Harvard University
Computer Science 121

Problem Set 1

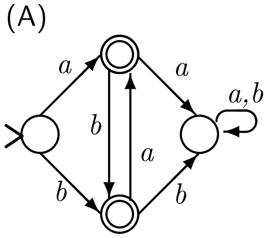
Due Friday, September 25, 2009 at 1:20 PM.
Late problem sets may be turned in until Monday, September 28, 2009 at 1:20 PM with a 20% penalty.
Please hand in Parts A and B separately; each part must be stapled.
All problem sets should be dropped off in the CS 121 box in the basement of Maxwell Dworkin.
See syllabus for collaboration policy.

PART A (Graded by Olga)

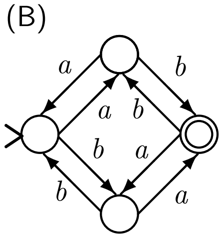
PROBLEM 1 (3+3 points)

Describe informally the language represented by each of the deterministic finite automata below.

(A)



(B)



PROBLEM 2 (4+4+4 points)

Assume that $\Sigma = \{a, b\}$ for the languages below.

(A) Draw a DFA that recognizes $\{w \in \Sigma^* : w \text{ begins with } a \text{ and ends with } a\}$

(B) Write the 5-tuple for the DFA you drew in part (A). (You may write the transition function separately using a table.)

(C) Draw a DFA that recognizes

$$\{w \in \Sigma^* : \text{ the difference between the number of } a\text{'s and } b\text{'s in } w \text{ is a multiple of } 3\}$$

PROBLEM 3 (6+10 points)

(A) For every $n \geq 6$ divisible by 3, prove that there is an undirected graph with exactly n nodes, each of which has degree 4.

(B) A *tree* is an undirected connected graph with no cycles (see Sipser, p.11). Prove by induction that for any $n \geq 1$, any tree with n nodes has exactly $n-1$ edges.
(Hint: for the inductive step, consider deleting an edge).

PART B (Graded by David)

PROBLEM 4 (4+4 points)

Are the following propositions true or false? Justify your answers with a proof or counterexample.

(A) Complementing all states in an NFA N (making the final states non-final and vice-versa) will result in a new NFA N' such that $L(N') = \Sigma^* - L(N)$.

(B) For any languages L_1 and L_2, $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$.

PROBLEM 5 (4+4+6 points)

(A) Draw an NFA that recognizes $\{w \in \Sigma^* : w \text{ ends in } aa \text{ or } w \text{ ends in } bab\}$

(B) Draw the complete computation tree for your NFA for the string $abaa$.

(C) Convert your NFA to an equivalent DFA using the subset construction.
(You may omit any states not reachable from the start state).

PROBLEM 6 (10 + 6 points)

An NFA M contains a *cycle* if there is a state q and a string x such that if M is in state q and reads string x, M can return to state q. Prove or disprove the following statements:

(A) If M recognizes an infinite language, then M has a cycle.
(B) If M has a cycle, then M recognizes an infinite language.

PROBLEM 7 (Challenge!! 3 points)

Let L_1 and L_2 be regular languages. Define the shuffle of L_1 and L_2 to be the language:

$$\{w \in \Sigma^* : w = a_1 b_1 ... a_k b_k, \text{ where } a_1 ... a_k \in L_1 \text{ and } b_1 ... b_k \in L_2, \text{ and each } a_i, b_i \in \Sigma^*\}$$

Show that the shuffle of L_1 and L_2 is regular.
(Note: by $a_1 ... a_k \in L_1$ we mean that the concatenation of all of the a_i's is in L_1, and not necessarily that the individual a_i's are in L_1. Similarly for the b_i's.)