Harvard University
Computer Science 121

Problem Set 2

Due Friday, October 2, 2009 at 1:20 PM.
Late problem sets may be turned in until Monday, October 5, 2009 at 1:20 PM with a 20% penalty.
Please hand in Parts A and B separately; each part must be stapled.
All problem sets should be dropped off in the CS 121 box in the basement of Maxwell Dworkin.
See syllabus for collaboration policy.

Unless specified otherwise, assume $\Sigma = \{a, b\}$

PART A (Graded by Jesse)

PROBLEM 1 (3+3+3+3 points)

Translate the following languages from English description to regular expressions, or vice versa:
(A) $L = \{w \in \Sigma^* : w = \sigma_0 \cdots \sigma_k,\ k \in \mathbb{N},$ where all characters with even indices are the same$\}$
(B) $L = \{w \in \Sigma^* : 3$ consecutive a's do not occur in $w\}$
(C) $((a \cup b)^* a) \cup (b(a \cup b)^*)$
(D) $(b^* \cup ba)^*$

PROBLEM 2 (4+8 points)

(A) Construct a DFA for $L = \{w \in \Sigma^* :$ there are an even number of a's and an even number of b's in $w\}$

(B) Convert your DFA for L to a regular expression using the GNFA construction described in lecture and in Sipser (p. 66). Show the steps of the construction. As you go along, use basic simplifications, such as $(a \cup \epsilon)^* \to a^*$ to make the REs simpler.

PROBLEM 3 (12 points)

Let L be a language over $\Sigma = \{a, b\}$. Define $\textsc{Mumbles}(L)$ as follows:

$$\textsc{Mumbles}(L) = \{w : \sigma w \in L \text{ or } w\sigma \in L \text{ for some } \sigma \in \Sigma^*\}$$

Show that if L is regular, then $\textsc{Mumbles}(L)$ is regular. That is, given a DFA for L, construct an NFA for $\textsc{Mumbles}(L)$. You need not prove your construction correct but you should explain how and why it works.

PART B (Graded by Victor)

For the remainder of the problem set, any constructions should be given formally, but their correctness can be established by a clear explanation rather than a formal proof.

PROBLEM 4 (10 points)

Classify the following sets as finite (in which case state the cardinality), countably infinite, or uncountable. Briefly justify the non-finite cases.

1. $\mathbb{N}$
2. $\{a, b\}$
3. The set of all languages of strings longer than 1024 symbols
4. The set of all strings longer than 1024 symbols
5. $\{\emptyset\}$
6. The set of all strings shorter than 1024 symbols
7. The set of all languages of strings shorter than 1024 symbols
8. $\emptyset$
9. The set of languages over the alphabet $\{a\}$.

PROBLEM 5 (10+10 points)

(A) For any language L, define:

$$\textsc{Echo}(L) = \{\sigma_1^{n_1} \cdots \sigma_k^{n_k} : \sigma_i \in \Sigma \text{ for all } 1 \leq i \leq k,\ \sigma_1 \cdots \sigma_k \in L,\ n_i > 0 \text{ for all } 1 \leq i \leq k\}$$

For example, if $abab \in L$, then $aaaaaabbbbbaaabbbb \in \textsc{Echo}(L)$. Show that if L is regular, then $\textsc{Echo}(L)$ is regular.

(B) For any language L, define:

$$\textsc{Isolated}(L) = \{w \in L : w = \sigma_1 \cdots \sigma_k,\ \sigma_i \in \Sigma \text{ for all } 1 \leq i \leq k,\ \sigma_i \neq \sigma_{i+1} \text{ for all } 1 \leq i < k\}$$

For example, if $\{abab, baaba\} \subseteq L$, then $abab \in \textsc{Isolated}(L)$ but $baaba \notin \textsc{Isolated}(L)$. Show that if L is regular, then $\textsc{Isolated}(L)$ is regular.

PROBLEM 6 (Challenge! 2+1 points)

(A) Let $L/A = \{x : wx \in A \text{ for some } w \in L\}$. Show that if A is regular and L is *any* language, then L/A is regular.
(B) Suppose $L = \{a^n : n \text{ is greater than 2, is even, and cannot be expressed as the sum of two primes}\}$, and $A = \Sigma^*$. Why doesn't this contradict your proof?