

# ES 275 Nanophotonics

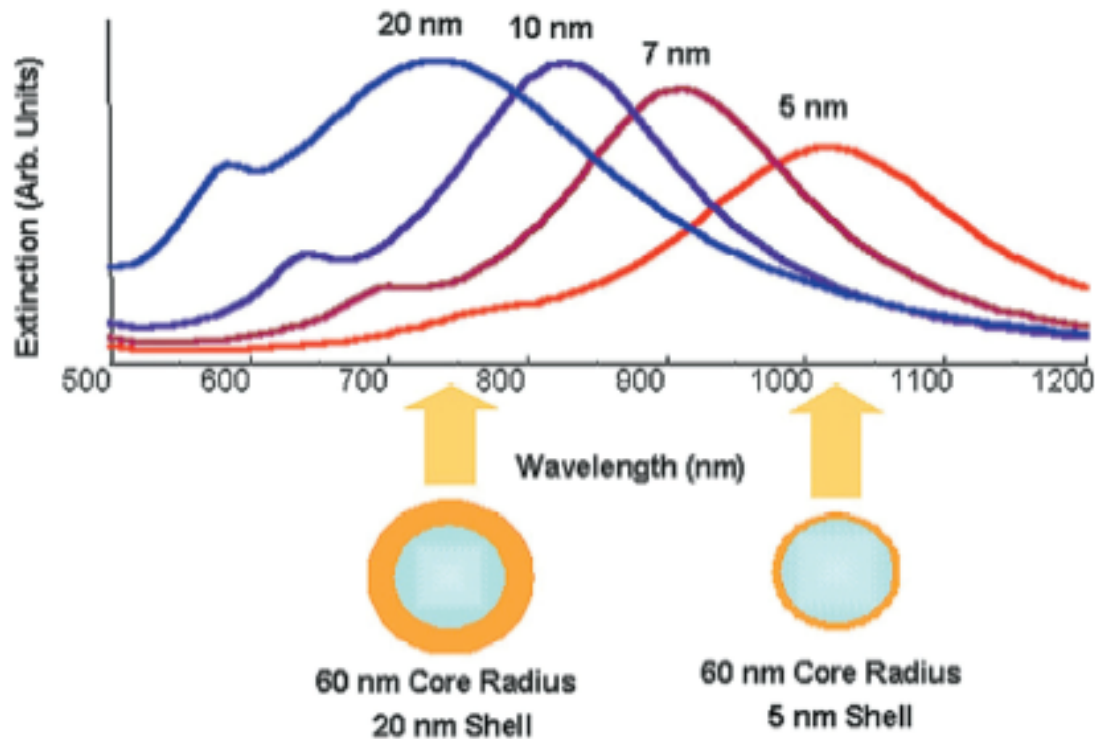
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## Lecture 19

### Gold Nanoshells Nanoshells – Electrostatic Approximation

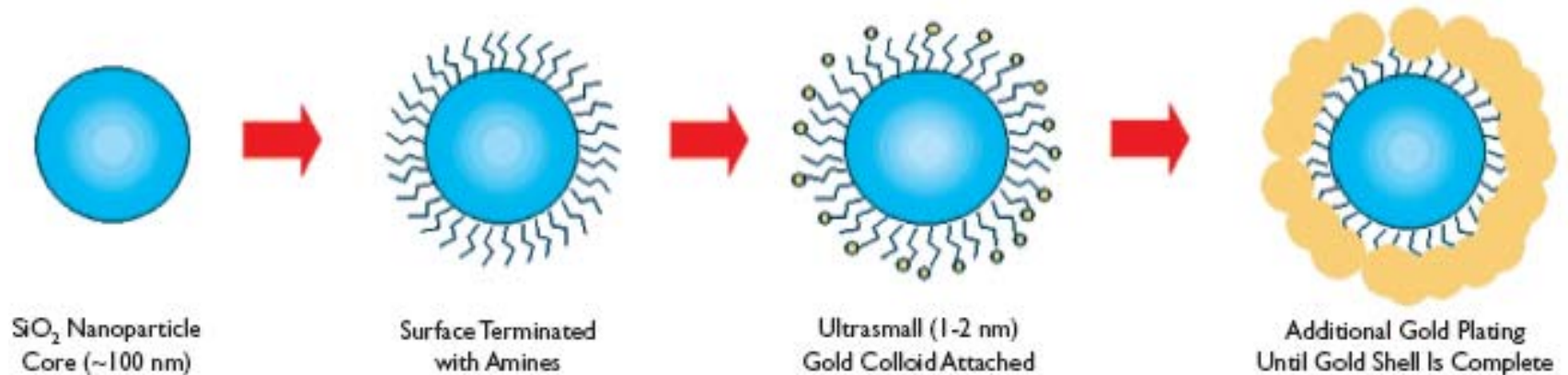
# Optical Resonances of Gold Nanoshells

## Calculated Optical Resonances of Silica Core, Gold Nanoshells



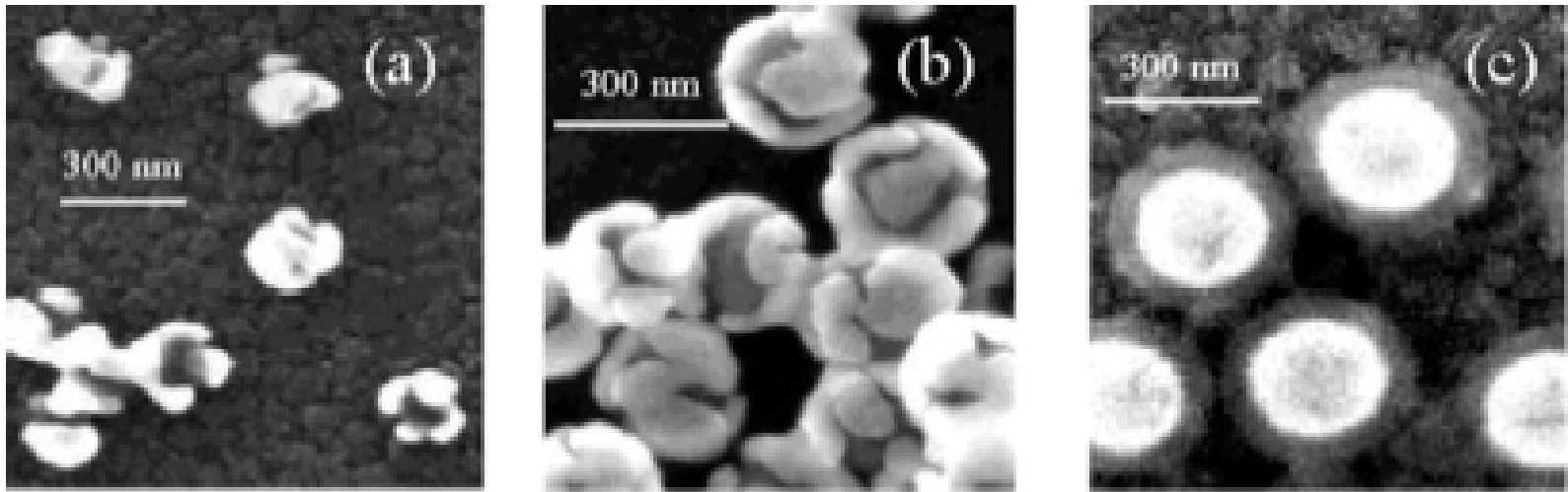
- Resonant wavelength tunability is useful: for example gold nanoshells could be an alternative to IR absorbing dyes which are limited by thermal stability, toxicity, and lightfastness

# Optical Resonances of Gold Nanoshells



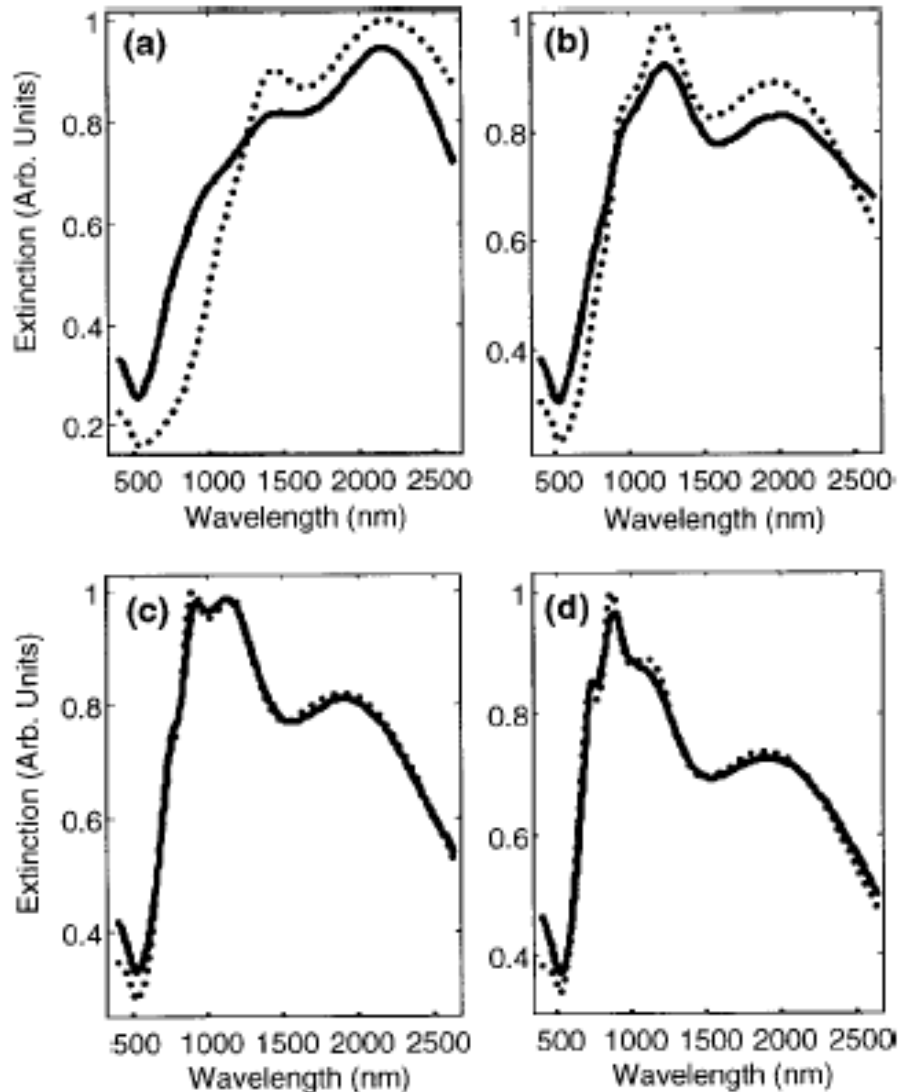
- Nanoparticles chemically modified so that gold colloids will attach to their surface
- Nanoparticles coated with gold colloid (1-2 nm), covering ~25% of the surface
- Nanoparticle then immersed in electroless plating solution
- Minimum metal thickness achievable is ~ 5nm

# Optical Resonances of Gold Nanoshells



- The melting temperature of metallic nanostructures are typically lower than that of the bulk material
- Silica encapsulation layers of 60-70 nm raise the effective melting temp. from 325 deg. C to greater than 625 deg. C
- In these SEMs, silica core, gold nanoshells have been heated to 325 deg C:
  - (a). Uncoated nanoshell
  - (b). Coated with 5nm silica
  - (c). Coated with 50nm silica

# Infrared Extinction Properties of Gold Nanoshells



- With gold nanoshells, the optical resonance can be tuned from 2.3 eV to  $< 0.45$  eV

- Spectra are of gold nanoshells with silica cores

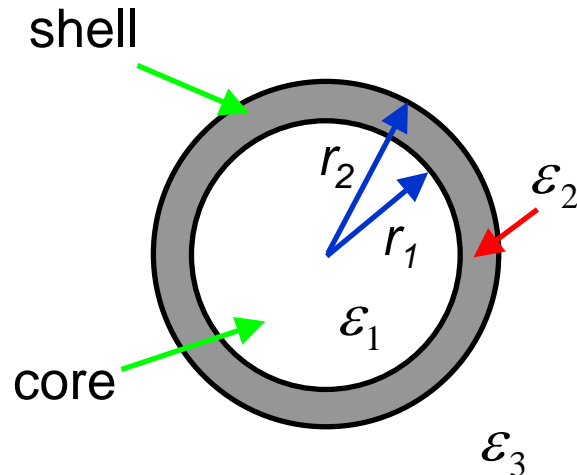
Solid line: experimental data  
Dotted line: theory

- Theoretical spectra calculated for 210 nm radius silica core with:

- (a). 6 nm gold shell
- (b). 10.5 nm gold shell
- (c). 15.0 nm gold shell
- (d). 20.0 nm gold shell

# Nanoshells – Electrostatics Approximation

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- It is assumed that the particle diameter is much smaller than the wavelength
- The electrostatic solution is obtained by solution of Laplace's equation for the potential

- General solution for potential in each region is given by:

$$\Phi_i(r, \theta) = [A_i r + (B_i / r^2)] \cos \theta \quad (19.1)$$

where  $A_i$  and  $B_i$  are constants multiplying the monopole and dipole terms

&  $i = 1, 2, \text{ or } 3$  for the core, shell and medium, respectively

# Nanoshells – Electrostatic Approximation

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- Recall that Laplace's Equation in spherical coordinates is given by:

$$\begin{aligned}\nabla^2\Phi &= \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial\Phi}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta} \left( \sin\theta \frac{\partial\Phi}{\partial\theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2\Phi}{\partial\phi^2} \\ &= 0\end{aligned}\quad (19.2)$$

- It may be shown that (19.1) satisfies Laplace's equation (19.2)
- How do we find the potential in the core, shell, and embedding medium ?
- Use boundary conditions at the core/shell and shell/medium interfaces
- Continuity of tangential component of the electric field:

$$\left. \frac{\partial\Phi_i}{\partial\theta} \right|_{r=r_i} = \left. \frac{\partial\Phi_{i+1}}{\partial\theta} \right|_{r=r_i} \quad (19.3)$$

# Nanoshells – Electrostatic Approximation

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- Continuity of normal component of displacement field:

$$\epsilon_i \left. \frac{\partial \Phi_i}{\partial r} \right|_{r=r_i} = \epsilon_{i+1} \left. \frac{\partial \Phi_{i+1}}{\partial r} \right|_{r=r_i} \quad (19.4)$$

- In region 1 (the core),  $B_1 = 0$
- In region 3 (surrounding medium), far from the shell we have:

$$\Phi_3 = -E_0 r \cos \theta \quad (19.5)$$

- Therefore, we have  $A_3 = -E_0$
- Therefore, the remaining unknowns are  $A_1, A_2, B_2$  and  $B_3$

# Nanoshells – Electrostatic Approximation

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- Once the potential is found, the electric field in each region may be found using the definition of the potential:

$$E_i = -\nabla\Phi_i(r, \theta) \quad (19.6)$$

- This gives us the electric field in the core as:

$$E_1 = \frac{9\varepsilon_2\varepsilon_3}{\varepsilon_2\varepsilon_a + 2\varepsilon_3\varepsilon_b} E_0 (\cos\theta \hat{r} - \sin\theta \hat{\theta}) \quad (19.7)$$

- Similarly, the field in the shell is given by:

$$E_2 = \frac{3\varepsilon_3}{\varepsilon_2\varepsilon_a + 2\varepsilon_3\varepsilon_b} \left\{ \left[ (\varepsilon_1 + 2\varepsilon_2) + 2(\varepsilon_1 - \varepsilon_2) \left(\frac{r_1}{r}\right)^3 \right] E_0 \cos\theta \hat{r} \right. \\ \left. - \left[ (\varepsilon_1 + 2\varepsilon_2) - (\varepsilon_1 - \varepsilon_2) \left(\frac{r_1}{r}\right)^3 \right] E_0 \sin\theta \hat{\theta} \right\} \quad (19.8)$$

# Nanoshells – Electrostatic Approximation

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- The electric field in the surrounding medium is then given by:

$$E_3 = \left(2 \frac{\varepsilon_2 \varepsilon_a - \varepsilon_3 \varepsilon_b}{\varepsilon_2 \varepsilon_a + 2\varepsilon_3 \varepsilon_b} \frac{r_2^3}{r^3} + 1\right) E_0 \cos \theta \hat{r} \\ + \left(\frac{\varepsilon_2 \varepsilon_a - \varepsilon_3 \varepsilon_b}{\varepsilon_2 \varepsilon_a + 2\varepsilon_3 \varepsilon_b} \frac{r_2^3}{r^3} - 1\right) E_0 \sin \theta \hat{\theta} \quad (19.9)$$

- In equations (19.7), (19.8) & (19.9) we have:

$$\varepsilon_a = \varepsilon_1(3 - 2P) + 2\varepsilon_2 P \quad (19.10)$$

$$\varepsilon_b = \varepsilon_1 P + \varepsilon_2(3 - P) \quad (19.11)$$

$$P = 1 - (r_1 / r_2)^3 \quad (19.12)$$

# Nanoshells – Electrostatic Approximation

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- Examination of (19.9) shows us that the field in the surrounding medium is the same as a dipole with an effective dipole moment given by:

$$\mathbf{p} = \varepsilon_3 \alpha \mathbf{E}_{ind} \quad (19.13)$$

- The polarizability is given by:

$$\alpha = 4\pi\varepsilon_0 r_2^3 \left[ \frac{\varepsilon_2 \varepsilon_a - \varepsilon_3 \varepsilon_b}{\varepsilon_2 \varepsilon_a + 2\varepsilon_3 \varepsilon_b} \right] \quad (19.14)$$

- Therefore, resonance is obtained when the real portion of the denominator of (19.14) goes to zero:

$$\text{Re}\{\varepsilon_2 \varepsilon_a + 2\varepsilon_3 \varepsilon_b\} = 0 \quad (19.15)$$

# Nanoshells – Electrostatic Approximation

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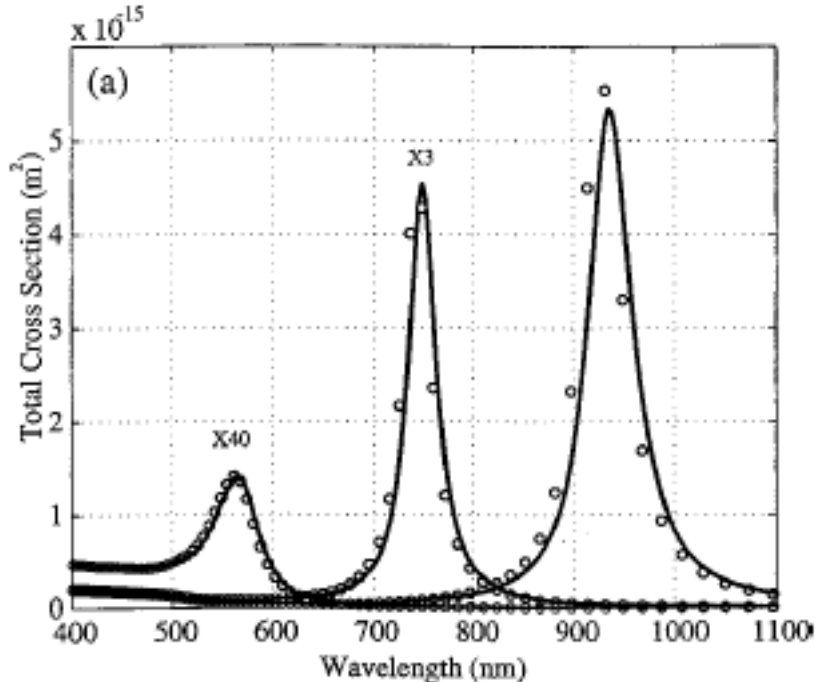
- Let us assume that the core and embedding medium are dielectrics
- Therefore, the resonance condition as a function of wavelength is:

$$\frac{r_1}{r_2} = \left[ 1 + \frac{3 \varepsilon_2'(\lambda)(\varepsilon_1 + 2\varepsilon_3)}{2 [\varepsilon_2'(\lambda)]^2 - \varepsilon_2'(\lambda)(\varepsilon_1 + \varepsilon_3) + \{\varepsilon_1 \varepsilon_3 - [\varepsilon_2''(\lambda)]^2\}} \right]^{1/3} \quad (19.16)$$

- This expression gives the ratio of the core radius to the total radius needed to obtain a resonant condition at a particular wavelength
- This expression demonstrates that metallic nanoshells possess geometric tunability of the resonance wavelength

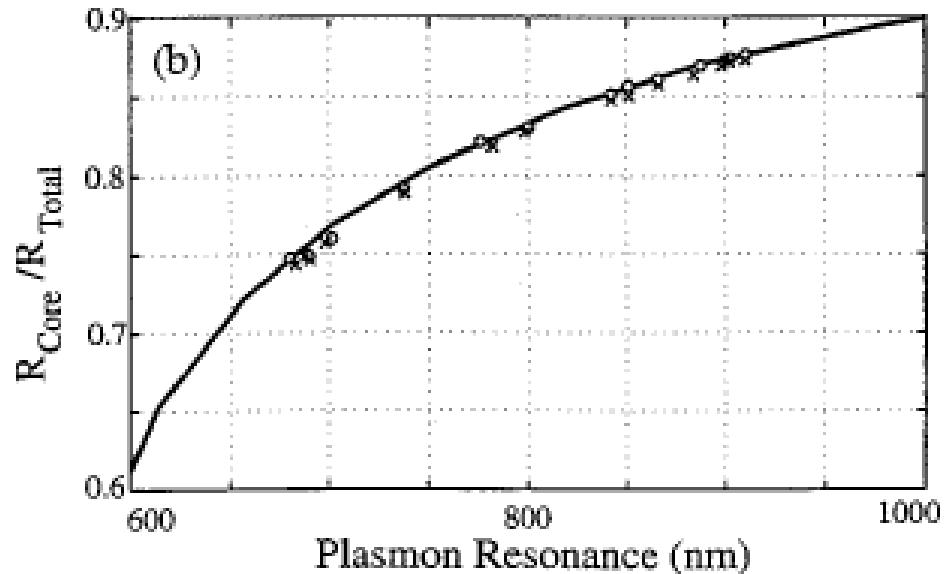
# Nanoshells – Electrostatic Approximation

## Total Cross-section vs Wavelength



- In going from shorter to longer wavelengths, the total radii are: 4.0, 10.0, 17.0 nm

## Ratio of Core Radius to Total Radius Required to Give Plasmon Resonance at a Particular Wavelength



- Au-coated Au<sub>2</sub>S nanoshells