ES 272 Assignment #3

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(1*) Spiral inductor (30pt)

In silicon integrated circuits, inductors are constructed by shaping metals into a planar spiral geometry. Figure 1(a) shows the top view of an example planar spiral inductor, realized using the top metal layer while the metal layer below the top metal layer is used for an interconnection for terminal 2. Figure 1(b) shows an equivalent electrical circuit model for the spiral inductor, obtained using an electromagnetic simulation. This is a narrow band model which is valid between 2.2 and 2.8 GHz. L, R_s, R_p, and C_p represent inductance, metal loss due to the skin effect, substrate loss, and metal-substrate capacitance, respectively. C_s accounts for the overlap capacitance between the top metal and the metal below. Using Cadence, measure the quality factor, Q, of the spiral inductor (See the note below.) over the frequency range between 2.2 and 2.8 GHz.

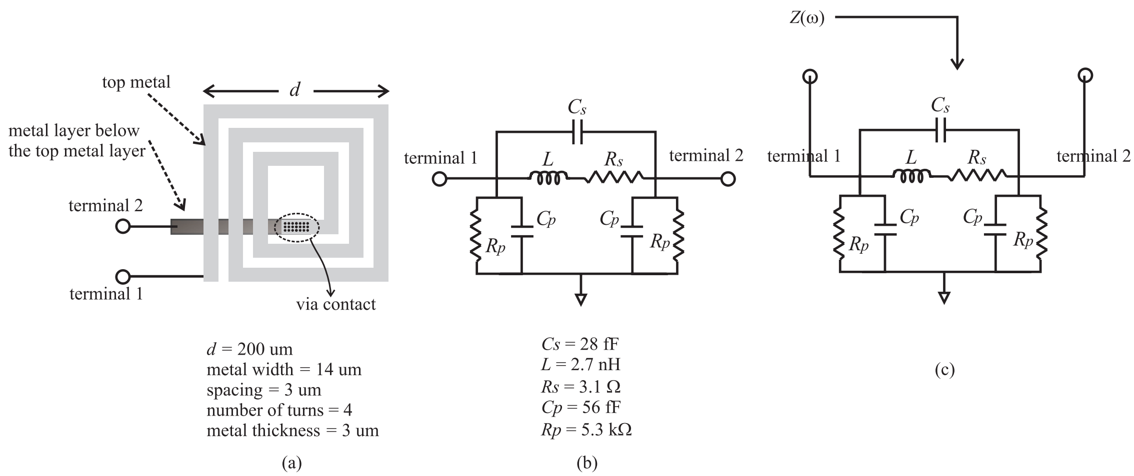

Figure 1: (a) Spiral inductor. (b) Circuit model. (c) Input impedance $Z(\omega)$.

(*Note*) As discussed in class, the quality factor, Q, is originally defined for a "resonator" as

$$Q \equiv \omega_0 \frac{\text{Stored energy}}{\text{Power dissipation}}\Big|_{\omega=\omega_0} \tag{1}$$

where ω_0 is the resonance frequency. For a resonator whose resonance frequency is fixed, Q is *not* a function of frequency. Since inductors are not resonators, the original Q definition above cannot be used for inductors. However, the following frequency-dependent Q definition may be used instead as the quality factor for inductors:

$$Q(\omega) \equiv \omega \frac{\text{Stored energy }(\omega)}{\text{Power dissipation }(\omega)} \tag{2}$$

It can be easily shown that $Q(\omega)$ in (2) is on the same order as, but not exactly the same as,

$$Q(\omega) \equiv \frac{Im\{Z(\omega)\}}{Re\{Z(\omega)\}} \tag{3}$$

where $Z(\omega)$ is the frequency-dependent input impedance of a given inductor. RF engineers traditionally choose to use (3) over (2) due to the simplicity of (3). In the problem above, you can evalulate the Q of the spiral inductor using (3) while the input impedance $Z(\omega)$ shown in Fig. 1(c) can be simulated in Cadence.

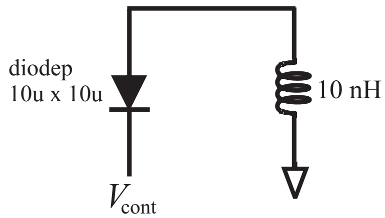

Figure 2: Tunable parallel LC tank

(2*) Tunable parallel LC tank (30pt)

Figure 2 shows a reverse-biased *pn* junction diode ($V_{cont} \geq 0$) in parallel with an ideal, lossless inductor, whose inductance is 10 nH. The reverse-biased diode can be modeled as a capacitor in parallel with a resistor, where the resistor accounts for parasitic loss of the junction. The capacitance of the *pn* diode varies with the voltage across the junction, and hence the circuit of Fig. 2 forms a tunable parallel LC tank where the frequency tuning is achieved by varying the control voltage, V_{cont}. Using *diodep* in the *tsmc18rf* library for the *pn* junction (dimension: 10 μm $\times$ 10 μm) and resorting to Cadence *ac* analyses, find the resonance frequency and quality factor, Q, of the LC tank when $V_{cont} = 0$ volt and $V_{cont} = 1$ volt. While this *ac* analysis is valid only for small signals due to the high nonlinearity present in the diode, the analysis conveys a general idea about how the tunable LC tank can be modeled.

(3*) Narrowband RF tuned-tank amplifier design (60pt)

Figure 3(a) illustrates a narrow-band RF amplifier. The MOS device is the *nmos2v* from the *tsmc18rf* library, and its size is L=0.18 μm and W=40 μm while the number of fingers is 20. The gate *dc* bias is 1.2 V.

(a) Calculate the resonance frequency and the quality factor of the LRC tank in the load.

(b) Using Cadence, find the transconductance, g_m, of the MOS transistor.

(c) Ignoring all the MOS device parasitics, calculate the small-signal voltage gain at the resonance frequency calculated in (a).

(d) Using Cadence, plot the small-signal voltage gain versus frequency. What are the resonance frequency and the corresponding gain at the resonance frequency? Why discrepancy between (a)/(c) and (d) (one sentence will do)?

(e) Using Cadence, find the input 1-dB compression point (in terms of voltage) at the resonance frequency of (d). Also find the IIP3 point (in terms of voltage) using two harmonic tones of your choice around the resonance frequency.

(f) In class, we discussed that the capacitive coupling between the gate and the drain of the transistor can cause parasitic (unintentional) oscillations, which we will simulate now. Figure 3(b) shows the same amplifier as Fig. 3(a), but now with inclusion of the 2-nH interconnect inductance which models a 2-mm long gold bonding wire.[1] The interconnect inductance is connected to the MOS gate at one end, and to the *dc* 1.2 V bias at the other end. Apart from the *dc* bias, there is no input (stimulus) to the system.

Using the Cadence transient analysis for various values of W between 40 μm and 200 μm, see if oscillation builds up and sustains itself. To this end, you can measure the drain voltage over time. You should be able to see self-sustained oscillation with W in excess of a certain critical value. This is because the increased gate-drain capacitive coupling with the increased $C_{gd} \propto W$ results in a larger negative resistance (magnitude-wise) seen at the amplifier input as discussed in class.

Note: Fig. 3(b) explicitly shows a current-pulse component called *ipulse* from the *analogLib* library. As you already know from the oscillator simulation problems, this current pulse is to start up an oscillation at the initial stage.

(g) Repeat the simulation in (f), now with a fixed W of 40 μm but with an external feedback capacitance, C_{feed}, between the gate and the drain (Fig. 3(c)), for its various values between 10 fF and 100 fF. C_{feed} was added to see the gate-drain coupling effect more explicitly. You should be able to observe parasitic oscillations with C_{feed} in excess of a certain critical value.

[1] A typical 1-mil-thick gold bonding wire introduces a 1 nH inductance every mm.

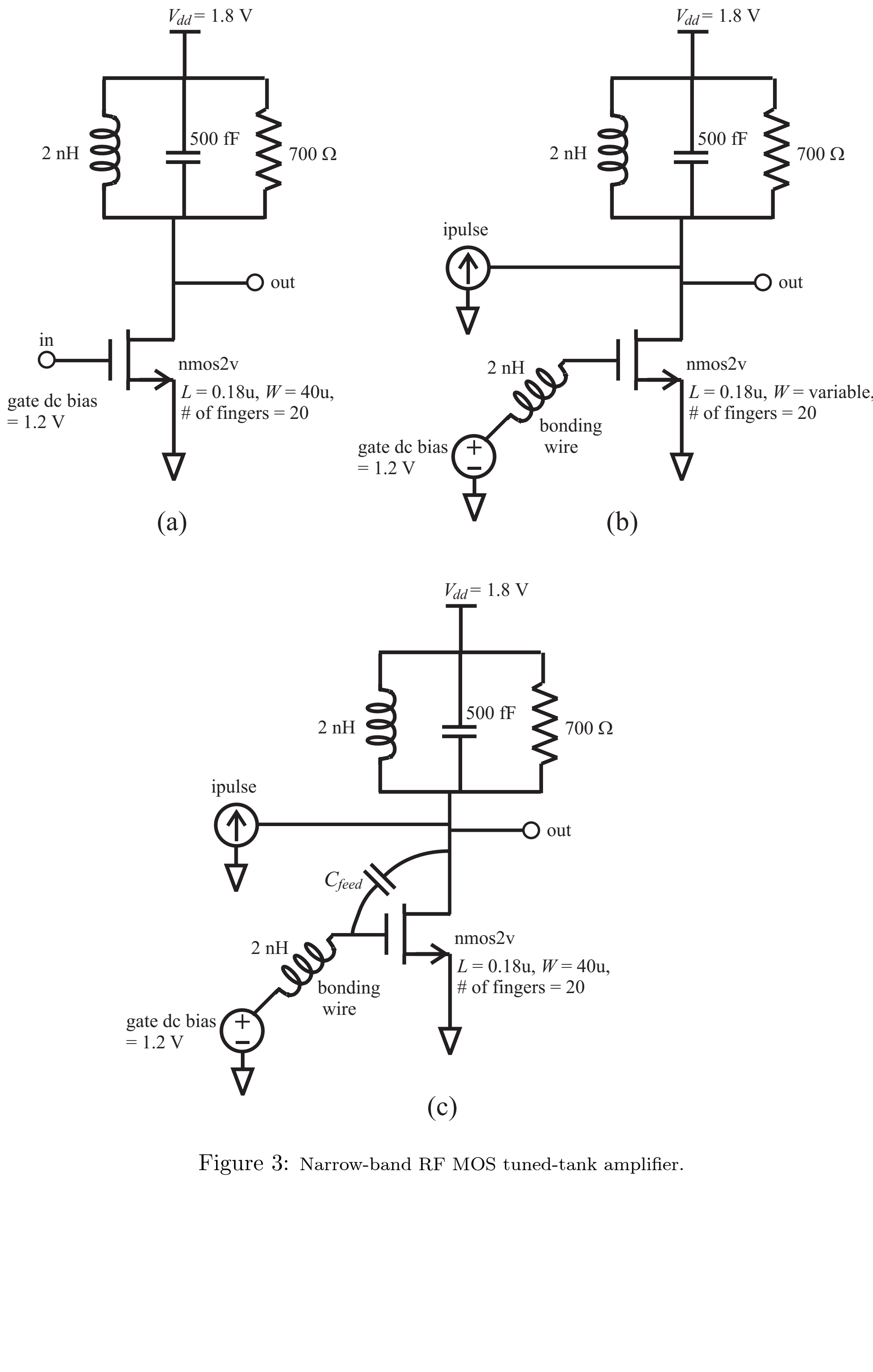

Figure 3: Narrow-band RF MOS tuned-tank amplifier.

(4) The kT/C-Noise (30pt)

As we discussed in class, in a simple RC circuit, the capacitor stores a mean squared noise voltage of kT/C when the resistor R generates the $4kTR$ Nyquist noise. This so-called "kT/C-noise" is indeed expected from the *equipartition theorem* of thermodynamics, which states that each degree of freedom of a system in thermal equilibrium has a mean thermal energy of $kT/2$. Since the capacitor's mean energy, $\langle Cv_C^2/2 \rangle$ (v_C: voltage across the capacitor), should be equal to $kT/2$ according to the equipartition theorem, we can easily see $\langle v_C^2 \rangle = kT/C$.

Now let us consider a little more complicated network shown in Fig. 4. The ambient temperature is T. The lossy elements, R_1, R_2, and R_3 in the network, generate Nyquist noise, which are shown as current sources in the figure. Thanks to the equipartition theorem, we know that the mean squared noise voltage across the capacitor must be again kT/C, independent of the values of L, R_1, R_2, and R_3. Derive the kT/C-noise without appealing to the equipartition theorem, but by integrating the power spectral density of the voltage noise across the capacitor.

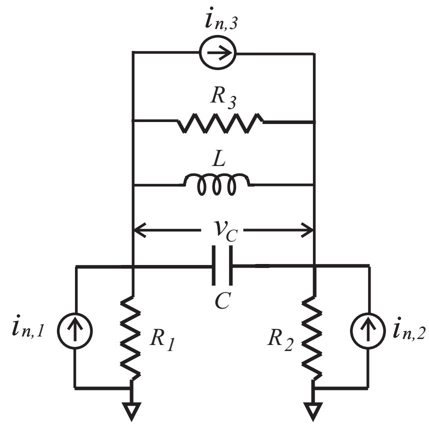

Figure 4: kT/C noise.

[Suggested Reading on Noise]

[1] H. Nyquist, *Phys. Rev.*, **32**, 110, 1928. — Nyquist's original paper explaining the $4kTR$ noise using 1-D blackbody radiation calculation.

[2] B. Oliver, "Thermal and quantum noise," *IEEE Proceedings*, May 1965. — Barney Oliver was a brilliant man who headed the HP research as well as the SETI for many years. This is a great article on noise.

[3] N. Wax, *Selected Papers on Noise and Stochastic Processes*, Dover Publication, 1954. — Collection of classic papers on random processes and noise.

[4] A. Einstein, *Investigation on the Theory of the Brownian Motion*, Dover Publication, 1956. — English translation of the Einstein's seminal 1905 paper on Brownian motion.

[5] A. Van der Ziel, *Noise in Solid-State Devices and Circuits*, John Wiley & Sons, 1986. — Van der Ziel explains how to obtain the 2/3 γ factor in the channel thermal noise spectral density for long channel MOSFET's.