Calculating the Time-Mean Oceanic General Circulation and Mixing Coefficients from Hydrographic Data

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ABSTRACT

The relation between the circulation calculated from averaged hydrographic data (such as the Levitus data), and the actual time average circulation is examined using a CTD dataset which provides both time and space coverage of a region of the Mediterranean Sea. The connection between eddy mixing coefficients calculated from hydrographic data and the eddy fluxes ($\overline{u'T'}$) they are intended to parameterize is also considered.

An inverse model is used to calculate circulation and mixing coefficients from the time average data. Then, the actual time average circulation is estimated by averaging six realizations of the instantaneous velocity field, and mixing coefficients are calculated by directly parameterizing the eddy fluxes of heat and salt.

Comparing the results obtained by the different procedures, it is concluded that the horizontal time average circulation can be reliably estimated from averaged and smoothed climatological data, but that it is nearly impossible to obtain physically meaningful mixing coefficient from such data.

1. Introduction

Oceanic velocity fields calculated from hydrographic data are often discussed as if they represent the time average circulation. In some cases the datasets are a composite of many sections obtained at different times (Wunsch, 1978). In others, the data have been averaged and smoothed before calculating the velocity field (as in the use of the Levitus, 1982, data by Olbers et al., 1985, or Hogg, 1986). But the connection between the true time average circulation and that estimated from either the original sections or the smoothed data is far from clear. Eddy mixing coefficients calculated from hydrographic data are intended to parameterize mixing by the time-dependent eddy field, but do they?

It is our purpose here to try and advance towards a better understanding of these problems, therefore realizing the possibilities and limitations of using climatological hydrographic data.

The availability of an unusual CTD dataset from the eastern Mediterranean, providing *both time and space* coverage of the region, makes it possible to examine different procedures for calculating the average circulation. Although the region covered by the data is small (250 km × 250 km), it is close to the scale of the general circulation for the eastern Mediterranean, and it is much larger than the mesoscales for this region (15 km). The discussion here, therefore, ought to be relevant to the circulation of larger oceans.

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Section 2 briefly describes the dataset and an inverse model used to calculate absolute velocity field and mixing coefficients from the hydrographic data. Section 3 discusses different procedures of calculating mixing coefficients and time-mean circulation both from time-mean (climatological) hydrographic data, and when many realizations of the hydrography are available. In section 4, the different procedures are applied to the dataset from the eastern Mediterranean Sea, and the differences between them are examined. Conclusions are summarized in section 5. The focus here is on general procedures. The circulation of the region, as calculated from the data, is discussed by Tziperman and Hecht (1988).

2. The dataset and the inverse model

The data used for the inverse calculation is part of an extensive dataset acquired by Israeli Oceanographic and Limnological Research (IOLR) in the eastern Levantine Basin of the Mediterranean Sea from 1979 to 1984. The data were collected on 17 cruises, each about ten days long, separated by 3–4 months periods. During each cruise the 27 CTD stations shown in Fig. 1 were occupied. The stations were arranged in a 5 by 6 regular grid, with half-degree spacing in latitude and longitude.

Our purpose here is to examine procedures used to calculate the time mean circulation from averaged climatological hydrographic data. Because the seasonal signal in the eastern Mediterranean sea is very strong, one cannot meaningfully define a yearly averaged circulation. We have therefore chosen six cruises representing the summer hydrography of the region, and

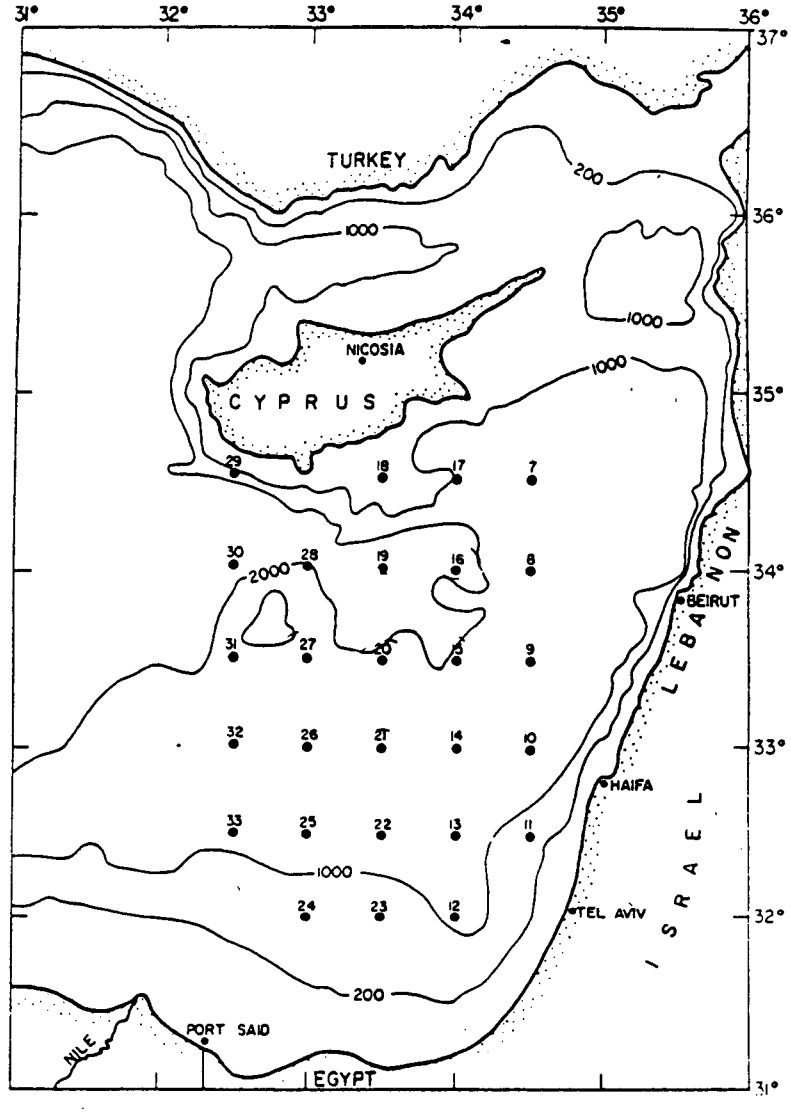

FIG. 1. Station location and bottom topography.

use them to calculate the average summer circulation. Within the summer season the hydrography is quite steady, so that time derivative terms may be neglected in the temperature and salt equations (this is further discussed by Tziperman and Hecht, 1988, referred to below as TH). By using summer data we also avoid difficulties due to convective events which occur in the region during the winter season.

The inverse model used is fairly standard, and is described in detail in TH. The velocity field is divided into a known relative part which is calculated from the density using the thermal wind equations with a reference level at 460 m, and an unknown reference velocity. This velocity field is substituted in the advection diffusion equations for the temperature and salinity fields, to obtain a set of linear equations for the unknown reference velocities and mixing coefficients. The equations are written in matrix form, with an additional set of linear inequalities requiring the mixing coefficients to be positive. The solution for the unknown velocities and mixing coefficients is obtained using singular value decomposition (SVD, Wunsch, 1978).

3. Calculating the average velocity field

Ideally, when considering the time-mean circulation, one divides all fields into average and time dependent parts

$$u = \bar{u}(x, y, z) + u'(x, y, z, t), \quad \overline{u'} = 0,$$
$$T = \bar{T}(x, y, z) + T'(x, y, z, t), \quad \overline{T'} = 0. \qquad (1)$$

Substituting these into the temperature equation, and averaging over time, we have

$$\bar{u} \cdot \nabla \bar{T} = -\nabla \cdot (\overline{u'T'}) + \mathcal{F}. \qquad (2)$$

The term $\nabla \cdot (\overline{u'T'})$ is the heat flux convergence due to mesoscale eddies and small scale turbulence, while $\mathcal{F}$ collects the molecular diffusion terms. Parameterizing the eddy fluxes by eddy mixing coefficients, and neglecting the molecular terms, we then have (Pedlosky, 1979)

$$\bar{u} \cdot \nabla \bar{T} = \nabla \cdot (\kappa \nabla \bar{T}), \tag{3}$$

where κ is in general a second order tensor, possibly a function of position (Redi, 1982). [In the calculations presented below only vertical mixing is included in (3), with a mixing coefficient which is possibly a function of depth. See TH for details.] Although the division into averaged and time dependent fields cannot normally be used in the analysis of hydrographic data, it is possible to follow it to some extent using the present dataset.

The average summer circulation of the eastern Levantine Basin is estimated in two ways. First, the averaged fields $(\bar{T}, \bar{S}, \bar{\rho})$ are calculated by averaging the data from the six cruises, e.g.,

$$\bar{T}(x, y, z) = \frac{1}{6} \sum_{i=1}^{6} T^{(i)}(x, y, z, t),$$

where $T^{(i)}$ are the data from the ith cruise, $i = 1, \cdots, 6$. The inverse model is then used to calculate a velocity field from the averaged data. This calculation mimics those based upon average dataset such as that of Levitus (1982).

The second way to calculate the average circulation is closer to the procedure outlined in Eqs. (1) to (3). The data from the six cruises are inverted separately, and then the six resulting velocity fields and mixing coefficients are averaged to obtain the time averaged velocity field $\bar{u}$, and the average mixing coefficient.

But according to (1)–(3) the average mixing coefficient is not what we are after. The mixing coefficient in (3) parameterizes the eddy mixing terms obtained by averaging the full time-dependent temperature equation, and this is also the way to calculate the mixing coefficients from the data. Having calculated the time-averaged velocity $\bar{u}$ (from the average of the six inversions) and tracer fields $\bar{T}$ and $\bar{S}$, we calculate the residuals left by the advection of the average temperature by the average velocity,

$$\bar{u} \cdot \nabla \bar{T} = r(x, y, z) \neq 0. \tag{4}$$

These residuals come from the eddy mixing terms, $\nabla \cdot (\overline{u'T'})$, not represented in the model yet. There is also, of course, a contribution to the residuals from model and data errors. These are ignored for now, and we come back to this problem later [see (13)]. Assume, as in (3), that the eddy term can be parameterized by eddy mixing coefficients

$$\bar{u} \cdot \nabla \bar{T} = r(x, y, z) = \nabla \cdot (\kappa \nabla \bar{T}). \tag{5}$$

The average velocity, $\bar{u}$, on the lhs in (5) is already known from the average of the six independent inversions, and so is $\bar{T}$. Equation (5) can therefore be used to obtain a set of linear equations for the mixing coefficients κ. Solving these equations by singular value decomposition, we find the mixing coefficients actually parameterizing the mixing by the time dependent eddies.

The different ways of calculating the average circulation and the mixing coefficients are summarized in Table 1, and the results are shown in Figs. 2 and 3. The results for the six individual cruises—which were used to obtain the average velocity field—are given in TH.

4. Results, discussion

A comparison of the velocity fields obtained by the two different procedures outlined above, ([inv{avg}], [avg{inv}] in Table 1) shows they are surprisingly similar—see, for example, the reference level velocity at 460 m depth. (Because of the linearity of the thermal wind equations the relative geostrophic velocities in [inv{avg}] and [avg{inv}] are identical. This guarantees that if the reference velocities are similar, so will be the velocity structure throughout the water column.) In contrast, the mixing coefficients calculated from the average tracer fields [inv{avg}] are significantly different from those from the average over all cruises [avg{inv}], and the coefficient parameterizing the eddy terms, obtained by inverting (5) [mix{avg}]. Understanding the differences between the circulation and eddy coefficients obtained by the different procedures would help in evaluating the usefulness of climatological hydrographic data.

This similarity of the velocity fields is somewhat surprising, because the inverse problem is nonlinear, and the procedure would be expected to give different results when the order of averaging and inversion are exchanged. The nonlinearity has two sources. The equations for the reference velocities are of the form

$$u_0 T_x + v_0 T_y + w_0 T_z - (\kappa_V T_z)_z = -(u_r T_x + v_r T_y + w_r T_z), \tag{6}$$

where u_0, v_0, w_0 and κ_V are the unknown reference velocities and mixing coefficient, and u_r is the known relative geostrophic velocity. The relative velocities (u_r) are a function of the density field, so that the rhs depends nonlinearly on the data. Write the system of equations for the reference velocities and mixing coefficients as $\mathbf{Ab} = \Gamma$, where the matrix $\mathbf{A}$ contains derivatives of the temperature and salinity fields, Γ is a column vector containing the rhs of (6), and $\mathbf{b}$ is the vector of unknowns, containing reference velocities and mixing coefficients. The solution for the reference velocities and mixing coefficients is, schematically, $\mathbf{b} = \mathbf{A}^{-1}\Gamma$, where a generalized inverse is implied. The solution for $\mathbf{b}$ is, therefore, cubic in the data

$$\mathbf{b} = \mathbf{A}^{-1}\Gamma \sim \left(\frac{\partial T}{\partial x_i}\right)^{-1} u_r \frac{\partial T}{\partial x_i} \sim \left(\frac{\partial T}{\partial x_i}\right)^{-1} \left(\frac{\partial \rho}{\partial x_i}\right)\left(\frac{\partial T}{\partial x_i}\right), \tag{7}$$

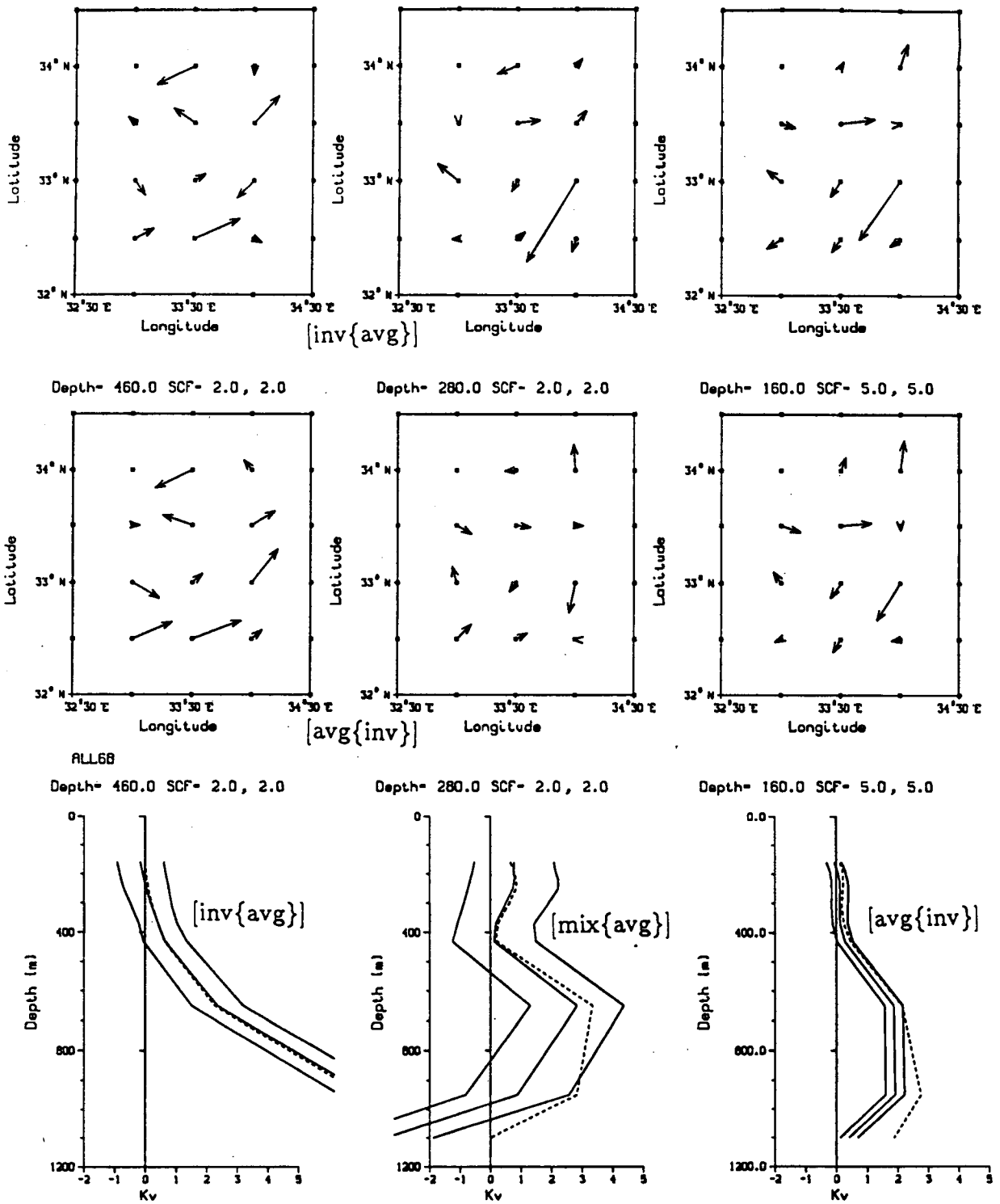

FIG. 2. Calculating the average circulation: The velocity field at three levels, and the mixing coefficients calculated by the three different methods given in Table 1. The distance between tick marks on the axes is equivalent to a velocity vector of 1.5 cm s^{-1} for the two deeper levels, and 5 cm s^{-1} for the upper level shown. In the profiles of the mixing coefficients the three solid lines are the SVD solution and error bars, and the dashed line is the value of the mixing coefficients when inequalities forcing it to be positive are applied (see TH for details). Units in the profiles of the coefficients are cm^2 s^{-1}.

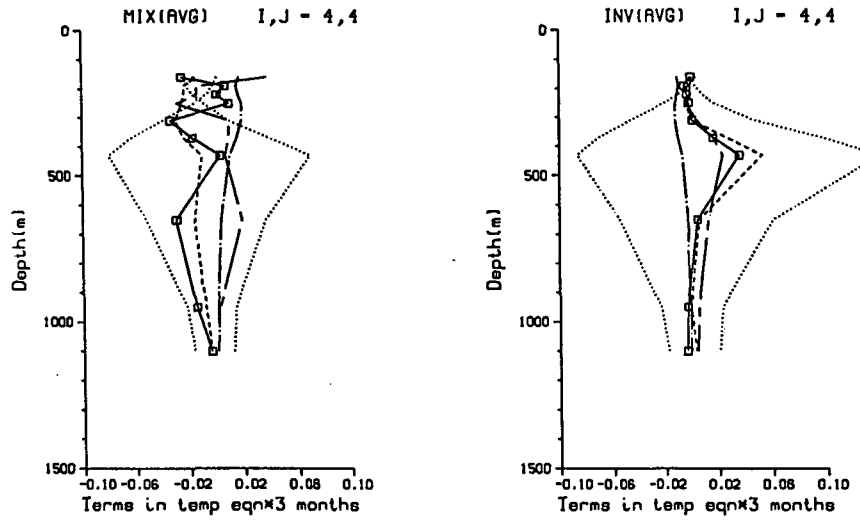

FIG. 3. Calculating the average circulation: Terms in the temperature equation, calculated by the inverse, as described in Table 1. Dotted lines are the horizontal advection terms, uT_x, vT_y; the dashed line is the sum of the horizontal advection terms, $uT_x + vT_y$; the chain-dotted line is the vertical advection term wT_z and the chain-dashed line is the vertical diffusion term $[\kappa_V(z)T_z]_z$.

and consequently one would expect the solution for $\mathbf{b}$ to depend on the order of averaging and inversion. Note that if the horizontal velocities can be shown in general to be independent of the order of inversion and averaging, it should be possible to obtain a good estimate of the time mean horizontal circulation by using an averaged hydrographic data, such as Levitus (1982).

• *Why is the solution for the horizontal velocities independent of the order of inversion and averaging?*

TABLE 1. Calculating the average circulation. The notation is descriptive: [inv{avg}] is the inverse of the averaged fields, [avg{inv}] is the average of the six inversions, [mix{avg}] is the calculation of mixing coefficients dynamically consistent with the average circulation.

Run	Description	Figures
[inv{avg}]	Calculating a velocity field and mixing coefficients from the averaged $\bar{T},\bar{S},\bar{\rho}$ fields. This calculation mimics the inversion of smoothed climatological data.	2, 3
[avg{inv}]	Average of the velocity fields and mixing coefficients obtained by separately inverting each of the six cruises. The resulting velocity field is the true time average velocity $\bar{u}$.	2, 3
[mix{avg}]	Calculating the mixing coefficients by using the average velocity $\bar{u}$ from [avg{inv}], and average temperature and salinity fields, $\bar{T}$ and $\bar{S}$, to form the equation $\bar{u}\cdot\nabla\bar{T} = \nabla\cdot(\kappa\nabla\bar{T})$ and solve it for κ. The resulting mixing coefficients parameterize the eddy fluxes $\overline{u'T'}$.	2

Denote the data and solution for the reference velocity and mixing coefficients of the ith cruise by $T^{(i)}$, $S^{(i)}$, $\rho^{(i)}$, $\mathbf{b}^{(i)}$. The average solution for the reference velocities and mixing coefficients is then

$$\bar{\mathbf{b}} = \frac{1}{6}\sum_{i=1}^{6}\mathbf{b}^{(i)} = \frac{1}{6}\sum_{i=1}^{6}(\mathbf{A}^{(i)})^{-1}\Gamma^{(i)}. \tag{8}$$

or

$$\bar{\mathbf{b}} = \overline{\mathbf{A}^{-1}u_r\cdot\nabla T} \tag{9}$$

where the average is over many realizations of the fields. The velocity and mixing coefficients calculated in [inv{avg}] by inverting the average data can be written schematically as

$$\text{“}\bar{\mathbf{b}}\text{”} = (\bar{\mathbf{A}})^{-1}\bar{u}_r\cdot\nabla\bar{T}. \tag{10}$$

The two solution $\bar{\mathbf{b}}$ and “$\bar{\mathbf{b}}$” are different:

$$\begin{aligned}\bar{\mathbf{b}} - \text{“}\bar{\mathbf{b}}\text{”} &\sim \overline{\mathbf{A}^{-1}u_r\cdot\nabla T} - \overline{\mathbf{A}^{-1}}\bar{u}_r\cdot\nabla\bar{T}\\ &\sim \overline{(\bar{\mathbf{A}}+\mathbf{A}')^{-1}(\bar{u}_r+u_r')\cdot\nabla(\bar{T}+T')} - \bar{\mathbf{A}}^{-1}\bar{u}_r\cdot\nabla\bar{T}\\ &\sim \bar{\mathbf{A}}^{-1}\overline{u_r'\cdot\nabla T'} + \bar{u}_r\overline{(\mathbf{A}')^{-1}\nabla T'}\\ &\quad + \nabla\bar{T}\overline{(\mathbf{A}')^{-1}u_r'} + \overline{(\mathbf{A}')^{-1}u_r'\cdot\nabla T'}.\end{aligned} \tag{11}$$

Roughly speaking, the difference between the average solution for the velocity field and mixing coefficients and the solution calculated from the average tracer fields is due to the transport of heat (salt) by the eddies. Equation (11) makes it clear why we expect the velocity and mixing coefficients calculated in the two different ways to be different. But the velocities in $\bar{\mathbf{b}}$ and “$\bar{\mathbf{b}}$” are actually very similar, and it is only the mixing coefficients that are different.

As has been mentioned above, the difference between $\bar{\mathbf{b}}$ and “$\bar{\mathbf{b}}$” is due to the effects of the time dependent

eddy field on the temperature and salinity fields. Considering for example the term $\mathbf{A}^{-1}\overline{u_r' \cdot \nabla T'}$ on the rhs of (11), we see that the difference between $\bar{\mathbf{b}}$ and "$\bar{\mathbf{b}}$" may be assumed small if the transport of heat by the eddies $\nabla \cdot (\overline{u' \cdot \nabla T'})$ is much smaller than the advection by the mean circulation $\bar{u} \cdot \nabla \bar{T}$. (Other terms are also second and third order eddy correlations, related to heat transport by the eddies, although the relation may not be as obvious as for the term singled out above.) The dominant physical process affecting the temperature and salinity fields in the ocean is the advection by the geostrophic horizontal velocity field. Transport of water properties by the eddy field is a second order effect, and therefore the eddy terms in the expression (11) for $\bar{\mathbf{b}}$ − "$\bar{\mathbf{b}}$" may be assumed to be small by comparison. As a result, the dominant horizontal advection terms in the T and S equations ought to be similar in the two estimates $\bar{\mathbf{b}}$ and "$\bar{\mathbf{b}}$" for the average circulation, and are indeed so (Fig. 2). Figure 3 shows profiles of the different terms in the temperature equation for the two estimates of the velocity field and mixing coefficients. The terms uT_x and vT_y are the dominant ones and they are quite similar for the two solutions $\bar{\mathbf{b}}$ (the average of the separate inversions), and "$\bar{\mathbf{b}}$" (the inverse of the average data).

• *Why does the solution for the mixing coefficients depend on the order of inversion and averaging?* The differences between the velocity field in the inverse of the average data and the average of the inversions was shown in (11) to be of the order of magnitude of the eddy fluxes. This difference is small relative to the horizontal advection terms, but of the same order as the diffusion terms in the tracer equations. As a result, the differences in the horizontal velocity field in the two estimates are relatively small, but the differences for the mixing coefficients may be of the order of magnitude of the mixing coefficients. (Compare the mixing coefficients calculated in [inv{avg}] and [avg{inv}] shown in Fig. 2.)

• *Mixing coefficients estimated from average (climatological) data: is it possible?* Note that, at least in principle, it might be possible for the mixing coefficients evaluated from the average tracer fields to be correct in spite of the difference between $\bar{\mathbf{b}}$ and "$\bar{\mathbf{b}}$". Remember that the coefficients actually representing the mixing by the time dependent eddy field are not the ones in the average solution $\bar{\mathbf{b}}$ calculated in [avg{inv}], but rather the ones calculated in [mix{avg}] by parameterizing the actual eddy fluxes.

Suppose the mixing coefficients are estimated from an average climatological dataset, and consider the following possibilities. By inverting the average fields, a solution "$\bar{\mathbf{b}}$" is found which is different from the actual average velocity field and mixing coefficients $\bar{\mathbf{b}}$, because of the nonlinearity of the inverse problem. This difference was shown above [see (11)] to be a result of the mixing by time dependent eddies. It is not impossible that the difference between the actual average solution $\bar{\mathbf{b}}$ and the one obtained from the average fields, "$\bar{\mathbf{b}}$", is *only in the mixing coefficients part of* $\mathbf{b}$, and that the velocities estimated in the two ways are the same. This is a reasonable possibility simply because the difference between the average of the inverse and the inverse of the average is due to the mixing, so that one would hope to find this difference expressed only in the part of the solution representing this mixing.

If this is the case, we can see that the mixing coefficients obtained from the average fields may actually be the correct ones, parameterizing the eddy fluxes. Note first that if the difference between $\bar{\mathbf{b}}$ and "$\bar{\mathbf{b}}$" is in the mixing coefficients only, then the velocity in "$\bar{\mathbf{b}}$", obtained from the average data, is the correct-time mean velocity $\bar{u}$. With the velocity field in "$\bar{\mathbf{b}}$" actually equal to the average velocity, inverting the time-mean data is equivalent to solving [see (6)]

$$\bar{u} \cdot \nabla \bar{T} = \nabla \cdot (\kappa \nabla \bar{T}) \tag{12}$$

for the mixing coefficients, where $\bar{u} = \overline{u_0 + u_{\text{rel}}}$ is the correct absolute time mean velocity. But this is exactly equation (5) which we indicated before to be the right way of calculating the mixing coefficients, actually representing the eddy fluxes. In this case the velocity field calculated from the average data [inv{avg}] is equal to the average velocity field from [avg{inv}]. The mixing coefficients calculated from the average data parameterize, then, the eddy term in (2), and are equal to those calculated in [mix{avg}] by directly parameterizing $\overline{u'T'}$.

In practice, of course, the inverse calculation cannot be expected to perfectly separate the advective effects from the diffusive effects. The difference between the average of the inverses $\bar{\mathbf{b}}$ and the inverse of the average "$\bar{\mathbf{b}}$" would probably affect the velocities in $\mathbf{b}$ as well as the mixing coefficients, therefore giving the wrong mixing coefficients which do not parameterize the eddy fluxes. Clearly, a necessary condition for the inverse model to be able to separate advection from diffusion is a correct parameterization of the mixing. If the eddy coefficients are not a good parameterization for mixing by the time dependent eddies, the model will not be able to separate advection from diffusion, and the solutions for the mixing coefficients may be completely wrong.

Any inaccuracy in the estimate for the horizontal velocities obtained from the climatological hydrography will, according to (12), affect the solution for the mixing coefficients as well. For the mixing coefficients to be of any value, the errors in the horizontal advection terms, $\epsilon[\bar{u}\bar{T}_x + \bar{v}\bar{T}_y]$, must be smaller than the mixing terms

$$\epsilon[\bar{u}\bar{T}_x + \bar{v}\bar{T}_y] \ll \nabla \cdot (\overline{u'T'}) \sim \nabla \cdot (\kappa \nabla \bar{T}). \tag{13}$$

Additional information on the time average velocities (e.g., from long term direct current measurements) can be inserted into the inversion to improve the estimate for $(\bar{u}, \bar{v})$, reduce their errors to the level required by (13), and therefore improve the separation between mixing and advection in the inverse solution.

The desired separation between advection and mixing in the solution calculated from the average data did not occur in the calculation presented in Fig. 2. The horizontal velocity calculated from the average data [inv{avg}] is slightly different from the time average velocity [avg{inv}], and the mixing coefficients calculated from the average data are quite different from those parameterizing the eddy fluxes [mix{avg}]. This failure of the inversion of the averaged data is not surprising considering the limitations of the dataset and the inverse model used. We had only six realizations of the hydrography in a region where the eddy activity is very strong (TH), and it is clear that an average over six realizations cannot give a good estimate for the time mean fields in the region. An additional problem is the parameterization of the mixing. Only vertical mixing was used in the inverse model, but in larger scale datasets one would probably need to use a long-isopycnal and cross-isopycnal mixing parameterization in order to properly model the mixing by the mesoscale eddies. We want to emphasis, though, that the above discussion of the difference between $\bar{\mathbf{b}}$ and "$\bar{\mathbf{b}}$" and the difficulties in calculating meaningful mixing coefficients is applicable in general to the analysis of climatological hydrographic data by inverse methods. More controlled experiments (perhaps with simulated data) are needed to decide whether a consistent inverse model, together with a high quality climatological dataset, may yield an accurate and physically consistent estimate for the mixing coefficients and the time-mean circulation.

5. Conclusions

The problem of estimating the time averaged general circulation, and the appropriate mixing coefficients from hydrographic data was considered. The above calculations and discussion seem to indicate that the time-averaged horizontal velocity field can be calculated from an estimate of the averaged density and tracer fields (such as the Levitus, 1982, dataset). It is more difficult, however, to obtain reliable and meaningful estimates for the mixing coefficients parameterizing the time dependent eddy terms $\overline{u'T'}$. Such an estimate requires the data to be very accurate, and—perhaps a much more restrictive condition—mixing coefficients must be the correct parameterization of the mixing by the eddy field, $\overline{u'T'}$. If the parameterization of the mixing is inappropriate, the model will not be able to correctly distinguish between advection and mixing, and the estimates for both velocities and mixing coefficients will be wrong. Additional information about time mean velocities, from long term direct current measurements, may improve the inverse estimate for the mixing coefficients.

Eddy mixing parameterization is probably not valid in many oceanic regions—strongly nonisotropic turbulent regions near western boundary currents, or regions of strong salt fingering activity to name two. Mixing coefficients calculated from climatological hydrographic data cannot be expected therefore to represent the eddy fluxes they are intended to parameterize. One is probably better off estimating the time mean horizontal circulation from smoothed climatological data, and relying as little as possible on eddy coefficients to explain the observed hydrographic fields.

The numerical values of the mixing coefficients are not what one is actually after. The value of these coefficients lies in helping to answer questions about the ocean circulation, such as what is the effect of ocean heat transport on climate changes, or what is the role of the oceans in the global CO_2 cycle, etc. It seems possible to explain the hydrography without mixing, by allowing the eddy field present in the nonsmoothed data to reduce the residuals left by the larger scale flows. The resulting flow is of a cellular character (Wunsch, 1978), and probably does not represent the time mean circulation. But such a nondiffusive model can be used to directly address the above questions about the oceanic circulation without relying on eddy coefficients parameterization, when the time mean circulation itself is not wanted (Wunsch, 1984).

In any case, questions about the ocean circulation are preferably answered using methods—numerical models, inverse methods, or data assimilation techniques—which do not depend on subgrid parameterization, at least as far as the information we are interested in is concerned.

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