Using the Adjoint Method with the Ocean Component of Coupled Ocean-Atmosphere Models

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Abstract

A Primitive Equation Ocean General Circulation Model (PE OGCM) in a global configuration similar to that used in coupled ocean-atmosphere models is fitted to climatological data using the adjoint method. The ultimate objective is the use of data assimilation for the improvement of the ocean component of coupled models, and for the calculation of initial conditions for initializing coupled model integrations. We argue that ocean models that are used for coupled climate studies are an especially appropriate target for data assimilation using the adjoint method.

1. Introduction

Ocean climate models that are used in coupled ocean-atmosphere model studies are an especially important target for data assimilation efforts. Data assimilation can be used to improve model parameterizations, and to calculate their poorly known parameters such as eddy coefficients, surface boundary forcing fields, *etc.* The assimilation methodologies can also be used to find optimal initial conditions for coupled model climate simulations. The improved ocean model parameterizations and the initial conditions obtained through data assimilation should clearly lead to an improved climate simulation of the coupled ocean-atmosphere model.

The combination of OGCMs and oceanographic data for the above purposes can be formulated as an optimization problem by minimizing a cost function which measures the degree to which the model equations are satisfied, as well as the distance of the model results to the data. The minimization of this cost function may be obtained using iterative algorithms such as the conjugate gradient algorithm, with the gradient of the cost function efficiently estimated using a numerical model based on the adjoint equations of the original model equations. Thus this optimization approach is often referred to as the "adjoint method" (*e.g.* Le Dimet and Talagrand, 1986; Thacker and Long, 1988; Wunsch, 1988; Tziperman and Thacker, 1989). The optimization approach to data assimilation is the only methodology allowing an efficient estimation of a large number of internal model parameters and thus has the potential of leading to the model improvement that is clearly required for climate studies. In addition, the computational cost of both the adjoint method and coupled ocean-atmosphere climate simulations limits them to about the same degree of model resolution (presently medium to coarse resolution ocean models). Hence the data assimilation problems related to coupled models are an excellent match to the capabilities of the adjoint method.

The work described here is a step towards the ultimate goal of using the adjoint method with the ocean component of coupled ocean-atmosphere models and has three specific objectives. First, we would like to investigate the issue of model formulation for such optimization problems, and in particular the surface boundary condition specification. Second, we shall examine the optimization problem formulation for PE models and its effects on the success of the minimization. Finally, we shall demonstrate methods for efficiently initializing the gradient-based optimization with solutions obtained using simpler, sub-optimal, assimilation methodologies. This report summarizes the main findings of Sirkes *et al.*, (1996, hereafter STT96) and the reader is referred to that work for details not provided here.

Throughout this report, we concentrate on technical issues that must be confronted if full-complexity ocean models are to be used with optimization approaches such as the adjoint method. Because of the coarse resolution of the ocean models used in climate studies, our objective cannot be the optimal estimation of the state of the ocean. We can only expect to improve the simulation of the these models as much as is allowed by their coarse resolution, and thus increase the reliability of coupled models incorporating such coarse ocean models. The estimation of an optimal ocean state from observations must be done using higher resolution models, and probably using simpler and less computationally demanding assimilation approaches.

While there is little doubt that future applications of the adjoint method to ocean GCMs should use time dependent models, we still maintain the steady state assumption in this work, because it provides a relatively simple framework for studying the many still un-encountered problems involved in formulating optimization problems based on a primitive equation OGCM.

Although the combination of 3D ocean climate models with data is of obvious interest, it is surprising to realize that there have only been very few efforts so far trying to apply the adjoint method to full complexity 3D ocean models. Tziperman *et al.* (1992a,b) have examined the methodology using simulated data and then real North Atlantic data; Marotzke (1992), and Marotzke and Wunsch (1993) have considerably improved on the methodology and analyzed a North Atlantic model; Bergamasco *et al.* (1993) used the adjoint method in the Mediterranean Sea with a full PE model, and Thacker and Raghunath (1993) have examined some of the technical challenges involved in inverting a PE model. All of these studies, including the present one, are based on the adjoint of the Geophysical Fluid Dynamics Laboratory (GFDL) model developed by Long *et al.*, (1989), and generously made publicly available. Recently, Schiller (1995) and Schiller and Willebrand (1995) used an approximate adjoint of the GFDL PE model to calculate surface fluxes and to estimate the North Atlantic hydrography. It is worthwhile noting that such adjoint models can also be used for somewhat different data assimilation approaches than used here, such as the approach of Bennett and McIntosh (1982).

In the following sections we describe the model and data used in this study (Section 2), and discuss the formulation of the optimization problem (Section 3). We then briefly present some results demonstrating the general issues discussed here (Section 4), and finally discuss the lessons to be learned for future work and conclude in Section 5.

2. Model and data

One of our main ultimate objectives is to use data in an effort to improve ocean models used in climate simulation runs. When choosing a model for the inverse/ assimilation, we therefore take the approach, that it should be possible to run independently in a simulation mode. This determines our choices of model and surface boundary condition (b.c) formulations. We use the GFDL PE model, in a coarse resolution global configuration (roughly a 4 degrees resolution in the horizontal and 12 levels in the vertical, see STT96 or Bryan and Lewis, 1979). The Arctic Ocean is not included in our model. The model geometry and resolution are also similar to those presently used by coupled ocean-atmosphere models.

The choice of surface boundary condition formulation turns out to be a crucial factor in the optimization problem that we have set out to solve here. There are two commonly used surface boundary condition formulations: flux conditions, in which the heat flux is specified independently of the model sea surface temperature (SST), and restoring conditions in which the heat flux is calculated by restoring the model SST to a specified (possibly to the observed) SST distribution. Similarly, specified fresh water flux or restoring conditions may be used for the salinity surface boundary conditions.

Previous applications of the adjoint method to 3D GCMs used flux conditions in an effort to calculate the surface fluxes that result in a good fit to the observed SST as well as to the interior temperature. Unfortunately, the results were not satisfactory, and a large drift of the surface temperature from the observed one is found, up to 6 degrees at some places. Tziperman *et al.* (1992b) suggested that this drift is the result of using flux boundary conditions, rather than restoring conditions that are normally used in ocean modeling. Marotzke and Wunsch (1993) proposed that this drift might be a result of the use of a steady model which lacks the large seasonal signal in the SST, and that this problem might be resolved using a seasonal model.

It is well known that ocean GCMs give very poor results when driven by specified surface heat fluxes rather than using restoring to the observed surface temperature. It is also known that when run under restoring conditions to observed SST and surface salinity, ocean models fit the observed temperature and salinity quite well, yet produce very poor estimate of the surface fluxes of heat and fresh water, and therefore of the meridional fluxes of heat and fresh water (Tziperman and Bryan, 1993). Boning *et al.* (1994) suggested that the poor simulated meridional fluxes are due to the non realistic meridional circulation in most ocean models. We would like to suggest that this deficiency of ocean models, rather than the lack of a seasonal cycle, is the reason for the inability of previous inverse studies to obtain a reasonable solution for the temperature distribution.

Now, in coupled model studies, it is presently more crucial for the ocean model to get the SST right than the heat flux, as the latter may be corrected for, if needed, using the artificially added flux correction. This dictates our choice of surface boundary condition formulation that is different from what was used in previous applications of the adjoint method to similar models, namely a restoring boundary condition rather than flux boundary conditions. Under restoring boundary conditions the model is driven with an implied air-sea heat flux, H^{SST}, that is calculated at each time step from the difference between the model SST and the SST data. Similarly, an implied fresh water flux, $[E-P]^{SSS}$, is calculated from the difference of the model sea surface salinity (SSS) and the surface salinity data (see, for example, STT96 or Tziperman and Bryan (1993) for the explicit expressions for these fluxes).

With the above model and using the restoring boundary conditions, our model produces as good a simulation as one normally obtains in such coarse model simulations without data assimilation, and the detailed results are shown and analyzed in STT96. These results still leave plenty of room for improvements through data assimilation.

The data used in this study are the annually averaged temperature and salinity analysis of Levitus (1982); the annually averaged climatologies of heat flux from Esbensen and Kushnir (1981), of fresh water flux ($[E-P]$) from Baumgartner and Reichel (1975) and of winds from Hellerman and Rosenstein (1983). The large errors expected in these climatological data sets, especially in the surface flux data, dictate the values of the weights used in the cost function formulation (STT96).

3. Optimization problem

One of the main lessons that have been learned over the past few years while trying to combine simplified 3D ocean models and data, is that the correct formulation of the inverse problem is of crucial importance to the success of the optimization, and that much thought and understanding of the dynamics should enter the process of posing the optimization problem. We find that a Primitive Equations model is even more sensitive to the precise problem specification.

Once the data and model formulation have been specified, the next stage in the formulation of the inverse problem is to specify a measure (cost function) for the success of the optimization. In our case the cost includes dynamical constraints which require the solution to be as close as possible to a steady state of the model equations. These constraints are estimated following Marotzke's (1992) suggestion by running the model N time steps and including in the cost function the squared difference between the temperature at time step N and the temperature at the beginning of the integration ($n=0$) at each horizontal grid point (i,j) and level k, $(T_{ijk}^{n=N}-T_{ijk}^{n=0})^2$. Similar terms penalize the deviation from steady state of the salinity S, of the horizontal velocity field at all levels (u,v), and of the barotropic stream function ψ. The model integration time $N\Delta t$ should be of the order of the time scale of physically relevant processes in the model, and is set here to two years. The cost also includes data penalty terms that are simply the square of the difference between the data and the model solution. The complete cost function may be written as follows,

$$J(T^{n=0},S^{n=0},u^{n=0},v^{n=0},\psi^{n=0},H,[E-P])$$
$$=\sum_{ijk}\left[\overline{W}_k^T(T_{ijk}^{n=N}-T_{ijk}^{n=0})^2+\overline{W}_k^S(S_{ijk}^{n=N}-S_{ijk}^{n=0})^2\right] \quad \text{(I)}$$
$$+\sum_{ijk}\left[W_k^T(T_{ijk}^d-T_{ijk}^{n=0})^2+W_k^S(S_{ijk}^d-S_{ijk}^{n=0})^2\right] \quad \text{(II)}$$
$$+\sum_{ijk}\left[\overline{W}_k^U(u_{ijk}^{n=N}-u_{ijk}^{n=0})^2+\overline{W}_k^U(v_{ijk}^{n=N}-v_{ijk}^{n=0})^2\right] \quad \text{(III)}$$
$$+\sum_{ij}\left[\overline{W}^\psi(\psi_{ij}^{n=N}-\psi_{ij}^{n=0})^2\right] \quad \text{(IV)}$$
$$+\sum_{ij}\left[W^H\left(H_{ij}^{SST,n=0}-H_{i,j}^d\right)^2 + W^{[E-P]}\left([E-P]_{ij}^{SSS,n=0}-[E-P]_{i,j}^d\right)^2\right] \quad \text{(V)}$$
$$+\sum_{ij}\left[W^H\left(H_{ij}-H_{i,j}^d\right)^2 + W^{[E-P]}\left([E-P]_{ij}-[E-P]_{i,j}^d\right)^2\right]. \quad \text{(VI)}$$

In this equation, a superscript $()^d$ denotes the data; $T^{n=0},S^{n=0},u^{n=0},v^{n=0},\psi^{n=0}$ are the initial conditions for the temperature, salinity, horizontal velocities and barotropic stream function searched for by the optimization; H and $[E-P]$ are the optimal heat and fresh water fluxes also searched for by the optimization when flux conditions are used in the optimization, together with term [VI] which requires the optimal fluxes to be close to the observed ones. When restoring conditions are used as we suggest here, term [V] may be used instead of [VI] in order to force the fluxes calculated from the restoring conditions to be close to the flux data. $\overline{W}_k^T, \overline{W}_k^S, \overline{W}_k^U, \overline{W}^\psi$ are the weights for the steady penalties at each level; $W_k^T, W_k^S, W^H, W^{[E-P]}$ are the weights for the temperature, salinity, heat flux and $[E-P]$ data penalties. The cost weights for the data and steady terms are discussed in detail in STT96. Generally, each weight reflects the squared inverse expected error in the corresponding cost term, and is normalized by the number of penalties in each cost term. Due to the weight normalization, if the deviation of the model solution in each cost term from its corresponding data is of the order of its expected error, the value of each term is expected to be of order one. Once the cost weights are chosen, a given constraint can be said to be consistent with the assumed error level if the corresponding term in the cost function is less than one. Larger values of the temperature and salinity data penalties (term II in (1)), for example, would indicate that the solution is not consistent with the Levitus analysis. A large steady penalty contribution (terms I,III and IV) indicates that the solution is not consistent with the steady state model equations. An optimal solution that is consistent with the data and the model equations should therfore have all terms, representing dynamical constraints as as well as data constraints, of the same magnitude and smaller than one. In addition, the residuals in a consistent solution must be randomly distributed in space (assuming the errors are homogenous, as implied by our choice of weights).

This full cost function, treated as a function of the initial conditions for the temperature, salinity, velocities and stream function presents a difficult optimization problem. We now need to use our knowledge of the dynamics of the oceanic general circulation in order to simplify and reduce this optimization problem to a more manageable form, while not changing the actual problem to be solved. This reduction procedure has three parts: a reduction of the cost formulation, a reduction of the space of control parameters, and a calculation of an initial guess to the optimization solution using simpler assimilation techniques. The combination of all three steps, together with the careful model and boundary conditions formulation presented above, is a necessary condition for a successful assimilation effort.

3.1 Reducing the cost form: Dynamical constraints for velocities and barotropic stream function

Under the primitive equation approximation, there are 5 prognostic fields: temperature, salinity, two horizontal baroclinic velocities and the barotropic stream function. In principle, each of these needs to be required to be at a steady state if such a model solution is desired. But note that given the density stratification, the velocity field in a rotating fluid adjusts to the density stratification within a few pendulum days (equatorial regions may have a longer adjustment time due to the larger radius of deformation there). Therefore, there seems to be no point in penalizing the velocity field (terms III and IV in (1)) separately from the temperature and salinity fields (term I). Once the temperature and salinity steady penalties (I) are minimized by the optimization, the velocity field just adjusts to the optimal stratification. Indeed, removing the velocity and stream function penalties from the cost function resulted in our experiments in an immediate improvement of the convergence of the optimization. The steady velocity penalties were still reduced by the optimization to an acceptable level although not explicitly in the cost function.

It is interesting to note that this issue did not arise in the previous studies of Tziperman *et al.*, 1992a,b; Marotzke, 1992 and Marotzke and Wunsch, 1993. These studies all used the simpler GCM developed in Tziperman *et al.*, 1992a, in which the momentum equations were diagnostic, and therefore did not require separate steady velocity penalties. The issue of dynamical constraints for the velocity field in a PE model is one of the new insights we seem to have gained by going to a full PE model in the present study.

3.2 Reducing the space of control variables for a PE optimization

A primitive equation ocean model such as we use here requires the specification of temperature, salinity, horizontal baroclinic velocity field and the barotropic stream function as initial conditions. This multiplicity of initial conditions that must be calculated by the optimization algorithm poses two potential difficulties. First, the parameter space is significantly larger due to the addition of the baroclinic velocities and stream function as control variables, and hence more iterations are required to find the cost minimum. Second, the additional control variables are very different from the temperature and salinity initial conditions, and thus pose new conditioning problems (Thacker and Raghunath, 1993).

As in the previous sub-section, we can use our knowledge of the physics to formulate the optimization problem in a way that is more likely to result in an efficient solution. We noted above that given the density stratification, the velocity field in a rotating fluid must adjust to the density stratification within a few pendulum days. It seems most reasonable, therefore, that one would not need to calculate initial conditions for the velocities, and restrict the optimization problem to finding only the optimal temperature and salinity. The optimal velocity field will be found by the model after a very short initial adjustment period. This short adjustment period should not have a significant effect on the cost function that is based on the difference in temperature and salinity over an integration period of years. These considerations are not restricted to steady state problems. In time dependent problems, where the adjoint method is used to estimate the initial conditions, the above arguments still hold if the integration time is significantly larger than the velocity adjustment time.

This procedure indeed results in a significantly better conditioning of the optimization problem due to the significantly reduced number of control variables, as further discussed and demonstrated using specific examples in STT96. We note that Schiller (1995) and Schiller and Willebrand (1995) have also used only temperature and salinity as their control variables, as necessitated by their approximate adjoint approach. Their calculation seemed to converge successfully in spite of not using the velocity field as part of the control variables, in agreement with the above arguments.

3.3 Initial guess

If started too far from the absolute minimum of the cost function, the gradient based optimization could lead to a local minimum of the cost function which does not represent the optimal combination of dynamics and data (Tziperman *et al.*, 1992b). Initializing the optimization with a solution that is close to the optimal solution can reduce the possibility of falling into a local minimum, as well as save much of the effort of minimizing the cost function through the expensive conjugate gradient iterations.

Such an initial guess for the optimization solution can be obtained by using simpler assimilation methods that are not optimal in the least square sense, yet have been shown to produce very good approximations for the optimal solution. One such method is the robust diagnostic method (Sarmiento and Bryan, 1982), which has been shown by Tziperman *et al.*, (1992b) to produce a good approximation to optimization problems trying to combine temperature and salinity data with dynamical constraints. A second such approach (which we term "extended robust diagnostics") has been proposed by Tziperman and Bryan (1993) in order to produce an approximation to optimization problems that seek a solution consistent with both air-sea flux data and surface properties data (SST and surface salinity). In both cases the simple assimilation methods are straightforward to implement. They involve adding simple nudging terms to the temperature and salinity equations, and running the ocean model to a steady state in order to obtain the desired approximate optimization solution.

4. Results

We now briefly discuss specific optimizations demonstrating some of the above ideas concerning the formulation of the model, boundary conditions, and optimization problem, as well as the initialization using simpler assimilation methods. The full analysis of these runs is given in STT96, including many plots of the data, solution, residuals, *etc.*

We first note that both the Levitus data and the steady state model solution obtained with no assimilation are characterized by large values of the cost function (entries (a) and (b) in Table 1), indicating that both are far from being optimal as reflected by their large cost values. For the steady state solution this indicates that the solution is not sufficiently close to the observations, so that model improvement is indeed needed.

4.1 Boundary condition formulation

In order to examine the issue of boundary condition formulation for inverse problems involving ocean GCMs, we have performed two optimizations. In the first (entry (c) in Table 1), we have followed the procedure used in previous inverse studies and used fixed flux conditions. In this case, the air-sea heat and fresh water fluxes are among the control variables varied in this optimization in order to estimate the optimal heat flux resulting in minimal cost function. In the second optimization (entry (d) in Table 1) we have not included the surface fluxes as control variables, but have used instead a restoring flux formulation in which the surface fluxes are calculated at each model time step from the SST and SSS. In both cases the surface flux data penalties (that is, V and VI in (1)) are not included in the cost function. The cost function for these runs included only terms I and II in (1).

As can be seen from Table 1, entries (c) and (d), both runs result in consistent solutions of seemingly acceptable cost function (that is, each term smaller than one), and a well balanced distribution of the total cost between the different cost terms. However, it turns out that the optimization using flux conditions converged to a solution that is characterized by large deviations of the SST from the observed one. The large SST deviations are similar to those found by Tziperman *et al.*, 1992b and Marotzke and Wunsch (1993). These SST deviations are not reflected in the temperature data terms in the cost function (Table 1) because these terms include both the surface and deep temperature data penalties.

Table 1. Summary of model runs and assimilations used in this study. Terms in brackets "()" were not actually part of the cost function used in the optimization and are only given for comparison with the other runs.

Run	Cost Parts								Comments
	data T	data S	steady T	steady S	steady u, v	steady ψ	data H	data $[E-P]$	
(a)	0.00	0.00	9.18	8.98	11.97	3221.	0.15	1.87	data
(b)	19.41	61.29	0.01	0.02	0.00	0.00	0.25	1.92	steady state
(c)	0.28	0.32	0.58	0.75	(0.06)	(1.73)	(0.12)	(1.03)	optimization, flux b.c
(d)	0.31	0.32	0.32	0.42	(0.03)	(2.17)	(0.15)	(1.81)	optimization, restoring b.c
(e)	0.31	0.32	0.51	0.49	0.06	1.47	0.15	1.81	robust (restoring b.c)
(f)	0.31	0.35	0.51	0.50	0.06	1.48	0.10	0.66	extended robust

But an examination of the SST solution shows that it is, indeed, very far from the data. This problem does not exist when restoring boundary conditions are used (run (d)), supporting our suggestion that restoring conditions seem a preferable choice. More detailed results and plots are given in STT96. Note that the flux-conditions optimization of entry (c) was not included in STT96, where less direct ways of comparing the flux and restoring conditions were used. This additional optimization, and its comparison to (d) further strengthen STT96's conclusions concerning the boundary condition formulation in inverse models.

4.2 Using simple assimilation methods to initialize the optimization

Entries (e) and (f) in Table 1 demonstrate the efficiency of initializing the optimization with the robust diagnostics (Sarmiento and Bryan, 1983) and extended robust diagnostics (Tziperman and Bryan, 1993) approaches. Note that the simpler assimilation techniques provide a most significant cost reduction as compared with the cost value for both the data and the steady state model solution. Their efficiency and ease of implementation clearly justify their use for initializing the gradient-based adjoint optimization.

In fact, the additional cost reduction obtained by the optimization (entry (d) in Table 1) which was started from the robust diagnostics solution (entry (e) in Table 1) is not very large, although the improvement is significant when the residual fields are examined in detail (STT96). Clearly, when the optimization approach will be used for parameter estimation problems which cannot be done using the simpler assimilation methods, the inherent advantages of this approach will be made even more apparent.

5. Conclusions

This paper, which is a summary of the work more fully described by Sirkes *et al.* (1996), presents a step towards using the adjoint method of data assimilation with the ocean component of coupled ocean-atmosphere models. The ultimate purposes of such an effort are two: first, the improvement of ocean climate models so that their simulations are closer to the observed ocean even when they are run without data assimilation; and second, the calculation of an ocean state based on the available data and the model equations, which can then be used to initialize coupled ocean-atmosphere climate simulations. While we have not achieved these goals as yet, we believe that an important progress was made. Let us briefly summarize the main lessons we have learned here:

Because our goal is to work with ocean models that can also be run without data assimilation, we have taken the approach that the model used for the inverse calculation must be formulated so that it can run independently in a simulation mode. A particular consequence of this approach has been the formulation of the surface boundary conditions. Large SST drifts have been encountered in previous inversions using the adjoint method (Tziperman *et al.*, 1992b; Marotzke and Wunsch, 1993). These inversions used a fixed-flux surface boundary condition formulation. Marotzke and Wunsch have suggested that the drift in their model towards colder surface temperatures is due to a tendency of their model towards "winter conditions" which results from the lack of a seasonal cycle in their steady model. We have shown that the drift in SST can be eliminated in our model by using restoring surface boundary conditions. Such boundary conditions are also more physical because they reflect a feedback between the SST and heat flux, as in the actual ocean-atmosphere system. Previous inverse calculations ignored this feedback, resulting in solutions characterized by a combination of a cooling surface flux and very cold SST. In the real ocean-atmosphere system, cold SST would be balanced by surface heating, as will be the case when restoring conditions are used. We suggest, therefore, that the use of flux conditions, together with the poor simulation of the meridional circulation (and thus the meridional heat flux) by many ocean models (Boning *et al.*, 1994) may be the cause of the SST drifts, rather than the lack of a seasonal cycle in the inverse model.

Our optimization approach provided a better solution than both the steady state model solution obtained with no data assimilation and the original climatological data sets. This solution was much more consistent with both the data and steady constraints, and therefore significantly more optimal in the least square sense. In the context of coarse climate ocean models such as used here, we view "optimal" as a state of the ocean that can be used to initialize coupled model climate simulations in a better way than initializing from the data alone, or from the model steady state, as is done today.

The use of a PE model in this study, as opposed to simpler models used in our previous efforts, resulted in some novel findings concerning the formulation of inverse problems for such models. We have shown that it does not seem necessary to include explicit dynamical penalties for the baroclinic velocities and barotropic stream function, because of the fast adjustment of the velocity field to the stratification in a rotating fluid. It seems sufficient to penalize the deviations of the temperature and salinity from a steady state. The fast adjustment of the velocity field to the stratification has also led us to suggest that one can do well by using only the temperature and salinity as control variables to be calculated by the optimization. This suggestion is consistent with the experience of Schiller and Willebrand (1995) and should result in an improvement of the conditioning of optimization problems based on PE ocean models.

A most successful part of this study has been the use of simple assimilation method to obtain good approximations to the optimization problem. These approximations are then used to initialize the optimization, significantly reducing the minimization effort in the optimization itself, as well as the possibility of encountering local minima that may prevent the optimization from finding the desired optimal solution.

We feel that the technical aspects of inverting complex PE ocean models treated here, as well as the more general issues we dealt with, should be useful to future studies directed at using data assimilation with ocean climate models. There is a clear and urgent necessity of improving ocean models used for climate studies, and of using these models to estimate the ocean state as well as is allowed by the available data. We have argued here that the adjoint method is a most appropriate tool for obtaining these goals, and we feel that they should and can be achieved in the near future.

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