

QR48 Hour Exam
Friday, March 10, 2006

You may use notes, books, calculators, or any other non-human aids. Your use of the Internet is limited to Google calculator. Use the base-two versions of kilo-, mega-, giga-, etc. You have 50 minutes to take the exam, and the exam totals 50 points.

1. (a) (4 points) Find a value of x satisfying $7^x = 3 \pmod{5}$.
(b) (4 points) Can you calculate 7^{1000} in much fewer than 1000 multiplications? About how many multiplications does it take?
(c) (4 points) Professor Abelson has a 7 kilobyte file that he needs to transmit using his 1200 bits per second modem. How long will that take?
(d) (4 points) A television show downloaded from iTunes is 212,510 kilobytes. Assuming that is the typical size for an episode of a television program, how many television shows would you be able to fit on your 60 gigabyte iPod?
2. The Harvard Office of Student Housing keeps records and statistics on where each Harvard undergraduate lives. Each can be characterized as living in one of five categories: the river houses (56%), the Quad houses (19%), the Yard dormitories (17%), the Union dormitories (6%), and off-campus housing (2%).
(a) (2 points) How many bits would be required to encode this information using a fixed-length encoding scheme?
(b) (5 points) Design a Huffman code for the housing information.
(c) (3 points) Compare the average code length for the two codes. How much better or worse is the Huffman code, or is it the same?
3. Jell-O makes 18 different varieties of flavored gelatin. Although each variety is supposed to have a distinct flavor, most people tend to differentiate Jell-O by color rather than by flavor. For example, one would say, "I had red Jell-O with my lunch today," not, "I had Strawberry-Kiwi Jell-O." Of the 18 flavors, 1 is blue, 1 is green, 1 is purple, 2 are yellow, 3 are orange, and 10 are red. Assume that all flavors occur with equal frequency.
(a) (4 points) How much information is contained in the fact that one ate red Jell-O? What about yellow Jell-O?
(b) (4 points) What is the entropy of Jell-O colors?
4. (a) (2 points) What is Moore's law?
(b) (4 points) Sue encrypts all her photographs using an encryption algorithm that would take a typical desktop computer about 1 million years to crack. Assume that the performance of desktop computers scales with Moore's law. How long would it take Sue's granddaughter to decrypt Sue's photographs using brute force 50 years from now?
(c) (3 points) If Sue wants to make her photos more secure so that even 50 years from now it will take 1 million years to crack her encryption using brute force, how many bits does she need to add to her encryption key?
5. (7 points) Suppose that you intercepted ten messages encoded using the same one-time pad. Which specific steps would you take to break the code? Please limit your answer to one page.