation, but also for determining the basic vertical density stratification of the wind driven circulation.

2) The air–sea heat fluxes affect the interior circulation in two ways. They determine the surface density (together with the surface circulation); and they produce masses of water of different densities which determine the basic interior stratification together with the mixing processes. As a result, one has to specify the heat fluxes *in addition* to specifying the surface density, as the thermal boundary conditions for the thermocline problem. These boundary conditions account for the full effects of the heat fluxes on the interior *without* explicitly considering the mixed layer physics, and allow one to determine the basic stratification of the circulation, as shown in the previous sections.

Recent studies (Rhines and Young, 1982b; see also Pedlosky and Young, 1983) have demonstrated the importance of the small frictional dissipation due to the mesoscale eddies to the dynamics of the wind driven circulation. Together with the results here, it seems that the physics of the general circulation, and of the thermocline problem in particular, is more intricate than anticipated from simple scaling arguments. Both friction and mixing (diffusion) are small, but their effects are crucial.

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