Harvard University
Computer Science 121

Problem Set 4

Due Monday, October 19, 2009 at 1:20 PM.
Late problem sets may be turned in until
at 1:20 PM with a % penalty.
Please hand in Parts A and B separately; each part must be stapled.
All problem sets should be dropped off in the CS 121 box in the basement of Maxwell Dworkin.
See syllabus for collaboration policy.

NOTE: For this problem set, there will be NO LATE SUBMISSIONS ACCEPTED

PART A (Graded by Victor)

PROBLEM 1 (10 points)

Draw the state diagram for a PDA for the language of all strings with three times as many as as bs over the alphabet $\{a, b\}$. Use the state diagram notation for PDAs given in Sipser.

PROBLEM 2 (10 points)

Prove that $L = \{a, b, c\}^* - \{a^n b^n c^n\}$ is context free. (Hint: Break it into cases. What are the different ways in which a string can $fail$ to be of the form $a^n b^n c^n$?)

PROBLEM 3 (15 points)

Let G be the following grammar:

$$S \to aB|bA|\varepsilon$$
$$B \to bS|aBB|b$$
$$A \to aS|bAA|a$$

This problem asks you to prove that $L(G) = \{w : w \text{ has equal numbers of } a\text{'s and } b\text{'s}\}$. A meta-hint: the proof is a bit long. Try to be concise–write everything that's necessary, and nothing that isn't.

First, we define the CFGs $G_X = (V, \Sigma, R, X)$ for all $X \in \{S, B, A\}$

By induction on the length of $w \in L(G)$, prove the following (Note: You'll need to prove these all together rather than separately as they depend on each other):

1. w has the same number of as and bs if and only if $w \in L(G_S)$
2. w has one more a than bs if and only if $w \in L(G_A)$
3. w has one more b than as if and only if $w \in L(G_B)$

PART B (Graded by Jesse)

PROBLEM 4 (6+6+6 points)

Are the following languages context-free? Prove or disprove. *When giving a formal construction, no correctness proof is required: just an explanation.*
(A) $\{a^i b^j c^k : i, j, k \in \mathbb{N}, \text{ and if } i = 1 \text{ then } j = k\}$ over $\Sigma = \{a, b, c\}$
(B) $\{a^{n^2} : n \in \mathbb{N}\}$ over $\Sigma = \{a\}$
(C) $\{a^n b^* a^n b^* a^n : n \in \mathbb{N}\}$ over $\Sigma = \{a, b\}$

PROBLEM 5 (15 points)

If L is a language, define $\text{PERMUTATION}(L) = \{x : \text{there exists a string } w \in L \text{ such that } |x| = |w|$ and the numbers of occurrences of any letter in w and x are the same$\}$. Show that if L is regular, then PERMUTATION(L) is context free when $\Sigma = \{a, b\}$. (Note that this isn't true when $|\Sigma| > 2$. Can you see why?)

PROBLEM 6 (Challenge! 2+1 points)

We saw *right regular grammars in lecture and on PS3*, and one can similarly define *left regular grammars* as CFGs with the restriction that all of the productions are of the form $A \to Bx$ or $A \to x$ where A and B are variables and x is a string in Σ^*. In this problem we ask you to consider *two-way regular grammars.*

(A) A *two-way regular grammar* is exactly like a CFG with the restriction that all of the productions are of the form $A \to xB$, $A \to Bx$, or $A \to x$ where A and B are variables and x is a string in Σ^*. Prove that the class of languages generated by two-way regular is distinct from both the class of regular languages and context-free languages. (That is, show that there are two languages L_1 and L_2 that exhibit the following properties: (1) L_1 is generated by a two-way regular grammar yet is not regular, and (2) L_2 is context free yet no two-way regular grammar generates it.)

(B) What would the corresponding machine model be for the languages generated by two-way regular grammars?