ES 154 Assignment #1

Due: 2:00pm, September 24th, 2009

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Problem 1: **Density of States** (30 pt)

In class, we saw that the density of electron states $N(E)dE$ between energe E and $E + dE$ behaves as

$$N(E)dE \propto E^{1/2}dE \tag{1}$$

Derive the entire expression for $N(E)dE$, including the proportional constant, which is given by

$$N(E)dE = \frac{\sqrt{2}}{\pi^2}\left(\frac{m}{\hbar^2}\right)^{3/2} E^{1/2}dE \tag{2}$$

where m is effective electron mass. This formula takes into account spin states, and is for a unit volume of a (semi)conductor.

— *Hint* —

The energy of an electron with a wave vector $\vec{k}$ is given by $E = \hbar^2 k^2/2m$, where $k = |\vec{k}|$. As discussed in class, the k-space volume sandwiched by two concentric $\vec{k}$-space spheres whose radi k and $k + dk$ correspond to energy E and $E + dE$ is given by $4\pi k^2 dk$. So the problem boils down to counting how many available quantized $\vec{k}$ states exist within the sandwiched k-space volume (don't forget to count the spin states!).

Consider that a cubic (semi)conductor with the length of L for each side. If you regard electron waves as standing waves as we did in class, quantized $\vec{k} = (k_x, k_y, k_z)$ states are given by $k_x = \pi/L \cdot m_x$, $k_x = \pi/L \cdot m_y$, and $k_x = \pi/L \cdot m_z$, where each of m_x, m_y, and m_z assumes 0, 1, 2, 3, ... (they cannot be negative, as we are dealing with standing waves). Therefore, the $\vec{k}$-space volume per one $\vec{k}$-state is given by π^3/V where $V = L^3$ is the volume of the (semi)conductor, and thus, the total number of available electron states between energy E and $E + dE$ is given by

$$N(E)dE = \frac{4\pi k^2 dk}{\pi^3/V} \times 2 \times (1/8) = \frac{V}{\pi} \cdot k^2 dk \tag{3}$$

Do you understand why we have a factor of 2 and another factor of 1/8? Therefore, the density of states for a unit-volume (semi)conductor is given by

$$N(E)dE = \frac{1}{\pi} \cdot k^2 dk \tag{4}$$

The rest problem is to express this formula in terms of E. Since Eq. (2) is for a unit volume material, if you evaluate $\int f(E)N(E)dE$, it becomes a concentration of electrons, as opposed to a total number of electrons.

You may instead regard electron waves as traveling waves (not standing waves). In this approach, the electron wave $e^{j\vec{k}\cdot\vec{r}}$ at $x = 0$ and at $x = L$ should assume the same value, and the same for y- and z-direction. With this boundary condition, known as Born-Karman boundary condition, quantized $\vec{k} = (k_x, k_y, k_z)$ states are given by $k_x = 2\pi/L \cdot m_x$, $k_x = 2\pi/L \cdot m_y$, and $k_x = 2\pi/L \cdot m_z$, where each of m_x, m_y, and m_z assumes 0, ±1, ±2, ±3, ... (they are now both positive and negative, as we deal with traveling waves). The rest goes essentially the same as in the standing wave approach. The $\vec{k}$-space volume per one $\vec{k}$-state is now given by $8\pi^3/V$ and hence the quantized states are less densely distributed than in the standing wave case. But since quantized $\vec{k}$ states are not limited to $k_x > 0$, $k_y > 0$, and $k_z > 0$ any more, we have an 8-time larger $\vec{k}$-space volume over which we get to count quantized $\vec{k}$ states, so the result will come out the same as in the standing wave approach.

I encourage you to do calculations using both standing and traveling wave approaches.

Problem 2: **Law of Mass Action** (30 pt)

In class, we showed that in thermal equilibrium, the product of the conduction band electron concentration (n) and the valence band hole concentration (p) is independent of the Fermi level. In other words, np is the same whether you have an intrinsic semiconductor, an n-doped semiconductor, or a p-doped semiconductor. Show that np is given by

$$np = n_i^2 = 4\left(\frac{kT}{2\pi\hbar^2}\right)^3 (m_n m_p)^{3/2} \exp\left(-\frac{E_g}{kT}\right) \tag{5}$$

where m_n and m_p are the effective electron mass and effective hole mass, respectively. The result of Problem 1 is essential in deriving this.

Problem 3 (50 pt)

(a) A silicon (Si) sample is doped with 10^{16} cm^{-3} boron (B) atoms and a certain number of shallow donors. The Fermi level is 0.36 eV above E_i at 300K. What is the donor concentration N_d?

(b) A Si sample contains 10^{16} cm^{-3} indium (In) acceptor atoms and a certain number of shallow donors. The In acceptor level is 0.16 eV above E_v, and E_F is 0.26 eV above E_v at 300 K. How many (per cm^{-3}) indium atoms are un-ionized?

(c) A Si sample is doped with 10^{17} As atoms/cm^3. What is the valence band hole concentration at 300 K? What is the conduction band electron concentration at the same temperature? Where is E_F relative to E_i?

(d) A Si sample is doped with 10^{17} boron (B) atoms/cm^3. What is the valence band hole concentration at 300 K? What is the conduction band electron concentration at 300 K? Where is E_F relative to E_i?

Problem 4 (30 pt)

A Si p-n junction has $N_a = 10^{17}$ cm^{-3} on the p side, and $N_d = 10^{16}$ cm^{-3} on the n side. At 300K, calculate the Fermi levels, draw an equilibrium band diagram, and find the built-in potential V_0 from the diagram.