

ES 272 Assignment #6

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(Problem 1) LNA and noise matching (60pt)

Protons act like tiny bar magnets and produce magnetic fields. In magnetic resonance imaging (MRI), these “proton magnets” in a human body (mostly from water molecules in our body) rotate at a certain radio frequency, and the corresponding rotating magnetic fields from the proton magnets induce an RF voltage signal (electromotive force) across a solenoidal coil surrounding the human body. Since the RF voltage signal is very weak, a low-noise amplifier (LNA) is used to enhance the signal. In this problem, we consider how to interface the coil with the LNA in order to minimize the LNA’s noise figure. The intention of this problem is to deepen your understanding of the noise figure, and MRI offers a nice context to that end.

The coil and induced RF voltage signal may be modeled as in Fig. 1(a), which includes: coil inductance L ; the induced RF voltage signal, whose *rms* value is V_{rms} ; parasitic coil resistance R ; thermal noise, $\overline{v_n^2}/\Delta f = 4kTR$, generated by R . You may think of the coil as an antenna like in a radio, but with a substantial inductance L contained in it. For numerical calculations in certain parts of this problem, use $L = 500$ nH, $R = 4.2$ Ω , and $f_0 = \omega_0/(2\pi) = 21$ MHz (signal frequency). The quality factor of the inductor is $Q = \omega_0 L/R \approx 16$. Assume that the signal bandwidth is 1.1 kHz (this non-zero signal bandwidth arises because the induced RF voltage signal is a very, very slowly damped sinusoid).

For the LNA shown in Fig. 1(b), assume that it has been already designed and has the following fixed properties: infinite input impedance, $Z_{LNA} = \infty$; input-referred voltage noise of $(\overline{v_i^2}/\Delta f)^{1/2} = 1.3$ nV/ $\sqrt{\text{Hz}}$; zero input-referred current noise (this zero current noise is a consequence of $Z_{LNA} = \infty$. Do you understand this statement?).

(a) We begin by directly connecting the LNA and the coil as in Fig. 1(c). Show that the LNA’s noise figure is given by

$$NF = 10 \cdot \log\left[\frac{\overline{v_n^2} + \overline{v_i^2}}{\overline{v_n^2}}\right] \quad (1)$$

where $\overline{v_n^2}$ is the coil noise over the entire signal bandwidth (1.1 kHz), *i.e.*, $\overline{v_n^2} = \overline{v_n^2}/\Delta f \times 1.1$ kHz, and $\overline{v_i^2}$ is the LNA’s input-referred voltage noise over the entire signal bandwidth, *i.e.*, $\overline{v_i^2} = \overline{v_i^2}/\Delta f \times 1.1$ kHz. Calculate the numerical value of the noise figure in dB. The noise figure should be very high. Can you explain this, without resorting to (1), but by arguing how the signal-to-noise ratio gets deteriorated through the LNA?

(b) To reduce the noise figure, we can insert a passive network between the coil and the LNA as shown in Fig. 1(d). The passive network won’t amplify its input power, but it can amplify its input voltage: both the signal V_{rms} and noise $\overline{v_n^2}$ of the coil are rescaled, or, passively amplified, by the network by a certain gain factor, $\alpha > 0$, whose value depends on a specific arrangement of the passive network (α is the magnitude of the transfer function of the passive network at the design frequency). By considering how the signal-to-noise ratio changes through the circuit, argue that the noise figure will be minimized when α is maximized. Quantitatively, show that the noise figure of the LNA is given by

$$NF = 10 \cdot \log\left[\frac{\alpha^2 \overline{v_n^2} + \overline{v_i^2}}{\alpha^2 \overline{v_n^2}}\right] \quad (2)$$

This equation confirms that a maximum α corresponds to a minimum noise figure.

(c) Now we go further and design an optimum passive network that yields a maximum α . Various topologies may be considered, and here, we try the C_1 - C_2 network shown in Fig. 1(e). Analytically express C_1

and C_2 that maximize α , in terms of given parameters. The expressions will be simplified if you use Q of the inductor as one parameter. Analytically express the corresponding maximum α . What physical situation does your optimum solution correspond to? What is the corresponding minimum noise figure, numerically (in dB), and how does it compare to part (a)? In this optimal case, what is the input impedance Z_c (analytically) shown in Fig. 1(e)? Is Z_c matched to Z_{LNA} ?

(d) Now imagine a situation where we cannot avoid using a transmission line (with fixed length l , characteristic impedance Z_0 , wave propagation velocity of v , and propagation constant β) in connecting the LNA and the coil, as shown in Fig. 1(f). This is a frequent scenario where the coil and the LNA cannot be placed in physical proximity. In such a case, the optimum passive network that minimizes the noise figure should include the transmission line. We also include a capacitor C_1 in the passive network, which is deliberately added in parallel to the coil [Fig. 1(f)]. Analytically express the optimum value of C_1 that maximizes α , in terms of the given parameters, and also, analytically express the corresponding maximum α . Assume that $\beta l \ll 1$ and use a first-order approximation in your calculation, and assume a negligible loss in the line. What is the minimum noise figure (numerically, in dB) corresponding to the maximum α ?

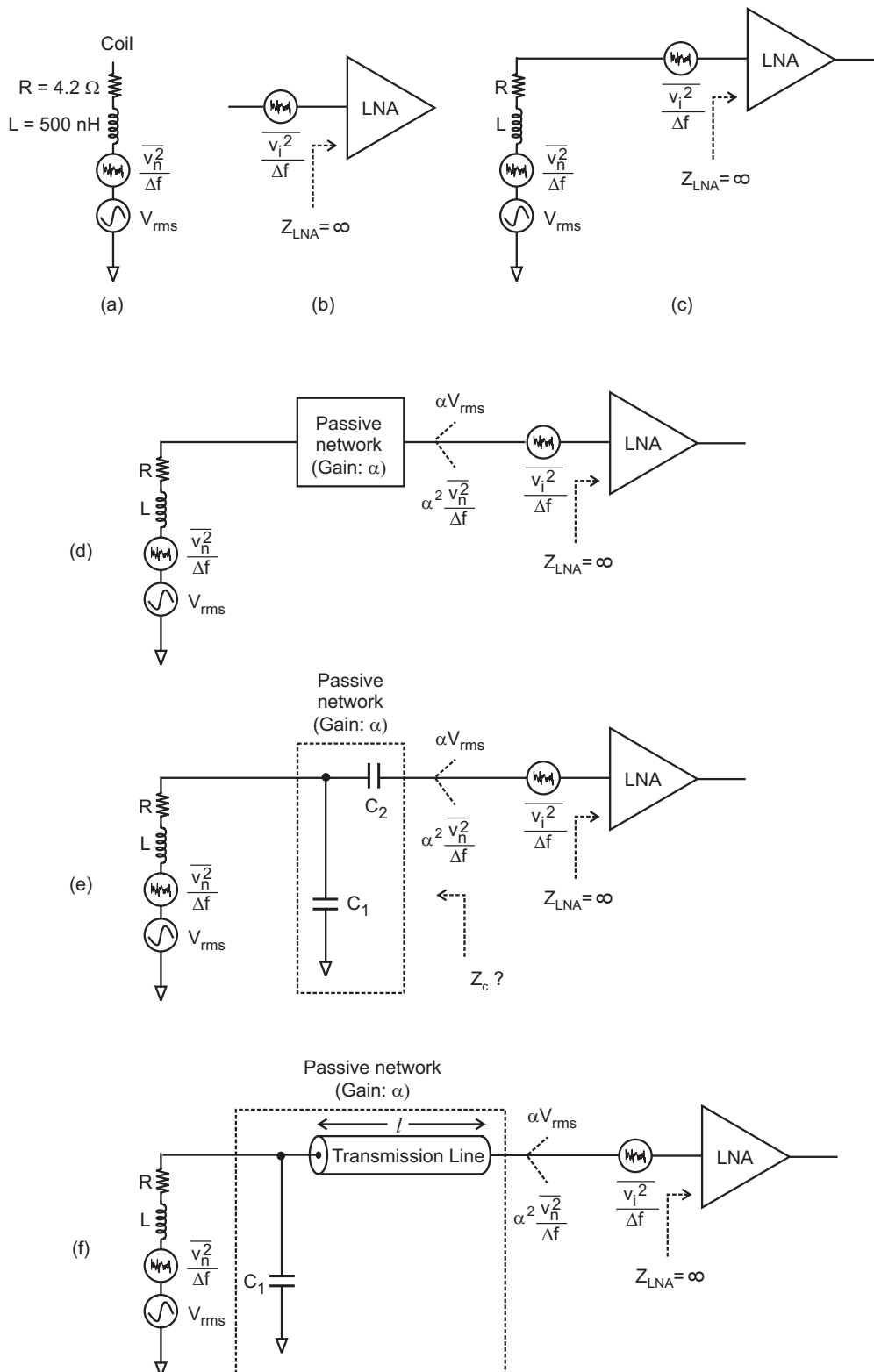


Figure 1: LNA noise matching.

(Problem 3) Diode Ring Mixer (30pt)

The diode ring circuit of Fig. 3 is a popular mixer topology. In this problem we analyze the diode ring mixer assuming that the RF signal, $V_1(t) = a \cos(\omega_{RF}t + \phi)$, is very small as compared to the local oscillator signal, $V_2(t) = b \cos(\omega_{LO}t)$, that is, $a \ll b$ ($a > 0$ and $b > 0$).

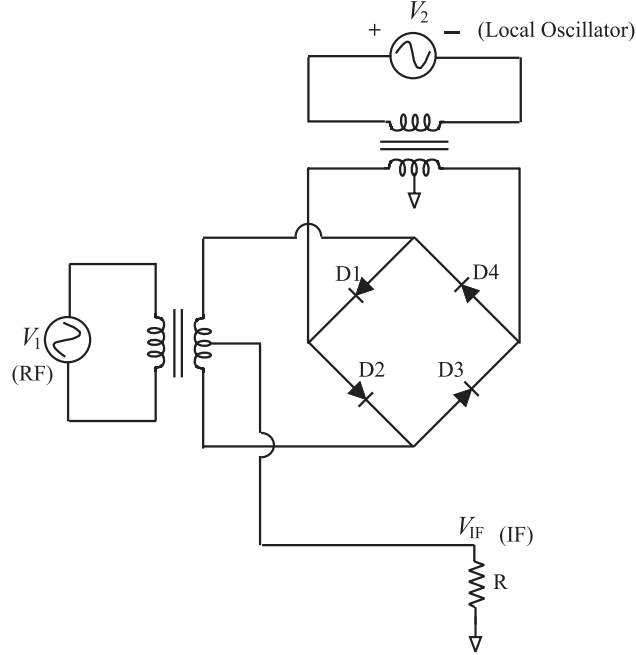


Figure 3: Diode ring mixer

Since $a \ll b$, when the oscillator output (V_2) is positive, diodes D2 and D3 conduct while the other two diodes are off. The situation turns the other way around when the oscillator output reverses its polarity. Overall diodes D2 and D3 conduct for positive half cycles of the oscillator while diodes D1 and D4 conduct for negative half cycles. Assuming that the diode resistance under the forward-biased condition is r_d and both of the transformers have 1:1 turn ratio, show that

$$V_{IF}(t) = V_1(t) \cdot \frac{R}{2R + r_d} \quad (3)$$

for a positive half cycle of V_2 , and

$$V_{IF}(t) = -V_1(t) \cdot \frac{R}{2R + r_d} \quad (4)$$

for a negative half-cycle of V_2 . Based on (3) and (4), explain how frequency downconversion is achieved in the diode ring mixer, and calculate the mixer's voltage conversion gain (ratio of the IF voltage amplitude to the RF voltage amplitude).