ES 154 Assignment #2

Due: 2:00pm, October 1st, 2009

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Problem 1 (30 pt)

A Si p-n junction has $N_a = 10^{17}$ cm^{-3} on the p side, and $N_d = 10^{16}$ cm^{-3} on the n side. At 300K, calculate the Fermi levels, draw an equilibrium band diagram, and find the built-in potential V_0 from the diagram.

Problem 2 (30 pt)

We have so far assumed that the Fermi level of intrinsic silicon, E_i, lies in the middle of the bandgap. This is a good approximation you can use in many problems, but to be precise, E_i is slightly off from the middle position. Calculate the exact E_i position in silicon — you can answer in the form of "xxx eV below E_c" or "xxx eV above E_v." To solve this problem, use the expressions for n and p you derived in Problem 2 of HW Set 1, in conjunction with $n_i = p_i$. Explain the physical reason for the deviation of E_i from the middle position (one or a couple sentences should suffice).

Problem 3 (30 pt)

Consider crystalline silicon. What concentration N_d of Arsenic (As) donors must be added to make the conductivity 10^4 times greater than the intrinsic conductivity at room temperature? Assume that the electron mobility is equal to the hole mobility, and that they stay the same after the doping (in other words, neglect carrier scattering due to donor impurities).

Problem 4 (30 pt)

Consider a pn junction in silicon with doping densities of $N_a = 8 \times 10^{15}/\text{cm}^3$ in the p-region and $N_d = 1 \times 10^{17}/\text{cm}^3$ in the n-region. Calculate the built-in potential, depletion layer depth into the p-region, depleition layer depth into the n-region, and maximum electric field strength within the depletion layer, at biases of -5 V (reverse bias), 0 V, and 0.3 V (forward bias). Plot the capacitance $C(V)$ of the pn-junction as a function of the bias voltage V, assuming that the cross-sectional area of the pn-junction is 25 μm^2. The silicon dielectric constant is $\epsilon \approx 12\epsilon_0$ where $\epsilon_0 = 8.854 \times 10^{-12}$ F/m.

Problem 5 (30 pt)

In class, we derived the expression for the built-in potential of a pn junction

$$V_0 = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} \tag{1}$$

arguing that the Fermi level of the p-type region and that of the n-type region should be the same once the junction is formed and equilibrium is reached. This problem seeks to derive Eq (1) using an alternative approach. Once the junction is formed and equilibrium is reached, the conduction band electron concentration will decrease as you walk from the n-region to p-region. Therefore, one could first think of an electron diffusion current in the conduction band, which would flow from the n-region to the p-region. This electron diffusion current (its density, to be accurate) would be expressed as

$$J_{diff}(x) = qD_n \frac{dn(x)}{dx} \tag{2}$$

where D_n is the diffusion constant of conduction band electrons, x is the spatial coordinate along the pn junction, and $n(x)$ is the conduction band electron concentration at position x. Now, the electric field produced in the depletion region opposes the diffusion of electrons. This may be thought of as an opposite-direction conduction band electron drift current that would cancel the electron diffusion current. The drift current would be expressed as

$$J_{drift}(x) = qn(x)v_n(x) = qn(x)\mu_n E(x) \tag{3}$$

where $v_n(x)$ is the drift velocity of electrons at position x, μ_n is the mobility of electrons, and $E(x)$ is the electric field at position x. Since $J_{diff}(x) = J_{drift}(x)$ in equilibrium, we have

$$qD_n \frac{dn(x)}{dx} = qn(x)\mu_n E(x) \tag{4}$$

or

$$\frac{1}{n(x)} \frac{dn(x)}{dx} = \frac{\mu_n}{D_n} E(x) \tag{5}$$

By integrating both sides of the equation above from one end of the depletion region to the other end of the depletion region, derive the built-in potential expression, Eq. (1). In this derivation, you may use the Einstein relation,[1] $D_n/\mu_n = kT/q$.

[1] The diffusion and mobility originate from the same physical event, electron scattering. Einstein quantified their relation in his famous 1905 paper on Brownian motion.