Name: ____________________ TF: ____________________

Harvard University
Computer Science 121
Professor Harry R. Lewis

Final Examination
Wednesday, January 17, 2007

You should answer the first problem directly on this sheet. Please write your name on this sheet and return it with the blue book. Do the rest of the problems in the blue book. If you can't easily do a problem, skip over it and come back to it later, skipping a page in the blue book so your answers appear in the correct order. The points total 150.

PROBLEM 1 (15 points)

Complete the following table with `YES`, `NO`, or `?? (CURRENTLY UNKNOWN)`. No explanations needed. M stands for a Turing machine, P_1 and P_2 are PDAs, and $\Sigma = \{a, b\}$.

Language:	regular	CF	recursive	r.e.	co-r.e.	$\mathcal{P}$	$\mathcal{NP}$	co-$\mathcal{NP}$
$\{a^n b^{2n} a^{3n} : n \geq 0\}$								
$\{a^n : n = 2^k \text{ for some } k \geq 0\}$								
$\overline{\text{3-SAT}}$ (the complement of 3-SAT)								
$\{\langle M \rangle : M \text{ does not accept } abab\}$								
$\{\langle P_1, P_2 \rangle : L(P_1) \cap L(P_2) \neq \emptyset\}$								

PROBLEM 2 (3+3+3+3+3+3+3+3 points)

State whether each of the following statements is true or false. Explain your answers *briefly*.
(A) If there exists $xyz \in L$ such that $y \neq \varepsilon$ and $xy^n z \in L$ for each $n \geq 0$, then L is regular.
(B) The complement of any finite language is recursive.
(C) There exists a general grammar G such that $L(G) = \{\langle M, w \rangle\}$: M is a Turing Machine that halts on input $w\}$.
(D) If D is a DFA, then $L(D)$ is infinite if and only if the state diagram of D contains a cycle.
(E) If L is a context-free language, then $\{w : w \in L \text{ and } |w| > 2007\}$ is also context-free.
(F) The class of r.e. languages is the complement of the class of co-r.e. languages.
(G) If L is countably infinite, then L^* is uncountably infinite.
(H) $\{\langle M \rangle$: M is a Turing Machine and $aba \in L(M)\}$ is r.e. but not recursive.

PROBLEM 3 (10 points)

Find two context-free languages such that neither is regular, but their intersection and union are both regular. Justify your answer.

PLEASE TURN OVER

PROBLEM 4 (5+5+10+5 points)

(A) Define $\mathcal{NP}$-hard and $\mathcal{NP}$-complete.
(B) Draw a diagram of the likely relationships among $\mathcal{P}$, $\mathcal{NP}$, co-$\mathcal{NP}$, $\mathcal{PSPACE}$, $\mathcal{NPSPACE}$, and the recursive, r.e., and co-r.e. sets, on the assumption that $\mathcal{P} \neq \mathcal{NP}$.
(C) Suppose we define $\text{Counting-Sat} = \{\langle F\rangle\$a^n\colon$ F is a boolean formula with at least n satisfying truth-assignments$\}$. Prove that Counting-Sat is $\mathcal{NP}$-complete.
(D) Comment on whether there is any important difference between the language of part (C) and $\{\langle F, n\rangle\colon$ F is a boolean formula with at least n satisfying truth-assignments$\}$. Why or why not?

PROBLEM 5 (5+5 points)

State whether each of the following languages is regular or non-regular, and prove your answers.
(A) $\{w \in \{a,b\}^* :$ the number of a's in w times the number of b's in w is odd$\}$
(B) $\{a^n(bc)^n : n \geq 0\}$

PROBLEM 6 (8 points)

Draw the state diagram for a DFA that accepts the language L consisting of all stings of 0s and 1s such that every 0 is both preceded and followed by a 1. For example, $10101 \in L$, but $10 \notin L$.

PROBLEM 7 (8 points)

Write a context-free grammar for the language over the four symbols (,), [, and] in which parentheses ( ) are properly balanced and brackets [ ] are also properly balanced but a matched pair of parentheses can't contain brackets. For example, ( )( ) and ( )[( )[[(( ))]]] are legal but ([]) is not.

PROBLEM 8 (3+9+9+9 points)

The *rotation* of a language L is $\{yx : xy \in L,$ for some $x, y \in \Sigma^*\}$.
(A) What is the rotation of a^*b^*?
(B) Prove that the r.e. languages are closed under rotation.
(C) Prove that $\mathcal{P}$ is closed under rotation.
(D) Prove that the regular languages are closed under rotation.

PROBLEM 9 (10+10 points)

(A) Show that determining whether two DFAs are equivalent (accept the same strings) is in $\mathcal{P}$.
(B) Show that determining whether two TMs are equivalent is undecidable.

THE END