**Harvard University**
**Computer Science 121**
**Professor Harry R. Lewis**

Final Examination
Saturday, January 19, 2008

**Please put your answers appear in order in the blue book, leaving a blank page if you skip a problem and plan to come back to it. The points total 150, so if you use one minute per point, you will have a half hour left over.**

PROBLEM 1 (5+10 points)

(A) Write a formal description of the set of all strings over $\{a, b\}$ that have odd length, begin and end with $a$, and have $b$ in the middle. That is, fill in the ellipsis in $L = \{w : \ldots\}$.
(B) Is this language regular? Context-free? Justify your answers.

PROBLEM 2 (5+5+5 points)

Let $L_0$ be regular, $L_1$ be recursive, and $L_2$ be r.e.
(A) Show that $L_1 - L_0$ is recursive.
(B) Show that $L_2 - L_1$ is r.e.
(C) Show that $L_1 - L_2$ might not be r.e.

PROBLEM 3 (5+20 points)

(A) Draw a 3-state NFA that accepts a string over $\{a, b\}$ if and only if its next to last symbol is $b$.
(B) Use the subset construction to convert it to a DFA.

PROBLEM 4 (10+10 points)

(A) Draw a DFA that accepts the language $L(G)$, where

$$G = (\{S, X, Y\}, \{a, b\}, \{S \to aX \mid bY, X \to aX \mid bX \mid a, Y \to aY \mid bY \mid b\}, S).$$

(B) Write a context-free grammar for the following language over the 3-symbol alphabet $\{a, (, )\}$: all nonempty strings in which left and right parentheses are matched, and there are no empty pairs of parentheses or single parenthesized expressions directly enclosed by another pair of parentheses. So $aaa$, $a(a)$, $(a(aa))$, and $((a)(a))$ are all OK, but not $)(a$, $()$, $a((a))$, or the empty string.

PROBLEM 5 (5+5+5+5 points)

Prove or disprove each part, by giving an example or explaining why none can exist.
(A) There is a regular language $L$ such that $LL^R$ is not regular.
(B) There is a regular language $L$ such that $\{ww^R : w \in L\}$ is regular.
(C) There is a regular language $L$ such that $\{ww^R : w \in L\}$ is not regular.
(D) There exist two countably infinite sets whose intersection is not infinite.

PROBLEM 6 (5+5+5 points)

(A) Define $\mathcal{NP}$-complete.
(B) Draw a diagram illustrating the relations of the regular, context-free, recursive, r.e., and co-r.e. sets, as well as $\mathcal{P}$, co-$\mathcal{P}$, $\mathcal{NP}$, co-$\mathcal{NP}$, and the $\mathcal{NP}$-complete and co-$\mathcal{NP}$-complete sets, assuming $\mathcal{P} \neq \mathcal{NP}$.
(C) Repeat part (B), assuming $\mathcal{P} = \mathcal{NP}$.

PROBLEM 7 (10+10 points)

State whether the following languages are (i) r.e., (ii) co-r.e., and (iii) recursive, and in each case, explain why or why not.
(A) $\{\langle G \rangle : G$ is a general grammar that generates at least one string of length at least $1000\}$.
(B) $\{\langle M \rangle : M$ is a TM with at most 1000 states that halts on the empty string$\}$.

PROBLEM 8 (10+10 points)

Define a homomorphism $\phi$ to be *non-erasing* if for all $\sigma \in \Sigma$, $\phi(\sigma) \neq \varepsilon$.
(A) Show that $\mathcal{NP}$ is closed under non-erasing homomorphisms.
(B) Prove that $\mathcal{P}$ is closed under non-erasing homomorphisms if and only if $\mathcal{P} = \mathcal{NP}$.

**THE END**