

**ES 154 Assignment #4**  
**Due: 2:00pm, October 22nd, 2009**

Instructor: Donhee Ham  
 Teaching Fellow: Nan Sun

**Problem 1 (50 pt)**

In this problem, we once again study the emitter-degenerated common emitter amplifier: Fig. 1.

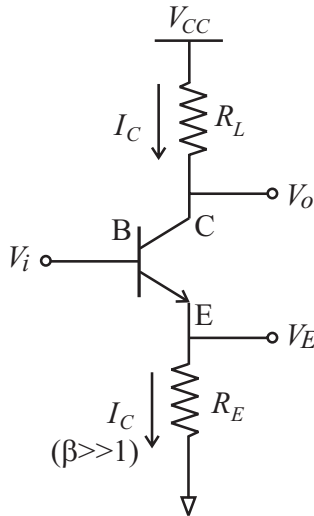


Figure 1: Common emitter amplifier with emitter degeneration.

(a) Using a small signal model and without ignoring the output resistance of the *npn* transistor,  $r_0$ , show that the low-frequency small signal gain of the amplifier is given by

$$G = -\frac{g_m(r_0 || R_L)}{1 + g_m R_E \times \{1 + (g_m r_0)^{-1}\} \times r_0 / (r_0 + R_L)} \quad (1)$$

In the derivation, assume that  $\beta \gg 1$ . For  $r_0 \gg R_L$  and  $r_0 \gg R_E$  (negligible Early effect for given  $R_L$  and  $R_E$ ), the equation above is reduced to

$$G = -\frac{g_m R_L}{1 + g_m R_E} \quad (2)$$

which is the small signal gain we derived in class, assuming  $r_0 = \infty$ . For  $g_m R_E \gg 1$ , or in other words, if the dc voltage drop across the resistor  $R_E$  at an operating point is much larger than the thermal voltage 26mV, the equation above is further reduced to

$$G = -\frac{R_L}{R_E} \quad (3)$$

(b) In principle, the accurate input-output transfer characteristics between  $V_i$  and  $V_o$  can be calculated from the following three simultaneous equations (can you derive them?):

$$V_o = V_{CC} - R_L I_s \left(1 + \frac{V_{CE}}{V_A}\right) \exp\left(\frac{V_{BE}}{V_T}\right) \quad (4)$$

$$V_i - V_{BE} = R_E I_s \left(1 + \frac{V_{CE}}{V_A}\right) \exp\left(\frac{V_{BE}}{V_T}\right) \quad (5)$$

$$V_o - V_{CE} = V_i - V_{BE} \quad (6)$$

Analytically solving these equations to express  $V_o$  as a function of  $V_i$  (by eliminating  $V_{BE}$  and  $V_{CE}$ ) is impossible, for we cannot analytically solve the transcendental equations involved, but still, we can use the three equations above to calculate the small signal gain  $G = \partial V_o / \partial V_i$  at a given operating point. Show that the small signal gain calculated in this way agrees with (1). To be consistent with (a) where  $\beta \gg 1$  was assumed, in writing down (5) it was assumed that  $I_E \approx I_C$ , which is equivalent to  $\beta \gg 1$  (negligible base current).

(Note 1) In the real design practice, one seldom calculates the small signal gain using the transfer characteristic equations as in (b). Using the small signal model as in (a) is a standard, and often much more convenient, method to calculate the small signal gain. In this connection, I won't ask you to do something like (b) any more after this problem. The intention of this problem is to check the consistency between the two methods one more time, in order to understand the foundation of the small signal theory.

(Note 2) As we discussed in class and in the last homework set, in obtaining the input-output transfer characteristic in the forward active region of the transistor, instead of using (4), (5), and (6), which are quite cumbersome to deal with, we often approximate  $V_{BE} \sim 0.6\text{V}$  (or any fixed value between 0.6 and 0.7V), which allows us to write down the following approximate input-output transfer function:

$$V_o \approx V_{CC} - R_L \cdot \frac{V_{in} - 0.6}{R_E} \quad (7)$$

Although this is not the most accurate expression like (4), (5), and (6), it is a good approximation, which provides an insight into how the input-output transfer characteristic would look. It also facilitates determining an operating point in an initial stage of design. In practice, accurate plotting of the transfer function is done using the universal circuit simulation tool called SPICE, which you will learn next week and use in subsequent homework sets. The SPICE, in plotting the transfer function, numerically solves the accurate expressions (4), (5), and (6).

(Note 3) From the approximate transfer function of (7), you can see that the small signal gain is given by  $-R_L/R_E$ . This agrees with (3).

**Problem 2** (40 pt)

Fig. 2(a) is a common-based amplifier, using an *npn* transistor.

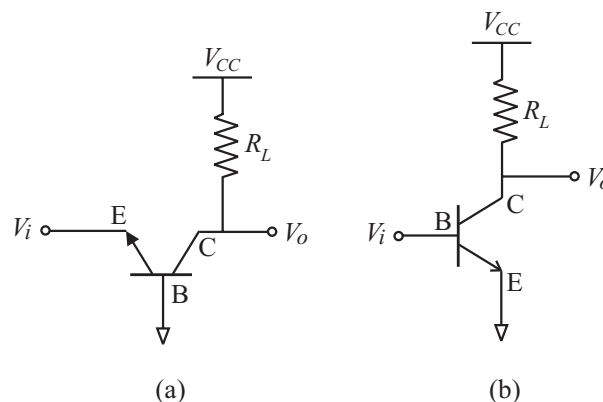


Figure 2: (a) Common base amplifier; (b) common emitter amplifier. For the *npn* BJT, use:  $\beta = 200$ ;  $I_s = 5 \times 10^{-15}\text{A}$ ;  $V_{CE,sat}=0.2\text{V}$ ;  $V_A=130\text{V}$ .

(a) Determine the input resistance, transconductance, small-signal gain, and output resistance of the common base amplifier. Assume  $I_C = 250\mu\text{A}$  and  $R_L=10\text{k}\Omega$ . Use the simplest small signal model (Page 6, Lecture Note #6) for these calculations, but neglect the base-charging capacitor  $C_b$  in the small signal

model. How do the input and output resistances, small signal gain, and transconductance of the common-base amplifier compare to those of the common-emitter amplifier of Fig. 2(b)?

(b) Now assume that  $R_L$  is made large compared to  $r_0$  of the transistor in otherwise the same common base amplifier of Fig. 2(a) ( $r_0$  should now be taken into account). Calculate the output resistance of the amplifier when the amplifier is driven by an ideal current source. Calculate the output resistance of the amplifier when the amplifier is driven by an ideal voltage source.

**Problem 3** (50 pt)

Fig. 3 is an emitter-degenerated common emitter amplifier (if output is taken at the collector of the transistor). Bias scheme is explicitly shown. The small signal ac input to the amplifier is shown as  $v_i$ .

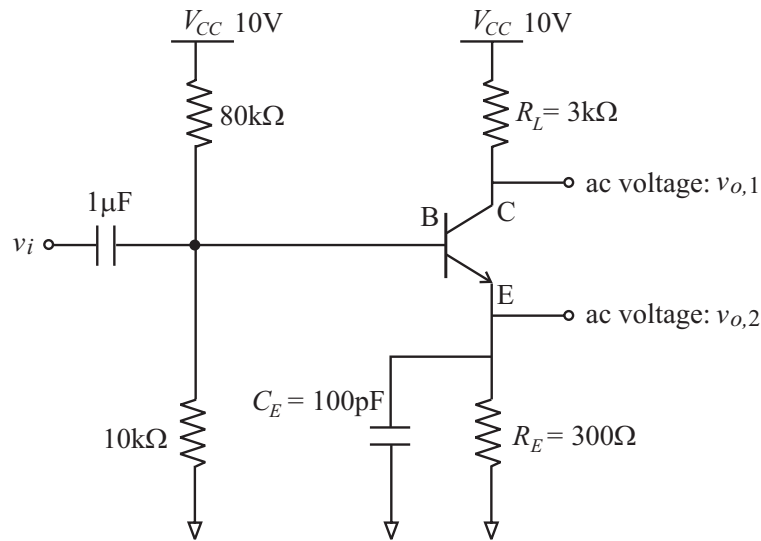


Figure 3: Common emitter amplifier with a resistor  $R_E$  and a capacitor  $C_E$  between the emitter and the ground. For the *npn* transistor, use:  $\beta = 200$ ;  $I_s = 5 \times 10^{-15} \text{ A}$ ;  $V_{C_E, sat} = 0.2 \text{ V}$ ;  $V_A = 130 \text{ V}$ .

(a) Calculate dc bias voltages at the base, collector, and emitter of the transistor. Calculate the collector current ( $\approx$  emitter current). Is the transistor biased in the forward active region?

(b) Let us take an output at the collector of the transistor. Sketch the small signal gain,  $v_{o,1}/v_i$ , of the amplifier as a function of frequency (it goes without saying that you should indicate important values on your sketch). Ignore parasitic capacitors of the transistor.

(c) Now we take an output at the emitter of the transistor. Sketch the small signal gain,  $v_{o,2}/v_i$ , of the amplifier as a function of frequency. Ignore parasitic capacitors of the transistor.