ES 272 Assignment #1

Due: February 19th, 2009

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Problem 1 (60pt)

Let's begin at the beginning with Fig. 1(a). Alice wants to measure the small-signal voltage gain $A(f)$ of the common-source MOS amplifier as a function of frequency, f. An RF generator and a high-bandwidth real-time oscilloscope (Oscilloscope 1) are available to her. The RF generator produces a sinusoidal signal whose frequency and power can be continuously adjusted over certain ranges, and has a 50-Ω source impedance. The oscilloscope has a 1-MΩ input impedance.

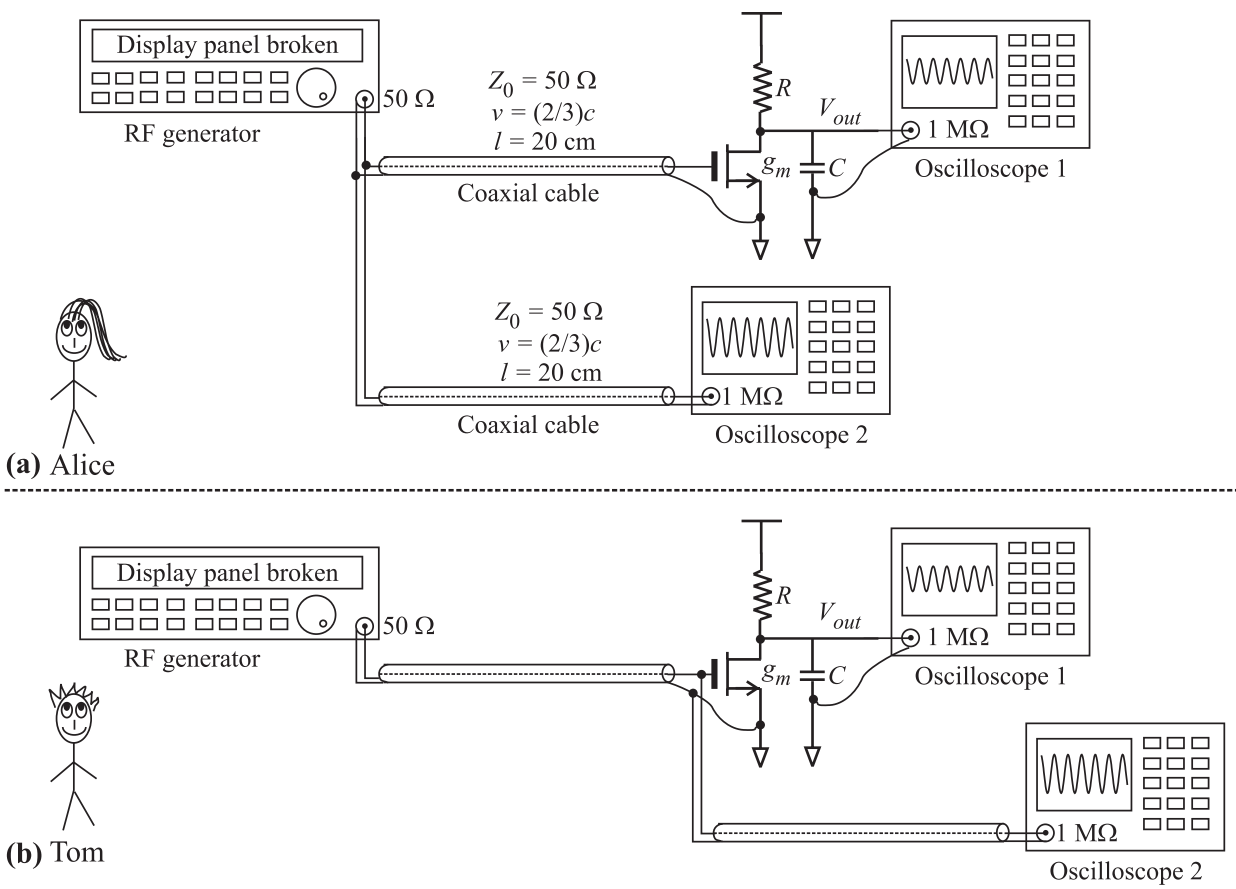

Figure 1: Understanding the transmission line effect.

Now, Alice connects the amplifier input to the signal generator through a coaxial cable ($Z_0 = 50\ \Omega$; $l = 20$ cm; $v = 2c/3 = 2 \times 10^8$ m/s). She does this somehow magically that the coaxial cable sees the gate of the MOS transistor directly with no parasitic elements in between (and we will pretend as if the MOS transistor had no gate parasitic capacitance). Alice then connects the amplifier output to Oscilloscope 1 through a very short, and hence, negligible, coaxial cable (therefore, not explicitly shown in the figure). Then she finds out that the RF generator's display panel is partly broken and the available power of the generated signal cannot be figured out. To circumvent this situation, Alice borrows another identical oscilloscope (Oscilloscope 2) from Tom, and connects it to the RF generator using another identical coaxial cable so that she can monitor the RF generator's signal in Oscilloscope 2.

Alice decides not to touch the power control knob of the RF generator, and hence, although unknown yet to Alice, the available power of the generated signal is maintained at a constant value. She now measures $O_1(f)$, the amplitude of the voltage sinusoid at Oscilloscope 1 (amplifier output) as a function of frequency, f, and measures $O_2(f)$, the amplitude of the voltage sinusoid at Oscilloscope 2 as a function of frequency, f, sweeping the frequency of the RF generator. She expects $O_2(f)$ to be constant or independent of f, and $O_1(f)$ to exhibit a standard 1-pole characteristic, but to her surprise, $O_2(f)$ periodically varies with f, and $O_1(f)$ exhibits a periodic modulation on top of a simple 1-pole characteristic.

(a) In Alice's experiment, calculate and plot $O_1(f)$ and $O_2(f)$. Assume that the available power of the RF generator (unknown to Alice) is -20 dBm (RMS), and assume that $g_m R = 10$ and $1/(2\pi RC) = 1$ GHz for the MOS amplifier. Can you provide the physical reason for the periodic variations/modulations of $O_2(f)$ and $O_1(f)$? Is $O_1(f)/O_2(f) = A(f)$ true in this case, where $A(f)$ is the voltage gain of the amplifier?

(b) Tom, helping Alice in her experiment, suggests to monitor the RF generator's signal at the amplifier input, and rearranges the measurement setup as shown in Fig. 1(b). Calculate and plot $O_1(f)$ and $O_2(f)$, and provide physical explanations on their behaviors. Is $O_1(f)/O_2(f)$ the same as $A(f)$ in this case?

Problem 2 (40pt)

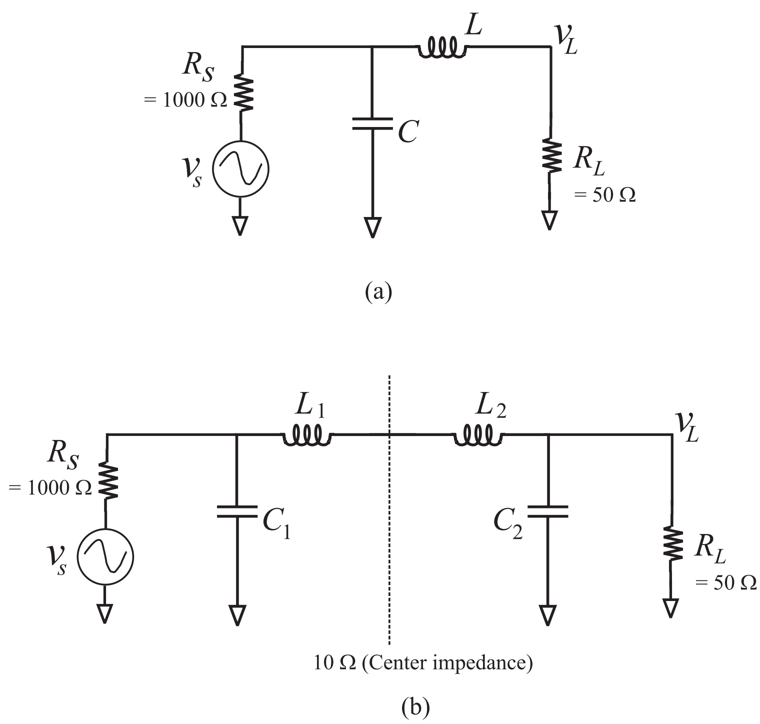

Figure 2: Impedance matching

Figures 2(a) and (b) show an L- and π- network, respectively, to impedance match the source resistance $R_s = 1000\ \Omega$ to the load resistance $R_L = 50\ \Omega$ at 5 GHz. Calculate C and L for the L-network and C_1, L_1, L_2, and C_2 for the π-network. For the π-network, use 10 Ω for the center impedance. Plot, for both cases, the normalized power at the load versus frequency from 1 to 10 GHz, while the power normalization is with respect to the available power of the source. You should be able to see that (1) the normalized power at the load becomes maximum at 5 GHz (with the maximum value of 1) in both cases, and (2) the π-network has a narrower passband than the L-network.

Problem 3 (40pt)

A lossy transmission line can be modeled using an $LRCG$ ladder network, and the propagation constant, $\gamma = \alpha + j\beta$, and the characteristic impedance, $Z_{0,lossy}$, of the line are given by

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{1}$$

$$Z_{0,lossy} = \sqrt{\frac{j\omega L + R}{j\omega C + G}} \tag{2}$$

where L, R, C, and G signify inductance, series resistance, capacitance, and shunt conductance per unit length, respectively (Supplemental Material # 1). The goal of this problem is to calculate the quality factor, Q, of the lossy line. We will assume that the losses, R and G, are small enough to satisfy $|j\omega L| \gg R$ and $|j\omega C| \gg G$; we will neglect second- and higher-order terms of $R/|j\omega L|$ and $G/|j\omega C|$ as well as their cross-product terms (*small-loss approximation*).

(a) Show that the propagation constant of the lossy transmission line can be approximated as

$$\gamma = \alpha + j\beta \approx \frac{1}{2v_0}(\frac{R}{L} + \frac{G}{C}) + j\frac{\omega}{v_0} \tag{3}$$

in the small-loss approximation, where $v_0 \equiv 1/\sqrt{LC}$ is the phase velocity in the lossless case.

(b) Show that the characteristic impedance of the lossy transmission line can be approximated as

$$Z_{0,lossy} \approx Z_0 e^{j\phi_0} \tag{4}$$

in the small-loss approximation, where $Z_0 \equiv \sqrt{L/C}$ is the characteristic impedance in the lossless case, and ϕ_0 is approximately given by $\phi_0 \approx 1/(2\omega) \cdot (G/C - R/L)$.

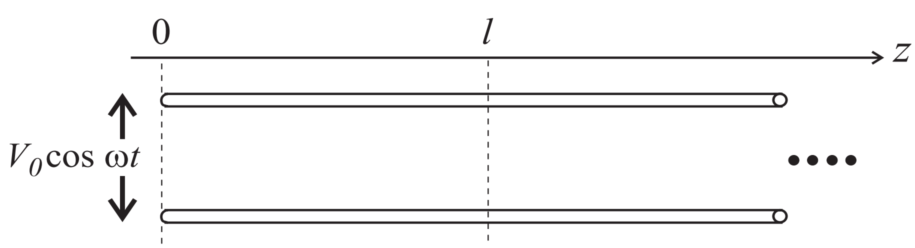

Figure 3: Q of a lossy transmission line.

(c) As shown in Fig. 3, an infinitely long, lossy transmission line is being driven by a sinusoidal source at $z = 0$ and as a result, the voltage in the transmission line at $z = 0$ is given by $V_0 \cos(\omega t)$. Using the phasor, $e^{-\gamma z + j\omega t}$, and relevant boundary conditions, show that the voltage and current at point z in time t are given by

$$V(z,t) = V_0 e^{-\alpha z} \cos(\beta z - \omega t) \tag{5}$$

$$I(z,t) \approx \frac{V_0}{Z_0} e^{-\alpha z} \cos(\beta z - \omega t + \phi_0) \tag{6}$$

Using these results, demonstrate that the total time-averaged energy stored in the transmission line segment between $z = 0$ and $z = l$ is given by

$$E_{tot} = \frac{1}{2} C V_0^2 \cdot \frac{1 - e^{-2\alpha l}}{2\alpha} \tag{7}$$

Also show that the total time-averaged power dissipation in the same line segment is given by

$$P_{diss} = \frac{1}{2} (\frac{R}{Z_0^2} + G) V_0^2 \cdot \frac{1 - e^{-2\alpha l}}{2\alpha} \tag{8}$$

(d) Using the results, (3), (7), and (8), show that the Q of the lossy transmission line is given by

$$Q = \frac{\omega L C}{RC + GL} = \frac{\beta}{2\alpha} \tag{9}$$

This is an important formula that is frequently used to characterize lossy transmission lines.

Problem 4 (40pt)

(a) An amplifier with 20 dB of power gain has a third-order intercept of 30 dBm at the *output*. If the input consists of a 0 dBm signal at 1 GHz and another 0 dBm signal at 1.05 GHz, what will be the output power of the third-order products at 1.1 GHz and 0.95 GHz?

(b) The same as Problem 4(a) except that input signal at 1 GHz increases in power to 10 dBm while the input signal at 1.05 GHz remains at 0 dBm.