

Section 3 Handout

CS 121

October 4, 2009

Today's Topics

- Regular and Non-regular Languages
- Context-Free Grammars

1 Regular and Nonregular Languages

There are countably many regular expressions over a language, but there are uncountably many languages—so some of these languages *must not* be regular! But how do we find an explicitly non-regular language? We have two techniques: the pumping lemma and the closure properties of regular languages. You can use either of these techniques to prove (by contradiction) that a language is non-regular.

Pumping Lemma for regular languages:

If L is a regular language, then there exists a constant $p > 0$ such that for any string $w \in L$ with $|w| > p$, there exist strings $x, y, z \in \Sigma^*$, such that $w = xyz$, $|xy| \leq p$, $y \neq \epsilon$, and $xy^n z \in L$ for all $n \geq 0$.

Closure Properties:

Recall from lecture (and from last week's section) that regular languages are closed under *union, concatenation, Kleene Star, intersection, difference, complement, reversal*.

Exercise 1.1. Which of the following are necessarily regular?

- A finite language.
- A union of finitely many regular languages.
- $\{x : x \in L_1 \text{ and } x \notin L_2\}$ where L_1 and L_2 are regular.
- A subset of a regular language.

Exercise 1.2. Show that $L = \{a^i b^j : 0 \leq i < j\}$ is non-regular using the pumping lemma.

Exercise 1.3. Let $L = \{ww \mid w \in \Sigma^*\}$. Show that L is non-regular using the pumping lemma.

Exercise 1.4. Let $L = \{a^i b^j : i > j \geq 0\}$. Show that L is nonregular using the closure of regular languages under reversal.

Exercise 1.5. Show that $L = \{b^n c^{2^k} : n \geq 1, k \geq 1\}$ is non-regular.

2 Context-Free Languages

Context-Free Grammars: A context-free grammar G is a four-tuple, defined as follows: $G = (V, \Sigma, R, S)$, where V (the set of variables) is an alphabet, Σ (the set of terminals) is a set disjoint from V , R is a finite set of rules, with each rule being a variable and a string of variables and terminals, and S (the start symbol) is an element of V .

Exercise 2.1. Give a context-free grammar for $L = \{w : w \text{ is an even-length palindrome} \}$

Exercise 2.2. (a) Give a context-free grammar for $L = \{w : w \text{ has three more } a\text{'s than } b\text{'s}\}$ over the alphabet $\Sigma = \{a, b\}$

(a) Draw a parse tree for the string $baabaaa \in L$.

Exercise 2.3. Let $L = \{wy : w, y \in L(a^* \cup b^*) \text{ and } |w| = |y|\}$. Is L regular? Is L context-free?