

# ES 275 Nanophotonics

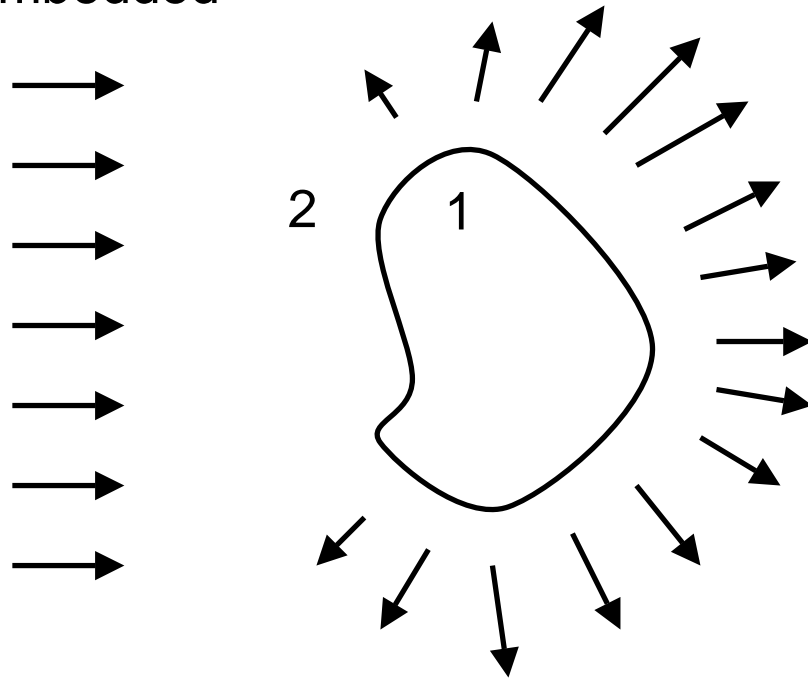
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## Lecture 17

Scattering, Extinction, Absorption  
Sphere in the Electrostatics Approximation – Revisited

# Scattering, Extinction and Absorption

- The Problem: given a particle of specified size, shape and optical properties, determine the electromagnetic field at all points in the particle and at all points of the homogeneous medium in which the particle is embedded



- Field inside particle is  $(\mathbf{E}_1, \mathbf{H}_1)$
- Field in medium surrounding particle is  $(\mathbf{E}_2, \mathbf{H}_2)$
- $(\mathbf{E}_2, \mathbf{H}_2)$  is the superposition of the incident field  $(\mathbf{E}_i, \mathbf{H}_i)$  and the scattered field  $(\mathbf{E}_s, \mathbf{H}_s)$ :

$$\mathbf{E}_2 = \mathbf{E}_i + \mathbf{E}_s \quad (17.1)$$

$$\mathbf{H}_2 = \mathbf{H}_i + \mathbf{H}_s \quad (17.2)$$

# Scattering, Extinction and Absorption

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- The time-averaged Poynting vector  $\mathbf{S}$  at any point in medium surrounding a particle is given by:

$$\mathbf{S} = \frac{1}{2} \text{Re}\{\mathbf{E}_2 \times \mathbf{H}_2^*\} \quad (17.3)$$

- It may be written as the sum of 3 terms:

$$\mathbf{S} = \mathbf{S}_i + \mathbf{S}_s + \mathbf{S}_{ext} \quad (17.4)$$

where

$$\mathbf{S}_i = \frac{1}{2} \text{Re}\{\mathbf{E}_i \times \mathbf{H}_i^*\} \quad (17.5) \quad \text{Poynting vector associated with the incident field}$$

$$\mathbf{S}_s = \frac{1}{2} \text{Re}\{\mathbf{E}_s \times \mathbf{H}_s^*\} \quad (17.6) \quad \text{Poynting vector associated with the scattered field}$$

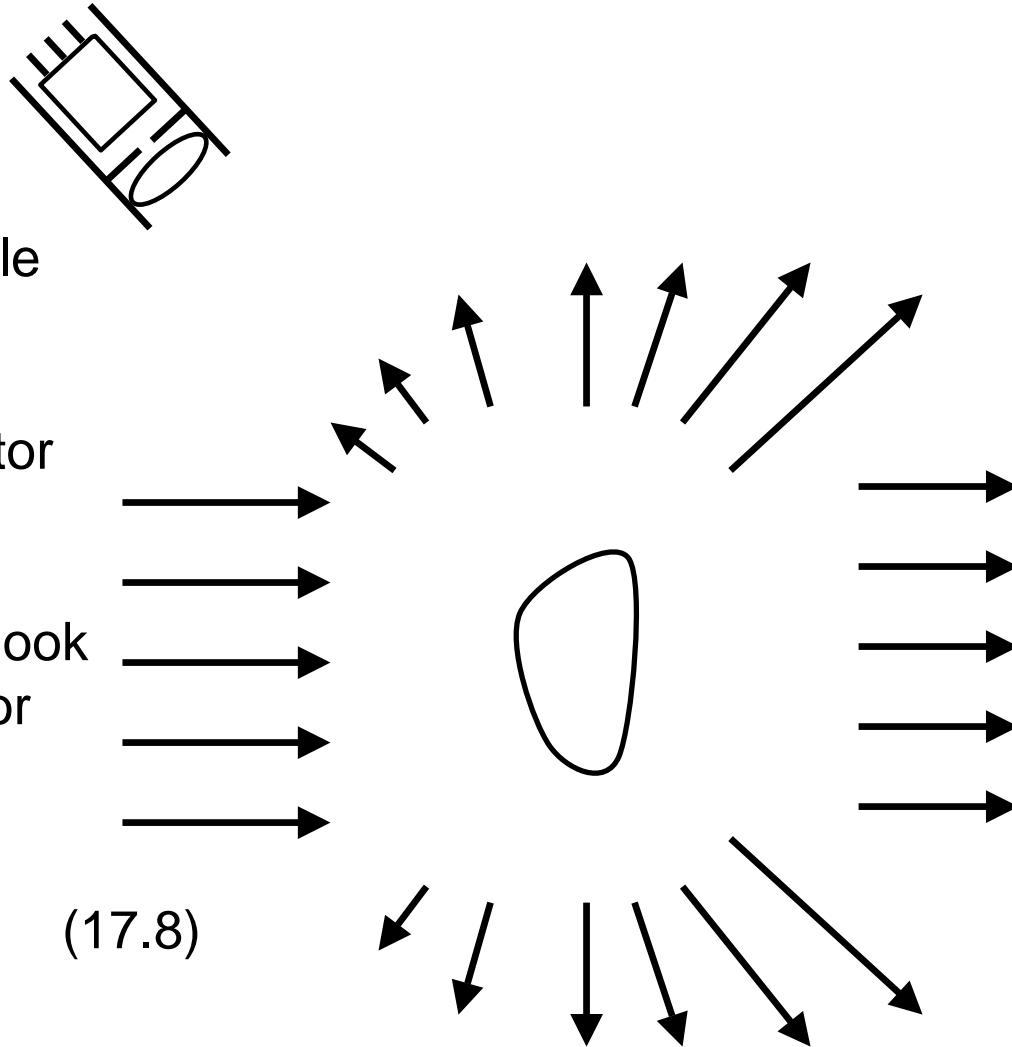
$$\mathbf{S}_{ext} = \frac{1}{2} \text{Re}\{\mathbf{E}_i \times \mathbf{H}_s^* + \mathbf{E}_s \times \mathbf{H}_i^*\} \quad (17.7)$$

# Scattering, Extinction and Absorption

## Detection of Scattered Light with Collimated Detector

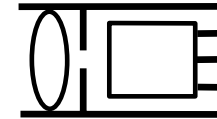
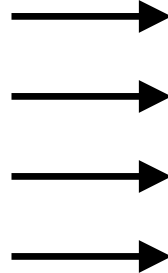
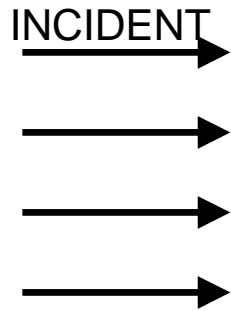
- Detector is placed a distance  $r$  from particle in far-field region
- Surface  $\Delta A$  of detector aligned normal to  $\hat{e}_r$
- If detector does not look at source, the detector power is:

$$P = S_s \cdot \hat{e}_r \Delta A \quad (17.8)$$



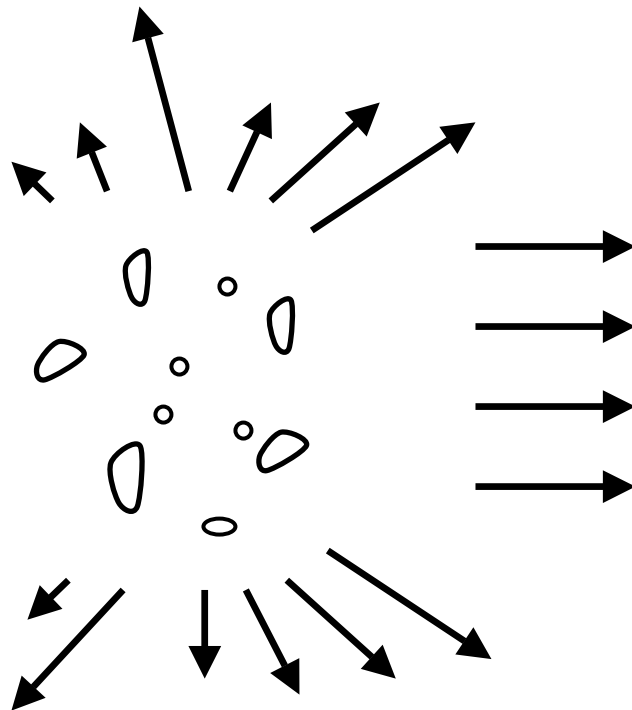
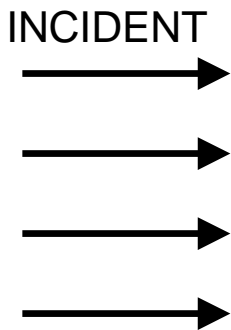
# Scattering, Extinction and Absorption

INCIDENT

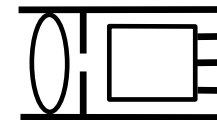


Power at detector:  
 $P = U_0$

INCIDENT



SCATTERED

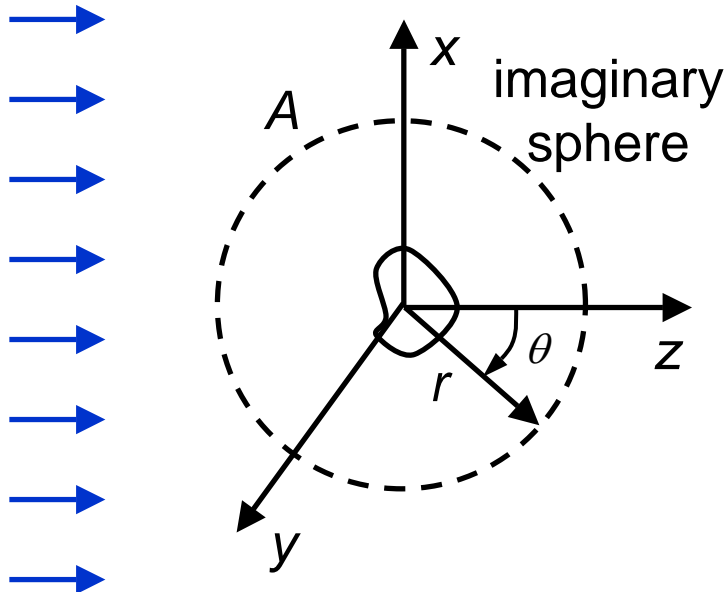


$P = U < U_0$

Difference  $(U - U_0)$  is due  
to scattering & absorption

# Extinction, Scattering and Absorption

Single Particle Embedded  
in a Nonabsorbing Medium  
Illuminated by Plane Wave



- Construct imaginary sphere of radius  $r$  around particle
- Net rate of electromagnetic energy crossing surface  $A$  of the sphere:

$$W_a = - \int_A \mathbf{S} \cdot \hat{\mathbf{e}}_r dA \quad (17.9)$$

If  $W_a > 0$

then energy is absorbed within the sphere

# Extinction, Scattering and Absorption

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•If the medium is nonabsorbing, then  $W_a$  is the rate at which energy is absorbed by the particle

•Recall from before: 
$$\mathbf{S} = \mathbf{S}_i + \mathbf{S}_s + \mathbf{S}_{ext} \quad (17.4)$$

•We can write:

$$W_i = - \int_A \mathbf{S}_i \cdot \hat{\mathbf{e}}_r dA \quad (17.10)$$

$$W_s = \int_A \mathbf{S}_s \cdot \hat{\mathbf{e}}_r dA \quad (17.11)$$

$$W_{ext} = - \int_A \mathbf{S}_{ext} \cdot \hat{\mathbf{e}}_r dA \quad (17.12)$$

•Then from (17.4) we have: 
$$W_a = W_i - W_s + W_{ext} \quad (17.13)$$

# Extinction, Scattering and Absorption

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- For a nonabsorbing medium, we have:

$$W_i = 0 \quad (17.14)$$

- From (17.13) we therefore have:

$$W_{ext} = W_a + W_s \quad (17.15)$$

- Now,  $W_s$  is the rate at which energy is scattered across surface  $A$
- So (17.15) says that  $W_{ext}$  is the sum of the energy absorption and the energy scattering rate

# Extinction, Scattering and Absorption

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- If  $I_i$  is the incident irradiance, then the ratio of  $W_{ext}$  to  $I_i$  is a quantity with the dimensions of area:

$$C_{ext} = \frac{W_{ext}}{I_i} \quad (17.16)$$

- This is known as the extinction cross section
- From (17.15) it follows that it may be written as the sum of the absorption cross section  $C_{abs}$  and the scattering cross section  $C_{sca}$ :

$$C_{ext} = C_{abs} + C_{sca} \quad (17.17)$$

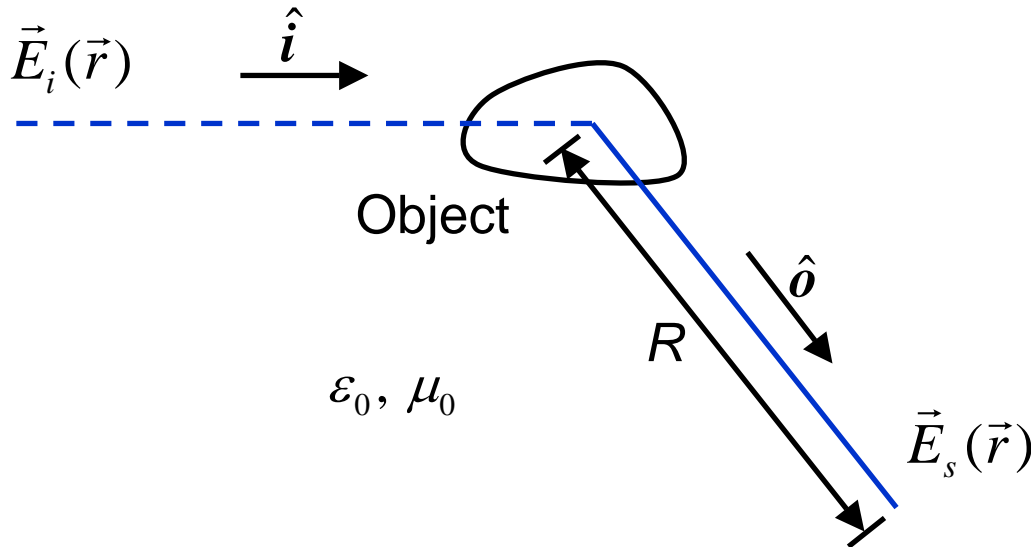
where

$$C_{abs} = \frac{W_{abs}}{I_i} \quad \& \quad C_{sca} = \frac{W_{sca}}{I_i} \quad (17.18a) \quad (17.18b)$$

# Scattering Amplitude

- Linearly polarized plane wave in medium with dielectric constant  $\epsilon_0, \mu_0$ :

$$\vec{E}_i(\vec{r}) = \hat{e}_i \exp(-j k \hat{i} \cdot \vec{r}) \quad (17.19)$$



where:

amplitude  $|E_i| = 1$  V/m,

$$k = \omega \sqrt{\mu_0 \epsilon_0} = 2\pi / \lambda$$

= wave number

$\hat{i}$  = unit vector in direction  
of wave propagation

$\hat{e}_i$  = unit vector in direction  
of polarization

# Scattering Amplitude

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- Total field  $\vec{E}$  at a distance  $R$  from a reference point on the object, in the direction of a unit vector  $\hat{\mathbf{O}}$ , consists of the incident field  $\vec{E}_i$  and the field  $\vec{E}_s$  scattered by the object
- When  $R < D^2 / \lambda$ , where  $D$  is a typical dimension of the object
  - scattered field has complicated amplitude and phase variations because of interference between contributions from different parts of the object
  - observation point  $\vec{r}$  is said to be in the near-field of the object

# Scattering Amplitude

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- When  $R > D^2 / \lambda$  , where  $D$  is a typical dimension of the object
- observation point  $\vec{r}$  is said to be in the far-field of the object
- scattered field  $\vec{E}_s$  behaves as a spherical wave:

$$\vec{E}_s(\vec{r}) = \vec{f}(\hat{o}, \hat{i}) \frac{\exp(-jkR)}{R} \quad (17.20)$$

where

$$\vec{f}(\hat{o}, \hat{i}) = \text{scattering amplitude}$$

- Represents amplitude, phase and polarization of scattered wave in far-field in direction  $\hat{o}$  when object is illuminated by plane wave propagating in direction  $\hat{i}$  with unit amplitude

# Scattering Amplitude

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- Consider the scattered power flux density  $S_s$  at a distance  $R$  from the object in the direction  $\hat{\boldsymbol{o}}$ , caused by an incident power flux density  $S_i$
- Let us define the differential scattering cross section by:

$$\begin{aligned}\sigma_d(\hat{\boldsymbol{o}}, \hat{\boldsymbol{i}}) &= \lim_{R \rightarrow \infty} \frac{R^2 S_s}{S_i} \\ &= |f(\hat{\boldsymbol{o}}, \hat{\boldsymbol{i}})|^2 \\ &= \frac{W_{ext}}{4\pi} p(\hat{\boldsymbol{o}}, \hat{\boldsymbol{i}}) \quad (17.21)\end{aligned}$$

where the dimensionless quantity  $p(\hat{\boldsymbol{o}}, \hat{\boldsymbol{i}})$  is called the phase function

# Scattering Amplitude

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## Interpretation of $\sigma_d(\hat{o}, \hat{i})$

- Suppose that the observed scattered power flux density in direction  $\hat{o}$  is extended uniformly over 1 steradian of solid angle about  $\hat{o}$

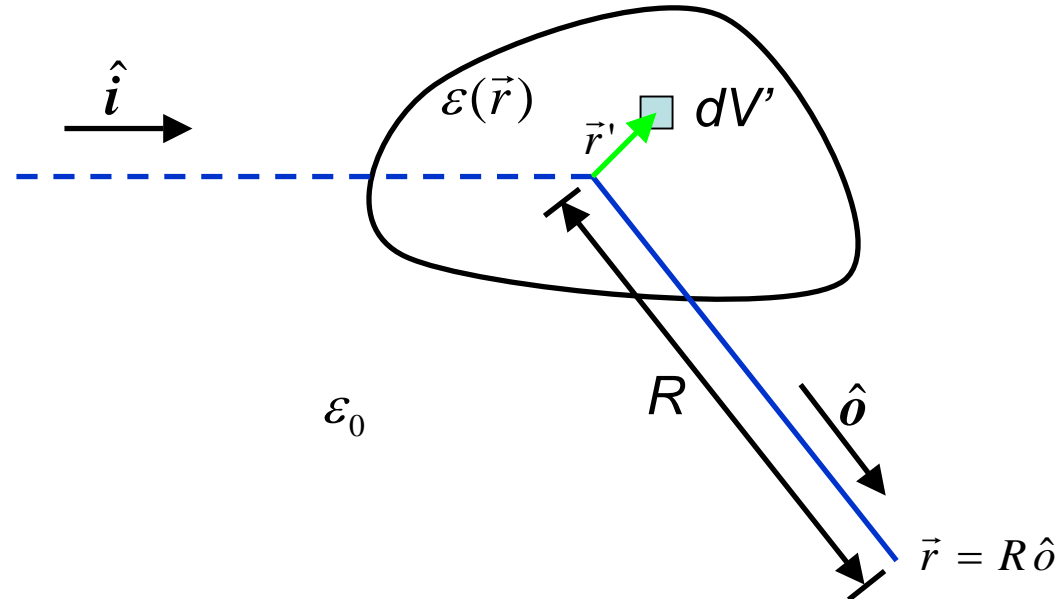
The cross section of an object that would cause just this amount of scattering would be  $\sigma_d(\hat{o}, \hat{i})$

# Integral Representation of Scattering Amplitude

- Consider a dielectric body whose relative dielectric constant is a function of position within the body:

$$\epsilon_r(\vec{r}) = \frac{\epsilon(\vec{r})}{\epsilon_0} = \epsilon'_r(\vec{r}) + j\epsilon''_r(\vec{r}) \quad \text{in } V \quad (17.22)$$

- The dielectric body occupies volume  $V$  and is surrounded by a medium whose dielectric constant is  $\epsilon_0$



# Integral Representation of Scattering Amplitude

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- Start with Maxwell's Equations:

$$\nabla \times \vec{E} = -j\omega\mu_0 \vec{H} \quad (17.23a)$$

$$\nabla \times \vec{H} = j\omega\epsilon(\vec{r}) \vec{E} \quad (17.23b)$$

- It is assumed that  $\mu = \mu_0$  everywhere

- We can rewrite (17.23b) as:

$$\nabla \times \vec{H} = j\omega\epsilon_0 \vec{E} + \vec{J}_{eq} \quad (17.24a)$$

where

$$\vec{J}_{eq} = \begin{cases} j\omega\epsilon_0[\epsilon_r(r) - 1]E & \text{in } V \\ 0 & \text{outside } V \end{cases} \quad (17.24b)$$

# Integral Representation of Scattering Amplitude

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- $\mathbf{J}_{eq}$  may be considered as an equivalent current source which generates the scattered wave
- We may write the electric and magnetic fields as the sum of incident waves and scattered waves:

$$\vec{E}(\vec{r}) = \vec{E}_i(\vec{r}) + \vec{E}_s(\vec{r}) \quad (17.25a)$$

$$\vec{H}(\vec{r}) = \vec{H}_i(\vec{r}) + \vec{H}_s(\vec{r}) \quad (17.25b)$$

- We can use the electric Hertz vector to find the scattered fields from the equivalent current source  $\mathbf{J}_{eq}$
- This will allow us to find the far-field scattering amplitude  $\vec{f}(\hat{o}, \hat{i})$
- Note that this is found in terms of the total electric field  $\vec{E}(\vec{r})$  inside the object, which must be found or approximated by some other means

# Electric Hertz Vector

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- The Electric Hertz Vector will allow us to calculate the scattered fields from the internal fields in a simple manner
- Begin with Maxwell's Equations:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (17.26a)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (17.26b)$$

$$\nabla \cdot \mathbf{D} = \rho \quad (17.26c)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (17.26d)$$

- Recall the vector identity:

$$\nabla \cdot \nabla \times \mathbf{A} = 0 \quad (17.27)$$

# Electric Hertz Vector

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- Now, (17.27) tells us that the divergence of the curl of any vector is zero
- From (17.26d) we note that the divergence of  $B$  is zero
- Therefore,  $B$  may be expressed by the curl of an arbitrary vector  $A$ , called the vector potential:

$$B = \nabla \times A \quad (17.28)$$

- Then (17.26b) becomes:

$$\nabla \times \left( E + \frac{\partial A}{\partial t} \right) = 0 \quad (17.29)$$

# Electric Hertz Vector

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- Now, we have the vector identity:

$$\nabla \times (\nabla \Phi) = 0 \quad (17.30)$$

which says that the curl of the gradient of any scalar function is zero

- Therefore we may write:

$$E + \frac{\partial A}{\partial t} = -\nabla \phi \quad (17.31)$$

where  $\phi$  is called the scalar potential

- Substituting (17.31) & (17.28) into (17.26a) gives us:

$$-\nabla \times \nabla \times A - \mu \epsilon \frac{\partial^2 A}{\partial t^2} - \mu \epsilon \nabla \frac{\partial \phi}{\partial t} = -\mu J \quad (17.32)$$

# Electric Hertz Vector

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•Now, let us define the electric Hertz vector  $\Pi$  such that:

$$A = \mu\varepsilon \frac{\partial \Pi}{\partial t} \quad (17.33)$$

$$\phi = -\nabla \cdot \Pi \quad (17.34)$$

•From (17.33) & (17.34) the Lorentz condition follows:

$$\nabla \cdot A + \mu\varepsilon \frac{\partial \phi}{\partial t} = 0 \quad (17.35)$$

In addition, we can combine  $J$  &  $\rho$  consistent with the continuity condition:

$$J = \frac{\partial P}{\partial t} \quad (17.36)$$

$$\rho = -\nabla \cdot P \quad (17.37)$$

# Electric Hertz Vector

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•Now, starting with:

$$-\nabla \times \nabla \times A - \mu\epsilon \frac{\partial^2 A}{\partial t^2} - \mu\epsilon \nabla \frac{\partial \phi}{\partial t} = -\mu J \quad (17.32)$$

•Recall the vector identity:

$$\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A \quad (17.38)$$

•Combining (17.38) with the Lorentz condition (17.35) gives us:

$$\nabla^2 A - \mu\epsilon \frac{\partial^2 A}{\partial t^2} = -\mu J \quad (17.39)$$

# Electric Hertz Vector

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- For the Electric Hertz Vector, we can write a similar expression:

$$\nabla^2 \Pi - \mu\epsilon \frac{\partial^2 \Pi}{\partial t^2} = -\frac{P}{\epsilon} \quad (17.40)$$

- For the time-harmonic case, we obtain:

$$\nabla^2 \Pi + \omega^2 \mu\epsilon \Pi = -\frac{J_s}{j\omega\epsilon} \quad (17.41)$$

where  $J_s$  is the  
source current  
density

- This allows us to find the electric and magnetic fields:

$$E = \nabla \times \nabla \times \Pi - \frac{J_s}{j\omega\epsilon} \quad (17.42)$$

$$H = j\omega\epsilon \nabla \times \Pi \quad (17.43)$$

# Integral Representation of Scattering Amplitude

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- For the scattered fields in the far-field region, we therefore have:

$$\vec{E}_s(\vec{r}) = \nabla \times \nabla \times \vec{\Pi}_s(\vec{r}) \quad (17.44a)$$

$$\vec{H}_s(\vec{r}) = j\omega\epsilon_0 \nabla \times \vec{\Pi}_s(\vec{r}) \quad (17.44b)$$

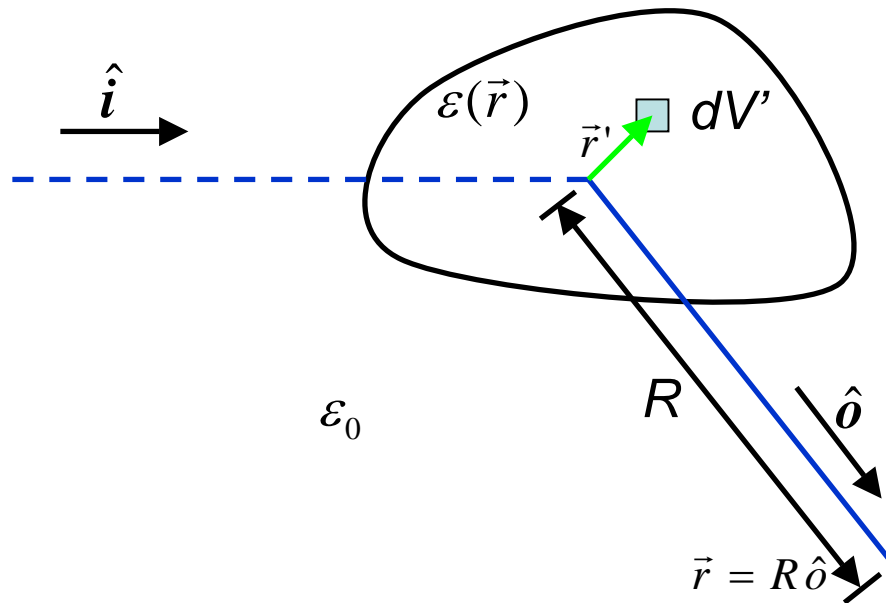
$$\begin{aligned} \vec{\Pi}_s(\vec{r}) &= \frac{1}{j\omega\epsilon_0} \int_V G_0(\vec{r}, \vec{r}') J_{eq}(\vec{r}') dV' \\ &= \int_{V'} [\epsilon_r(\vec{r}') - 1] \vec{E}(\vec{r}') G_0(\vec{r}, \vec{r}') dV' \end{aligned} \quad (17.44c)$$

where

$$G_0(\vec{r}, \vec{r}') = \frac{\exp(-jk|\vec{r} - \vec{r}'|)}{4\pi|\vec{r} - \vec{r}'|} \quad \begin{array}{l} \text{free space} \\ \text{Green's function} \end{array} \quad (17.44d)$$

# Integral Representation of Scattering Amplitude

- To obtain the scattering amplitude, consider  $\vec{E}_s(\vec{r})$  in the far-field of the object



Now, we have:

$$\vec{r} = R \hat{o} \quad (17.45)$$

In the far-field, we have:

$$\frac{1}{|\vec{r} - \vec{r}'|} \approx \frac{1}{R} \quad (17.46)$$

- However, we cannot approximate the phase  $k |\vec{r} - \vec{r}'|$  by  $kR$

# Integral Representation of Scattering Amplitude

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- We can expand  $|\vec{r} - \vec{r}'|$  in a binomial series and keep the first term:

$$\begin{aligned} |\vec{r} - \vec{r}'| &= (R^2 + r'^2 - 2R\vec{r}' \cdot \hat{o})^{1/2} \\ &\approx R - \vec{r}' \cdot \hat{o} \end{aligned} \quad (17.47)$$

- For large  $R$ , the Green's function becomes:

$$G_0(\vec{r}, \vec{r}') \approx \frac{\exp(-jkR + jk\vec{r}' \cdot \hat{o})}{4\pi R} \quad (17.48)$$

- In the far-field, we also have:

$$\begin{aligned} \nabla\left(\frac{e^{-jkR}}{R}\right) &\approx \frac{e^{-jkR}}{R}(-jk\nabla R) \\ &= -jk\hat{o}\frac{e^{-jkR}}{R} \end{aligned} \quad (17.49)$$

# Integral Representation of Scattering Amplitude

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- Therefore,  $\nabla$  is equivalent to  $-jk\hat{o}$
- Substituting (17.49) & (17.48) into (17.44a) & (17.44c), we obtain:

$$\vec{E}_s(\vec{r}) = \vec{f}(\hat{o}, \hat{i}) \frac{\exp(-jkR)}{R} \quad (17.50a)$$

$$\vec{f}(\hat{o}, \hat{i}) = \frac{k^2}{4\pi} \int_{V'} [\vec{E} - \hat{o}(\hat{o} \cdot \vec{E})][\epsilon_r(\vec{r}') - 1] \exp(jk\vec{r}' \cdot \hat{o}) dV' \quad (17.50b)$$

where we used:

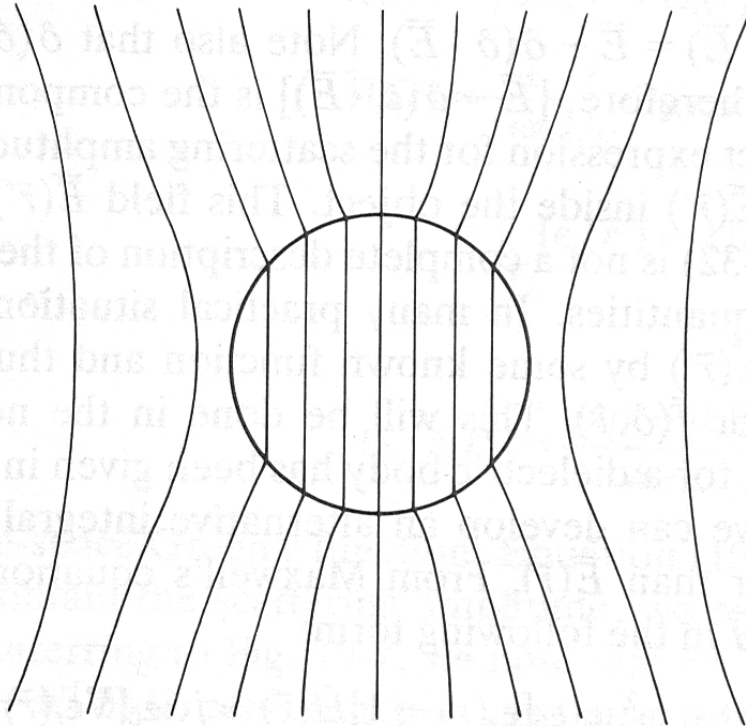
$$-\hat{o} \times (\hat{o} \times \vec{E}) = \vec{E} - \hat{o}(\hat{o} \cdot \vec{E}) \quad (17.51)$$

# Integral Representation of Scattering Amplitude

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- Now  $\hat{o}(\hat{o} \cdot \vec{E})$  is the component of  $\vec{E}$  along  $\hat{o}$
- Therefore,  $[\vec{E} - \hat{o}(\hat{o} \cdot \vec{E})]$  is the component of  $\vec{E}$  perpendicular to  $\hat{o}$
- Now, (17.49b) is an exact expression for the scattering amplitude in terms of the total electric field  $\vec{E}(\vec{r})$  inside the object
- This field  $\vec{E}(\vec{r})$  is not known in general
- Therefore, (17.46b) is not a complete description of the scattering amplitude in terms of known quantities
- However, in practical situations,  $\vec{E}(\vec{r})$  may be approximated by a known function and a useful approximate expression for  $\vec{f}(\hat{o}, \hat{i})$  may be found

# Rayleigh Scattering for a Spherical Object



- Consider a dielectric sphere much smaller than the wavelength
- Because of its small size, the electric field within and near the sphere must behave almost like an electrostatic field
- From electrostatics, when a constant electric field  $E_i$  is applied to a dielectric sphere, the electric field  $\vec{E}$  inside the sphere is uniform and given by:

$$\vec{E} = \frac{3}{\epsilon_r + 2} \vec{E}_i \quad \text{where} \quad \vec{E}_i = E_i \hat{e}_i \quad (17.52)$$

# Rayleigh Scattering for a Spherical Object

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- Because the object is much smaller than the wavelength, we have:

$$k\vec{r}' \ll 1 \quad (17.53)$$

- Therefore,

$$\exp(jk\vec{r}' \cdot \hat{o}) \approx 1 \quad (17.54)$$

- We can now substitute (17.52) into (17.50b) to obtain the scattering amplitude  $\vec{f}(\hat{o}, \hat{i})$

- However, we can write this more compactly if we note that the scattering is caused by an equivalent current  $J_{eq}$  given by (17.24b)

# Rayleigh Scattering for a Spherical Object

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- The polarization vector is given by:

$$\vec{P} = \vec{J}_{eq} / (j\omega) \quad (17.55)$$

- The equivalent dipole moment  $\vec{p}$  of the sphere is then given by:

$$\begin{aligned} \vec{p} &= \int_V \vec{P} dV' \\ &= \int_V \epsilon_0 (\epsilon_r - 1) \vec{E} dV' \\ &= \frac{3(\epsilon_r - 1)}{\epsilon_r + 2} \epsilon_0 V \vec{E}_i \end{aligned} \quad (17.56)$$

where  $V$  is the volume of the sphere

# Rayleigh Scattering for a Spherical Object

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- The scattering amplitude for Rayleigh scattering is then given by:

$$\vec{f}(\hat{o}, \hat{i}) = \frac{k^2}{4\pi\epsilon_0} [\vec{p} - \hat{o}(\hat{o} \cdot \vec{p})] \quad (17.57)$$

- Now  $[\vec{p} - \hat{o}(\hat{o} \cdot \vec{p})]$  is the component of  $\vec{p}$  perpendicular to  $\hat{o}$
- Therefore its magnitude is equal to  $p \sin \chi$ , where  $\chi$  is the angle between  $\vec{p}$  and  $\hat{o}$
- This makes sense as it represents the radiation pattern of the electric dipole
- Note that (17.56) & (17.57) are always valid, even when the object is lossy and  $\epsilon_r$  is complex

# Rayleigh Scattering for a Spherical Object

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- Recall from before that the differential scattering cross section is given by:

$$\sigma_d(\hat{o}, \hat{i}) = |f(\hat{o}, \hat{i})|^2 \quad (17.21)$$

- Then Rayleigh scattering for a spherical object is given by:

$$\sigma_d(\hat{o}, \hat{i}) = \frac{k^4}{(4\pi)^2} \left| \frac{3(\epsilon_r - 1)}{\epsilon_r + 2} \right|^2 V^2 \sin^2 \chi \quad (17.58)$$

where  $\sin^2 \chi = 1 - (\hat{o} \cdot \hat{e}_i)^2$  (17.59)

- Note that the cross section is inversely proportional to the fourth power of the wavelength and directly proportional to the square of the volume of the scatterer

# Rayleigh Scattering for a Spherical Object

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- The scattering cross section of the sphere is then given by:

$$\begin{aligned}\sigma_s &= \int_{4\pi} \sigma_d d\omega \\ &= \frac{k^4}{(4\pi)^2} \left| \frac{3(\epsilon_r - 1)}{\epsilon_r + 2} \right|^2 V^2 \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} \sin^2 \theta \sin \theta d\theta d\phi \\ &= \frac{24\pi^3 V^2}{\lambda^4} \left| \frac{\epsilon_r - 1}{\epsilon_r + 2} \right|^2 \\ &= \frac{128\pi^5 a^6}{3\lambda^4} \left| \frac{\epsilon_r - 1}{\epsilon_r + 2} \right|^2 \quad (17.60)\end{aligned}$$

- Note that  $d\omega$  is the differential solid angle
- In deriving (17.60), we made the substitution  $\theta = \chi$

# Rayleigh Scattering for a Spherical Object

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- The actual geometric cross section of the sphere is  $\pi a^2$
- The scattering efficiency is the ratio of the scattering cross section to the geometric cross section:

$$Q_s = \frac{\sigma_s}{\pi a^2} = \frac{8(ka)^4}{3} \left| \frac{\epsilon_r - 1}{\epsilon_r + 2} \right|^2 \quad (17.61)$$

- The Rayleigh equation (17.61) is only valid for small  $ka$
- The approximate upper limit of the radius of the scatterer is generally taken to be  $a = 0.05 \lambda$
- At this radius the percentage error of (17.61) is less than 4%

# Rayleigh Scattering for a Spherical Object

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- Recall that the absorption cross section is given by:

$$C_{abs} = \frac{W_{abs}}{I_i} \quad (17.18a)$$

where

$$W_a = - \int_A \mathbf{S} \cdot \hat{\mathbf{e}}_r dA \quad (17.9)$$

$$\mathbf{S} = \frac{1}{2} \text{Re}\{\mathbf{E}_2 \times \mathbf{H}_2^*\} \quad (17.4)$$

- Note that  $(\mathbf{E}_2, \mathbf{H}_2)$  refers to the fields outside the scatterer
- In (17.18a) we have expressed the absorption cross section in terms of the total power flux entering the object

# Rayleigh Scattering for a Spherical Object

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- We can also express the absorption cross section as the volume integral of the loss inside the particle:

$$C_{abs} = \frac{\int_V k\epsilon''_r(\vec{r}') |\vec{E}(\vec{r})|^2 dV'}{|E_i|^2} \quad (17.62)$$

where  $\vec{E}(\vec{r}) = \vec{E}_i(\vec{r}) + \vec{E}_s(\vec{r})$       &       $\vec{H}(\vec{r}) = \vec{H}_i(\vec{r}) + \vec{H}_s(\vec{r})$

are the total fields inside the object

Substituting (17.52) into (17.62), we have:

$$C_{abs} = k\epsilon''_r \left| \frac{3}{\epsilon_r + 2} \right|^2 V \quad (17.63)$$