Name: _____ TF: _____

Harvard University
Computer Science 121
Professor Salil Vadhan

Final Examination
Saturday, January 21, 2006

Time: Three hours. Solve ALL the problems. The points total to 150.
You may use any result proved in class or in Sipser, but you must identify it clearly. Good luck!
WRITE YOUR NAME AT THE TOP AND TURN IN THE EXAM WITH YOUR BLUEBOOK!

PROBLEM 1 (4+4+4 points)

For each of the following conditions, either state that no language satisfies it, or give an example of a language that satisfies it. No justifications are necessary in either case.
(A) L is decidable but not recognized by any PDA.
(B) L is Turing-recognizable but not decidable.
(C) L is in P but not in NP.

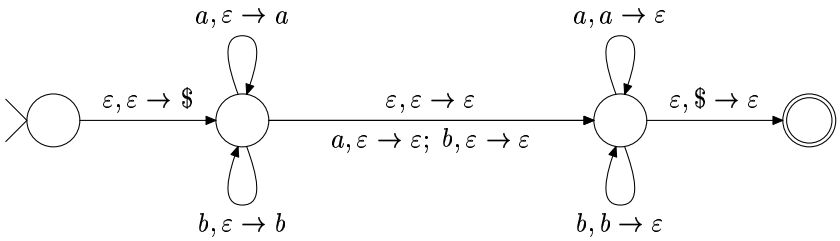
PROBLEM 2 (15 points)

Give the 4-tuple for a grammar that generates the following context-free language over the alphabet $\Sigma = \{a, b\}$:

$$\{x \in \Sigma^* : x \neq x^R \text{ and } |x| \geq 1\}$$

PROBLEM 3 (5+10 points)

(A) What language does the following PDA recognize?



(B) Prove that this language is not regular.

(TURN OVER!)

PROBLEM 4 (6+6+6+6+6+6 points)

Answer TRUE or FALSE. Justify your answers in a sentence or two.

(A) If L is regular, then $L' = \{x \in L : |x| > 2006\}$ is also regular.
(B) Every language is either Turing-recognizable or co-Turing-recognizable.
(C) If $f(n) = O(2^n)$, then P $\subseteq$ TIME($f(n)$)
(D) Simulating a 2-tape TM by a 1-tape TM requires an exponential slowdown.
(E) $n^3 = o(n^3 + n^2)$.
(F) The number of languages *not* in NP is countable.

PROBLEM 5 (10 points)

List all languages recognized by DFAs having exactly two states, over the alphabet $\Sigma = \{a\}$.

PROBLEM 6 (15 points)

Complete the following table with `YES`, `NO`, or `?? (CURRENTLY UNKNOWN)`. No explanations needed. In the following table, M always stands for a Turing machine and R always stands for a regular expression.
(One point per box.)

Language	decidable	Turing-recognizable	co-Turing-recognizable	P	NP
$\{\langle R\rangle : R$ generates $abba\}$					
$\{\langle M\rangle : \|L(M)\| < 2006\}$					
$\{\langle M\rangle : M$ halts on all inputs$\}$					

PROBLEM 7 (10+15 points)

For each of the following languages, say whether they are in P, are NP-complete, or neither. Prove your answers.
(A) 3-CLIQUE= $\{G : G$ has a clique of size 3$\}$.
(B) MULT3-CLIQUE= $\{(G, k) : G$ has a clique of size k, and k is a multiple of 3$\}$.

PROBLEM 8 (7+15 points)

Consider the "is polynomial-time reducible to" relation, denoted $\leq_p$, on the class of all languages in NP other than $\emptyset$ and Σ^*.
(A) In class, we showed that 3-SAT $\leq_p$ VERTEX COVER. Does VERTEX COVER $\leq_p$ 3-SAT? Justify your answer.
(B) Is $\leq_p$ reflexive? symmetric? transitive? If the answer to any of these properties is "uncertain," give an assumption that would make the property hold and an assumption that would make the property not hold. Justify your answers. (Remember that we explicitly excluded $\emptyset$ and Σ^* from the relation's domain.)