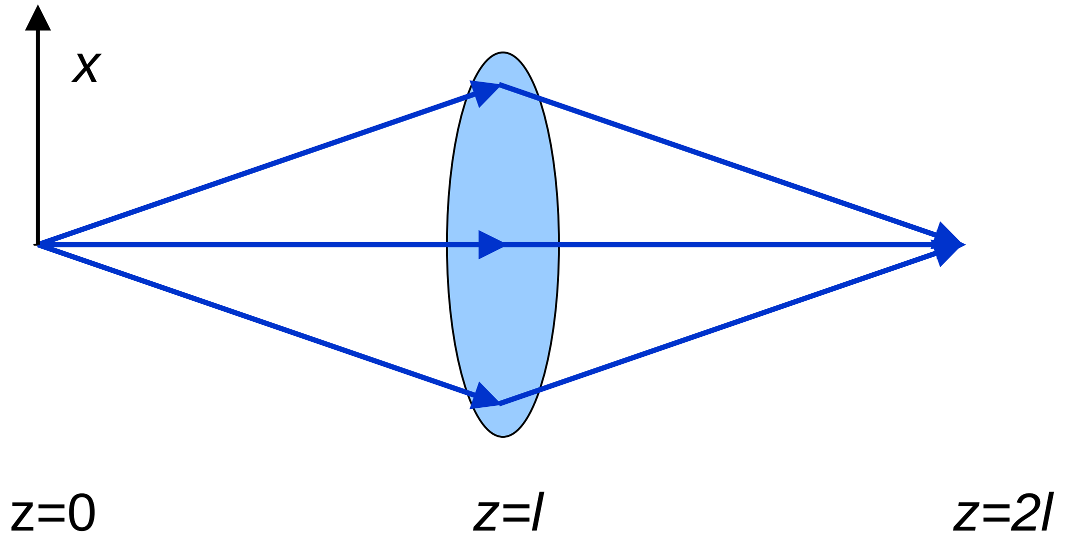
ES 275 Nanophotonics

Lecture 21

Resolution Limitation of Conventional Lenses
The Perfect Lens
Left-Handed Materials
Image Formation with Left-Handed Materials
Experimental Construction of a Left-Handed Material

Image Formation with a Conventional Lens



•What does a lens do ?

•Consider an infinitesimal dipole of frequency ω in front of the lens

•The electric field is given by a 2D Fourier expansion:

$$\boldsymbol{E}(r,t) = \sum_{\sigma,k_x,k_y} \boldsymbol{E}_\sigma(k_x,k_y)\exp(ik_z z + ik_x x + ik_y y - i\omega t) \qquad (21.1)$$

Image Formation with a Conventional Lens

•At the object, the electric field is therefore given by:

$$\boldsymbol{E}(x,y,z=0,t) = \sum_{\sigma,k_x,k_y} \boldsymbol{E}_\sigma(k_x,k_y)\exp(ik_x x + ik_y y - i\omega t) \tag{21.2}$$

•Just before the lens, the field is given by:

$$\boldsymbol{E}(x,y,z=l^-,t) = \sum_{\sigma,k_x,k_y} \boldsymbol{E}_\sigma(k_x,k_y)\exp(ik_z l + ik_x x + ik_y y - i\omega t)$$

$$= \sum_{\sigma,k_x,k_y} \boldsymbol{E}_\sigma(k_x,k_y)\exp(ik_x x + ik_y y - i\omega t)\exp(i\sqrt{(\frac{\omega}{c})^2 - (k_x^2 + k_y^2)}\, l) \tag{21.3}$$

because we know that:

$$k_x^2 + k_y^2 + k_z^2 = (\frac{\omega}{c})^2 \tag{21.4}$$

Image Formation with a Conventional Lens

•At *z=l*, the lens applies a phase delay to each of the Fourier components in such a way that at some distance an image is formed

•The phase-delaying action of the lens results in the electric field just after the lens being given by:

$$\boldsymbol{E}(x, y, z = l^{+}, t)$$

$$= \sum_{\sigma, k_x, k_y} \boldsymbol{E}_{\sigma}(k_x, k_y) \exp(ik_x x + ik_y y - i\omega t) \exp(i\sqrt{(\frac{\omega}{c})^2 - (k_x^2 + k_y^2)}\, l + i\phi(k_x, k_y)) \tag{21.5}$$

where $\phi(k_x, k_y)$ is the phase delay provided by the lens

Image Formation with a Conventional Lens

•At $z = 2l$, the electric field is then given by:

$$\boldsymbol{E}(x, y, z = 2l, t)$$
$$= \sum_{\sigma, k_x, k_y} \boldsymbol{E}_\sigma(k_x, k_y) \exp(ik_x x + ik_y y - i\omega t) \exp(i\sqrt{(\frac{\omega}{c})^2 - (k_x^2 + k_y^2)}\, 2l + i\phi(k_x, k_y)) \tag{21.6}$$

•To form an image, the phase delay must be given by:

$$\phi(k_x, k_y) = -\sqrt{(\frac{\omega}{c})^2 - (k_x^2 + k_y^2)}\, 2l \tag{21.7}$$

•Therefore, the electric field at $z=2l$ is given by:

$$\boldsymbol{E}(x, y, z = 2l, t)$$
$$= \sum_{\sigma, k_x, k_y} \boldsymbol{E}_\sigma(k_x, k_y) \exp(ik_x x + ik_y y - i\omega t) \tag{21.8}$$

Image Formation with a Conventional Lens

Conventional Lenses

- In conventional imaging, the summation in (21.8) is only over those values of k_x & k_y that correspond to propagating waves

- In other words, the evanescent waves are discarded

We have propagating waves when:

$$\left(\frac{\omega}{c}\right)^2 > k_x^2 + k_y^2 \qquad (21.9a)$$

We have evanescent waves when:

$$\left(\frac{\omega}{c}\right)^2 < k_x^2 + k_y^2 \qquad (21.9b)$$

Image Formation with a Conventional Lens

•Because the evanescent components are discarded, the maximum resolution in the image can never be greater than:

$$\Delta \approx \frac{2\pi}{k_{\max}} = \frac{2\pi c}{\omega} = \lambda \qquad (21.10)$$

•Therefore, with conventional lenses the resolution is limited by the wavelength

Image Formation with a Perfect Lens

•What does the lens need to do in order to achieve perfect imaging ?

•Like the conventional lens, our perfect lens will need to apply a (k_x, k_y) dependent phase shift to the propagating components of the field

•For the evanescent wave, the perfect lens will need to transform the exponentially-decaying wave into an exponentially-growing wave

•In other words, it will apply the transform

$$\exp(i\sqrt{(\frac{\omega}{c})^2-(k_x^2+k_y^2)}\,l) \quad \Rightarrow \quad \exp(-i\sqrt{(\frac{\omega}{c})^2-(k_x^2+k_y^2)}\,l) \tag{21.11}$$

to both the propagating and exponentially-decaying components (not just to the propagating components)

Image Formation with a Perfect Lens

- If the perfect lens can do this, then the field at the image is given by:

$$\boldsymbol{E}(x,y,z=2l,t) = \sum_{\sigma,k_x,k_y} \boldsymbol{E}_\sigma(k_x,k_y)\exp(ik_x x + ik_y y - i\omega t)$$
$$= \boldsymbol{E}(x,y,z=0,t) \qquad (21.12)$$

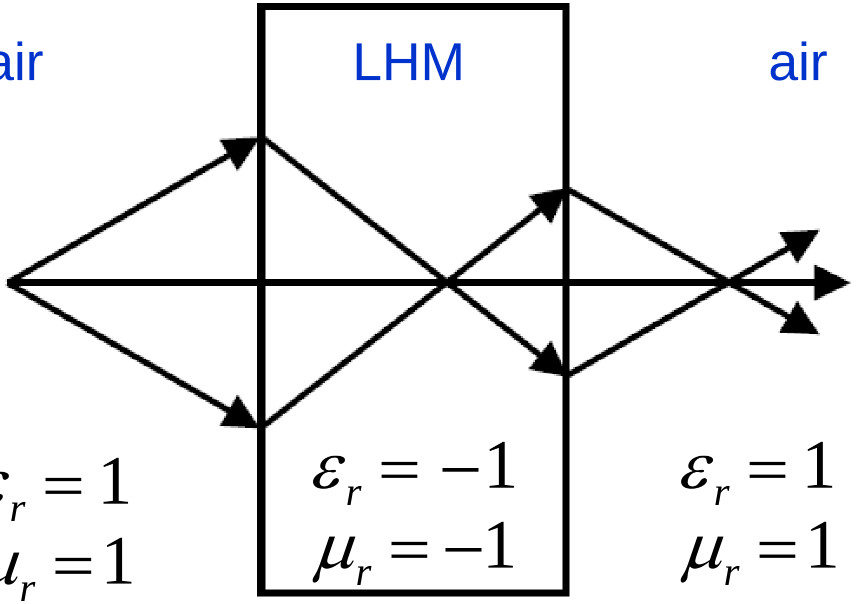
- In other words, the field at the image is the same as the object because now the summation is over all values of k_x & k_y because we no longer discard the exponentially-decaying components

Negative Refraction Makes a Perfect Lens

•A left-handed material (LHM) with $\varepsilon_r = -1$ & $\mu_r = -1$ will:

(a). apply phase correction to the propagating component of the wave
(b). amplify the evanescent component of the wave

•The left-handed material has a negative refractive index



$\varepsilon_r = 1$
$\mu_r = 1$

$\varepsilon_r = -1$
$\mu_r = -1$

$\varepsilon_r = 1$
$\mu_r = 1$

From: Pendry, Physical Review Letters, vol. 85, pp. 3966-3969 (2000)

Wave Propagation in Left-Handed Materials

•Recall Maxwell's Equations:

$$\nabla \times \boldsymbol{E} = i\omega\mu \boldsymbol{H} \qquad (21.13)$$

$$\nabla \times \boldsymbol{H} = -i\omega\varepsilon \boldsymbol{E} \qquad (21.14)$$

•The wave equation for the electric field is then given by:

$$\nabla^2 \boldsymbol{E} + \omega^2 \mu\varepsilon \boldsymbol{E} = 0 \qquad (21.15)$$

or

$$\nabla^2 \boldsymbol{E} + k^2 \boldsymbol{E} = 0 \qquad (21.16)$$

where *k* is the wavevector

Wave Propagation in Left-Handed Materials

- The dispersion relation is therefore given by:

$$k^2 = \omega^2 \mu\varepsilon$$

$$= (\frac{\omega}{c})^2 \mu_r \varepsilon_r \qquad (21.17)$$

- We can classify media according to whether ε & μ are positive or negative:

(a). $\varepsilon < 0$ & $\mu > 0$
Metals below the plasma frequency
$k^2 < 0$, so the wave is exponentially decaying

(b). $\varepsilon > 0$ & $\mu > 0$
Regular dielectric such as glass
$k^2 > 0$, so the wave is propagating

Wave Propagation in Left-Handed Materials

(c). $\varepsilon < 0$ & $\mu < 0$
Left-handed materials
$k^2 > 0$, so wave is propagating

(d). $\varepsilon > 0$ & $\mu < 0$
Magnetic plasmas
$k^2 < 0$, so wave is exponentially decaying

•Note that the light seems to propagate in the left handed material in the same way that it does in a regular dielectric

For example, if $\varepsilon_r = -1$ & $\mu_r = -1$, then:

•the dispersion relation in the LHM will be the same as that in vacuum with $\varepsilon_r = 1$ & $\mu_r = 1$

•the impedance of the wave will be the same as that of vacuum

Wave Propagation in Left-Handed Materials

•Difference between regular materials and LHMs:

-in LHMs, direction of Poynting vector is opposite to direction of ***k***

•If the incident plane wave has its electric field along the x-direction:

$$\boldsymbol{E} = E_x \exp(i(kz - \omega t))\,\hat{x} \qquad (21.18)$$

then from Maxwell's equation (21.13), the magnetic field is given by:

$$\boldsymbol{H} = \frac{1}{i\omega\mu}\nabla\times\boldsymbol{E} = \frac{1}{i\omega\mu}\left(\frac{\partial E_x}{\partial z}\right)\hat{y}$$

$$= \frac{k}{\omega\mu} E_x \exp(i(kz - \omega t))\,\hat{y} \qquad (21.19)$$

Wave Propagation in Left-Handed Materials

•The time-average Poynting vector is defined as:

$$\boldsymbol{P} = \frac{1}{2}\mathrm{Re}\{\boldsymbol{E} \times \boldsymbol{H}^*\} \qquad (21.20)$$

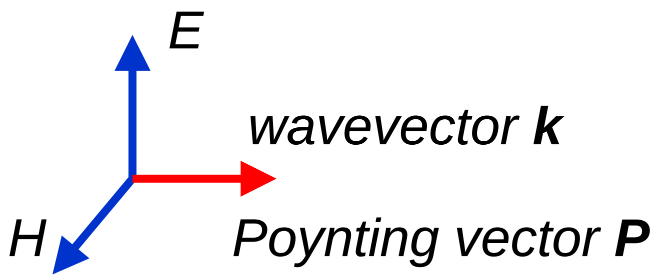
•It is therefore given by:

$$\boldsymbol{P} = \frac{1}{2}\frac{k}{\omega\mu}|E_x|^2\,\hat{z} \qquad (21.21)$$

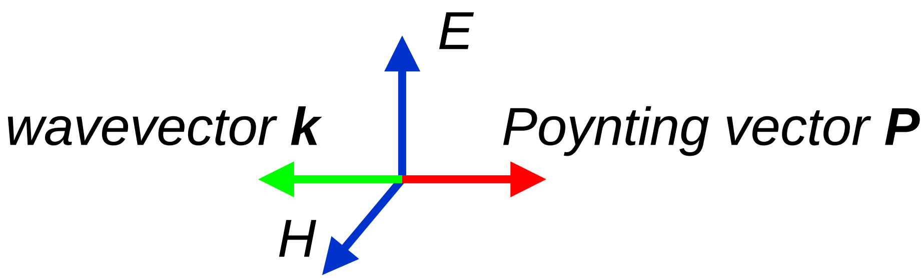
•Therefore, if we have a left-handed material with $\mu = -\mu_0$, then the direction of the Poynting vector is opposite to that of the direction of ***k***

Right-Handed and Left-Handed Materials

•In a right-handed material, the wavevector *k* and Poynting vector *P* are in the same direction:



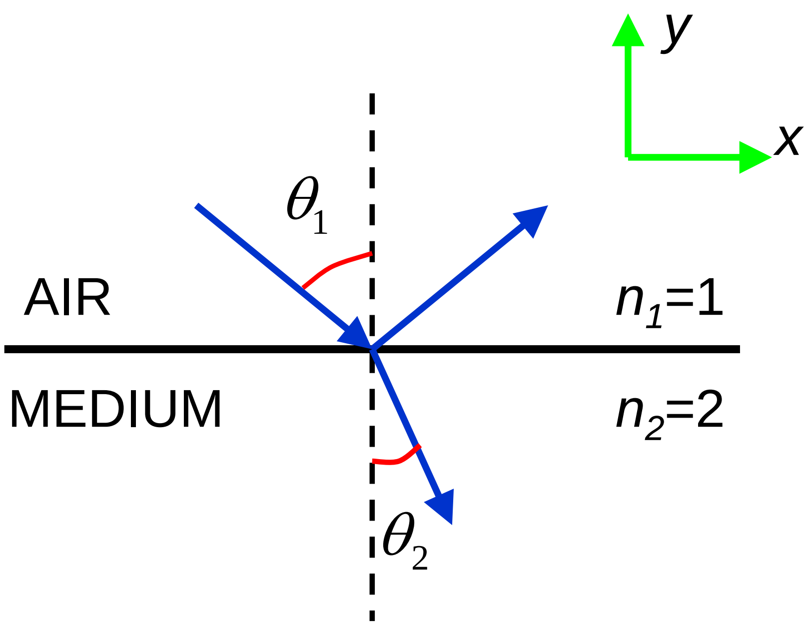
•In a left-handed material, the wavevector points in the opposite direction to $(\boldsymbol{E} \times \boldsymbol{H}^*)$:



i.e. *(E, H, k)* form a left-handed system for the LHM

Refraction between Conventional Materials

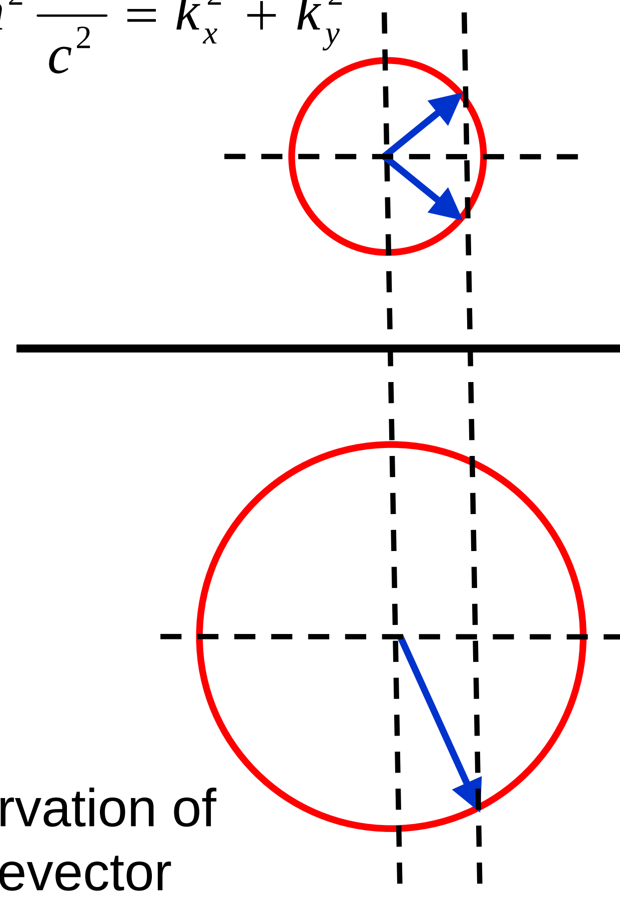
Reflection and Refraction at an Interface



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Constant Frequency Contours

$$n^2 \frac{\omega^2}{c^2} = k_x^2 + k_y^2$$

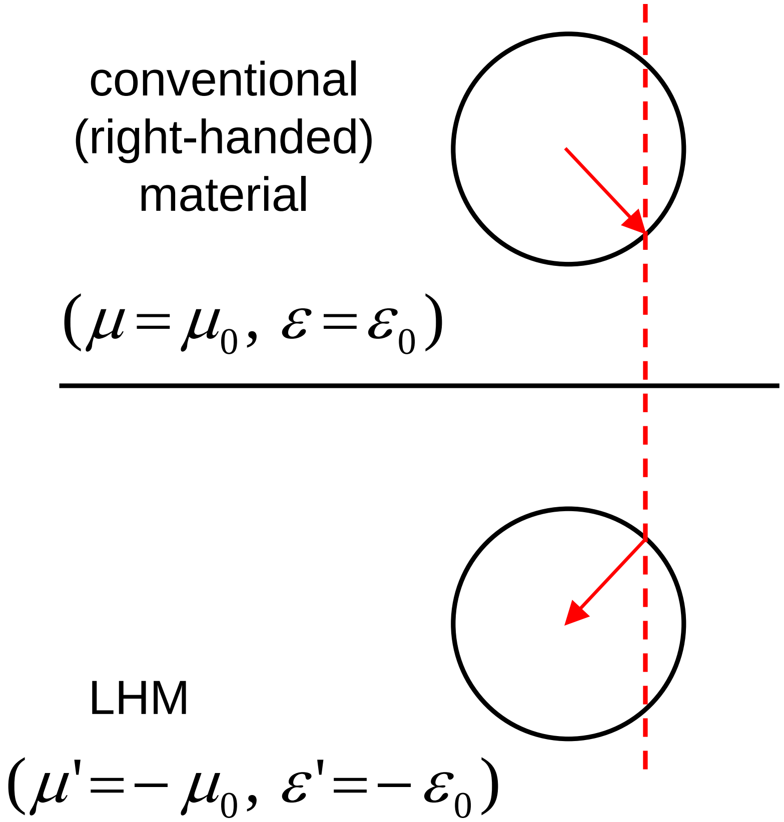


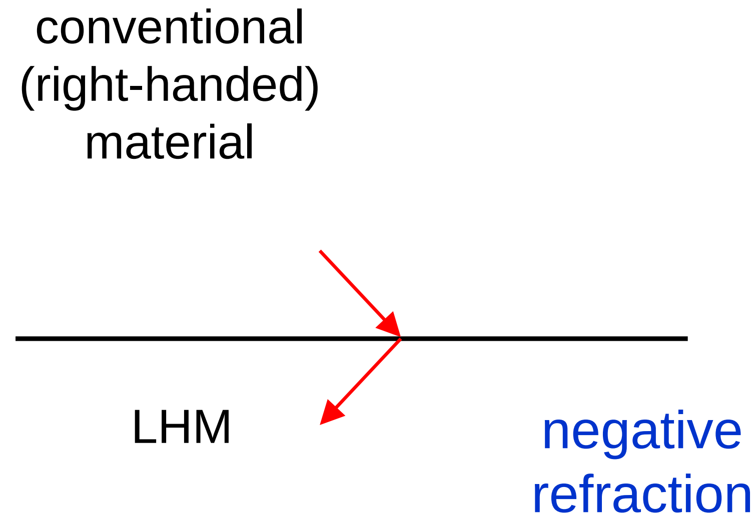
Conservation of wavevector parallel to interface

(from Lecture 13)

Negative Refraction

•Now consider refraction at the interface between a conventional material (right-handed material) and a left-handed material (LHM):





•The k-vector points outwards

•Therefore, in LHM, arrow representing energy flow direction points inwards

Note: there is no reflection since the two media are impedance-matched

Reflection & Transmission of Evanescent Wave Through Interface

•Assume that we have an S-polarized evanescent wave incident on the interface between free space and the LHM

•Incident electric field is given by:

$$\boldsymbol{E}_{0S+} = [0,\ 1,\ 0]\ \exp(ik_z z + ik_x x - i\omega t) \qquad (21.22)$$

•The wavevector k_z is imaginary, meaning that the wave decays exponentially:

$$k_z = i\sqrt{k_x^2 + k_y^2 - \omega^2 / c^2} \qquad (21.23)$$

Reflection & Transmission of Evanescent Wave Through Interface

- At the interface between vacuum and the LHM, some of the light is reflected:

$$\boldsymbol{E}_{0S-} = r[0,\ 1,\ 0]\exp(-ik_z z + ik_x x - i\omega t) \qquad (21.24)$$

- The transmitted field is given by:

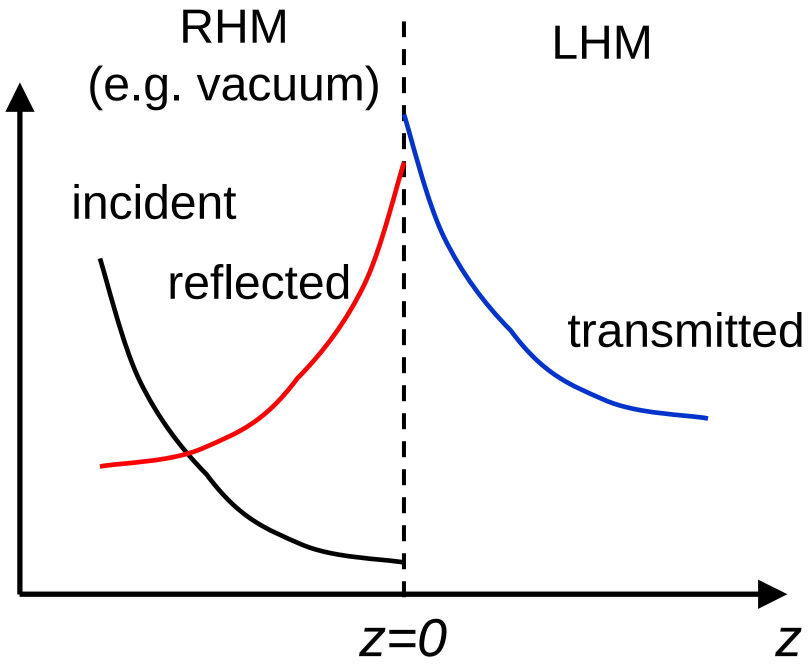
$$\boldsymbol{E}_{1S+} = t[0,\ 1,\ 0]\exp(ik_z^{'} z + ik_x x - i\omega t) \qquad (21.25)$$

- The wavevector in the LHM is given by:

$$k_z^{'} = i\sqrt{k_x^2 + k_y^2 - \varepsilon_r^{'}\mu_r^{'}\omega^2 / c^2}$$

$$= k_z \quad \text{if} \quad \varepsilon_r^{'} = -1 \ \ \& \ \ \mu_r^{'} = -1 \qquad (21.26)$$

Reflection & Transmission of Evanescent Wave Through Interface



•What are the amplitudes of the reflected and transmitted waves ?

Reflection & Transmission of Evanescent Wave Through Interface

- Match tangential components of electric & magnetic fields at interface

- Electric field gives us:

$$1 + r = t \qquad (21.27)$$

- Use Maxwell equation to find magnetic field from electric field

- The electric field only has a y-component, so magnetic field is given by:

$$H_x = \frac{-1}{i\omega\mu}\frac{\partial E_y}{\partial z} \qquad (21.28)$$

- From (21.22), (21.24) & (21.25), we have:

$$-\frac{k_z}{\omega\mu}(1-r) = -\frac{k_z^{'}}{\omega\mu'}t \qquad (21.29)$$

Reflection & Transmission of Evanescent Wave Through Interface

•Combining (21.27) & (21.29) gives us the reflection coefficient:

$$r = \frac{\mu' k_z - \mu k_z^{'}}{\mu' k_z + \mu k_z^{'}} \qquad (21.30)$$

•If the conventional medium is free space with $\mu = \mu_0$, then we have:

$$r = \frac{\mu_r^{'} k_z - k_z^{'}}{\mu_r^{'} k_z + k_z^{'}} \qquad (21.31)$$

•For the transmission coefficient we have:

$$t = \frac{2\mu' k_z}{\mu' k_z + \mu k_z^{'}} = \frac{2\mu_r^{'} k_z}{\mu_r^{'} k_z + k_z^{'}} \qquad (21.32)$$

Reflection & Transmission of Evanescent Wave Through Interface

•For the case of a wave incident from the LHM to the RHM (e.g. vacuum), we have similar expressions for the reflection & transmission coefficients:

$$r' = \frac{\mu k_z^{'} - \mu' k_z}{\mu k_z^{'} + \mu' k_z}$$

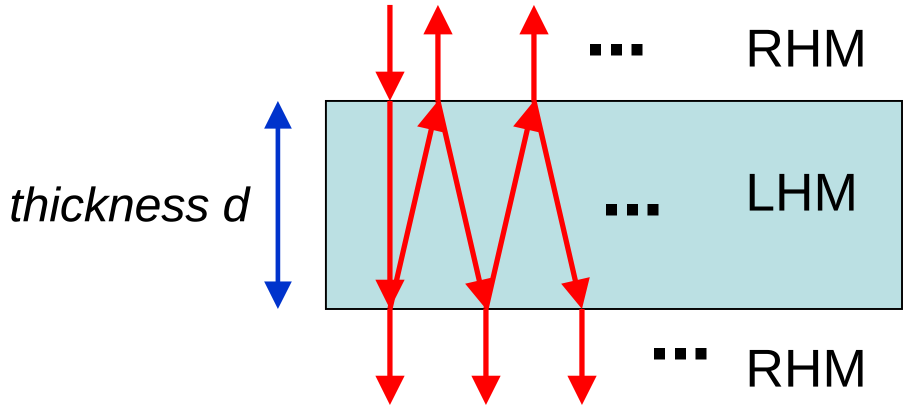
$$= \frac{k_z^{'} - \mu_r^{'} k_z}{k_z^{'} + \mu_r^{'} k_z} \qquad \text{(21.33)} \qquad \text{if the RHM is free space}$$

$$t' = \frac{2\mu k_z^{'}}{\mu k_z^{'} + \mu' k_z}$$

$$= \frac{2k_z^{'}}{\mu_r^{'} k_z + k_z^{'}} \qquad \text{(21.34)} \qquad \text{if the RHM is free space}$$

Evanescent Wave Transmission Through a LHM Slab

•To find the transmission of an evanescent wave through a LHM slab, we need to sum over multiple scattering events



•The transmission coefficient through the LHM slab is given by:

$$T_S = tt'\exp(ik_z'd) + tt'r'^2\exp(i3k_z'd) + tt'r'^4\exp(i5k_z'd) + ...$$

$$= \frac{tt'\exp(ik_z'd)}{1 - r'^2\exp(i2k_z'd)} \qquad (21.35)$$

Evanescent Wave Transmission Through a LHM Slab

•Let us find the transmission in the limit where $\mu' \to -\mu_0$ & $\varepsilon' \to -\varepsilon_0$:

$$\lim_{\substack{\mu_r' \to -1 \\ \varepsilon_r' \to -1}} T_s = \lim_{\substack{\mu_r' \to -1 \\ \varepsilon_r' \to -1}} \frac{tt' \exp(ik_z' d)}{1 - r'^2 \exp(2ik_z' d)}$$

$$= \lim_{\substack{\mu_r' \to -1 \\ \varepsilon_r' \to -1}} \frac{2\mu_r' k_z}{\mu_r' k_z + k_z'} \frac{2k_z'}{k_z' + \mu_r' k_z} \frac{\exp(ik_z' d)}{1 - (\frac{k_z' - \mu_r' k_z}{k_z' + \mu_r' k_z})^2 \exp(2ik_z' d)}$$

$$= \exp(-ik_z' d)$$

$$= \exp(-ik_z d) \qquad (21.36)$$

Evanescent Wave Transmission Through a LHM Slab

•The transmission of the p-polarized wave through a LHM slab is given by:

$$\lim_{\substack{\mu_r^{'} \to -1 \\ \varepsilon_r^{'} \to -1}} T_p = \lim_{\substack{\mu_r^{'} \to -1 \\ \varepsilon_r^{'} \to -1}} \frac{2\varepsilon_r^{'} k_z}{\varepsilon_r^{'} k_z + k_z^{'}} \frac{2k_z^{'}}{k_z^{'} + \varepsilon_r^{'} k_z} \frac{\exp(ik_z^{'} d)}{1 - (\frac{k_z^{'} - \varepsilon_r^{'} k_z}{k_z^{'} + \varepsilon_r^{'} k_z})^2 \exp(2ik_z^{'} d)}$$

$$= \exp(-ik_z^{'} d)$$

$$= \exp(-ik_z d) \qquad (21.37)$$

•Therefore, the LHM slab amplifies both the s- and p- polarizations by the same factor

Transmission of p-polarized Wave in Electrostatic Limit

•In the electrostatic limit, we have:

$$\omega << c_0 \sqrt{k_x^2 + k_y^2} \qquad (21.38)$$

•The wavevector k_z is then given by:

$$\lim_{k_x^2+k_y^2\to\infty} k_z = \lim_{k_x^2+k_y^2\to\infty} i\sqrt{k_x^2+k_y^2-(\omega/c_0)^2}$$

$$= i\sqrt{k_x^2+k_y^2} \qquad (21.39)$$

•The wavevector k'_z in the LHM is given by:

$$\lim_{k_x^2+k_y^2\to\infty} k_z^{'} = \lim_{k_x^2+k_y^2\to\infty} i\sqrt{k_x^2+k_y^2-\varepsilon_r^{'}\mu_r^{'}(\omega/c_0)^2}$$

$$= i\sqrt{k_x^2+k_y^2} = k_z \qquad (21.40)$$

Transmission of p-polarized Wave in Electrostatic Limit

•The transmission through the LHM is then given by:

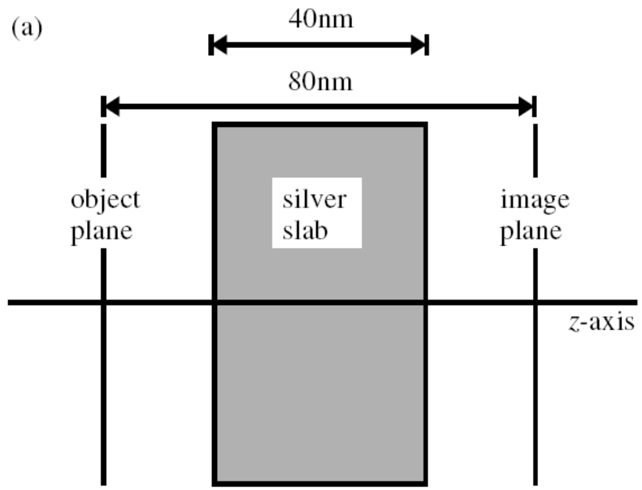
$$\lim_{k_x^2+k_y^2\to\infty} T_P = \lim_{k_x^2+k_y^2\to\infty} \frac{2\varepsilon_r^{'} k_z}{\varepsilon_r^{'} k_z + k_z^{'}} \frac{2k_z^{'}}{k_z^{'} + \varepsilon_r^{'} k_z} \frac{\exp(ik_z^{'} d)}{1-(\frac{k_z^{'} - \varepsilon_r^{'} k_z}{k_z^{'} + \varepsilon_r^{'} k_z})^2 \exp(2ik_z^{'} d)}$$

$$= \frac{4\varepsilon_r^{'} \exp(ik_z d)}{(\varepsilon_r^{'}+1)^2 - (\varepsilon_r^{'}-1)^2 \exp(2ik_z d)} \qquad (21.41)$$

•If we have $\varepsilon_r^{'} \to -1$, without placing any conditions on μ, we have:

$$\lim_{\varepsilon_r^{'}\to -1} \lim_{k_x^2+k_y^2\to\infty} T_P = \exp(-ik_z d) \qquad (21.42)$$

Silver Slab Lens

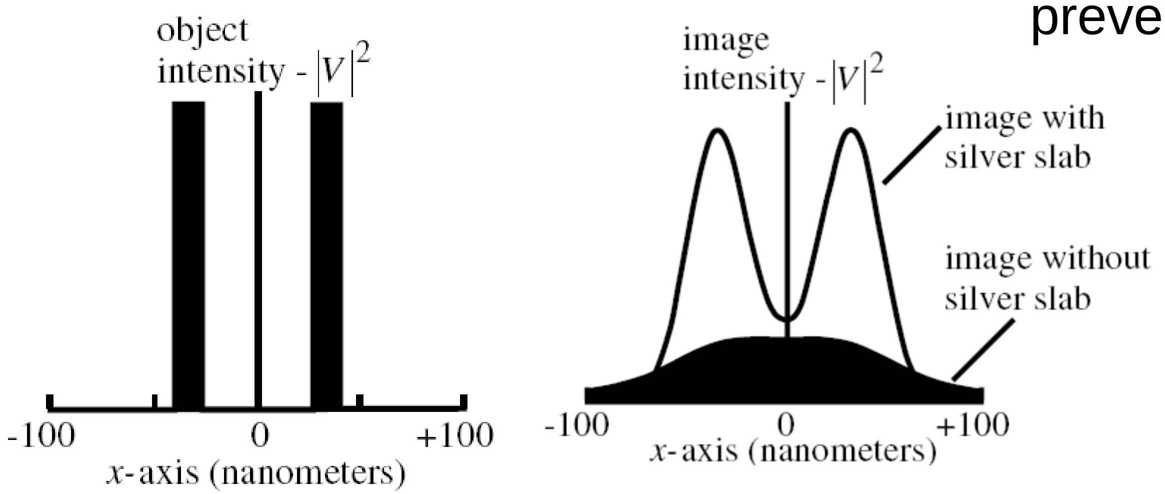


- At $\lambda = 356\,nm$, we have:

$$\varepsilon_r^{'} \approx -1 + 0.4i$$

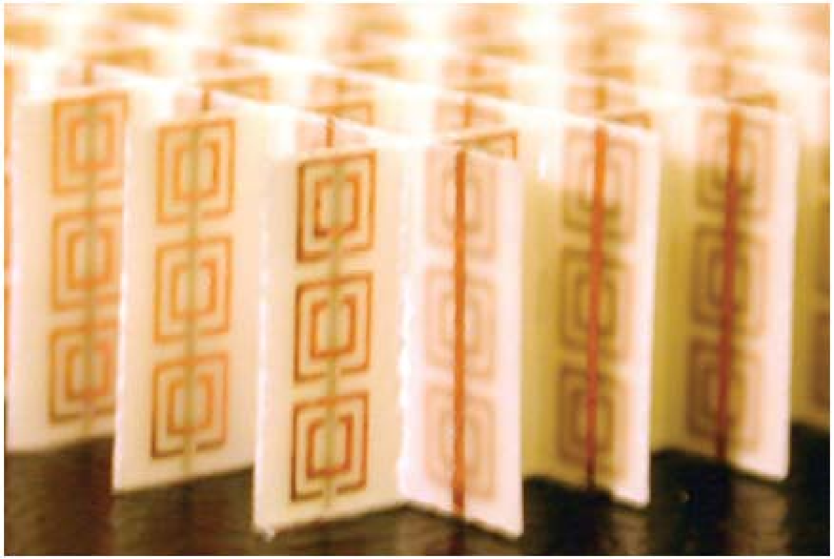
for silver

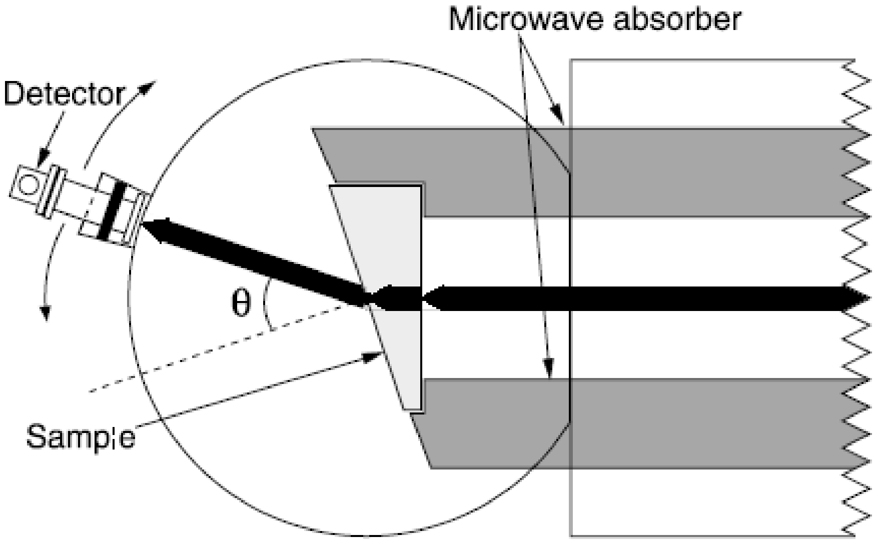
- Therefore, a silver slab may be used to focus light, but the imaginary part of permittivity prevents ideal reconstruction



From: Pendry, Physical Review Letters, vol. 85, pp. 3966-3969 (2000)

Negative Index of Refraction at Microwave Frequencies





•LHM consists of two-dimensionally periodic array of copper split ring resonators and wires

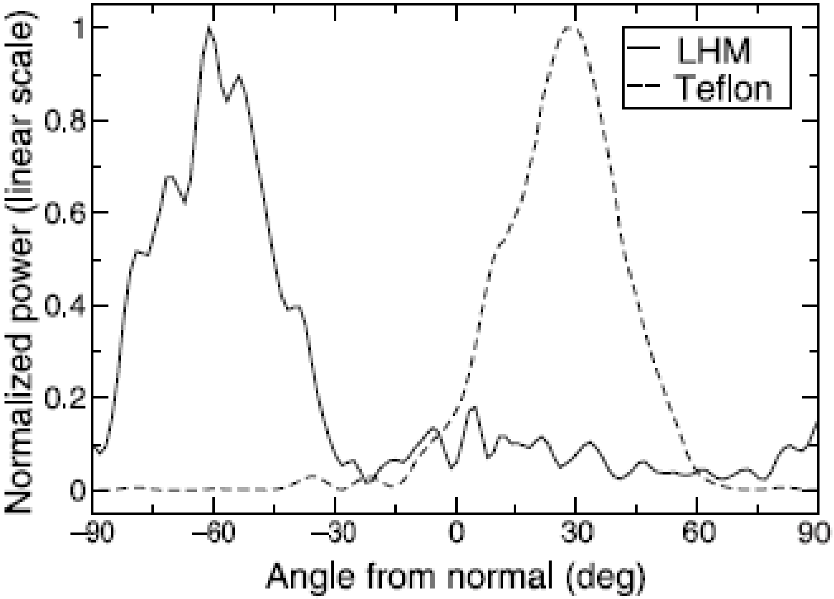
•Split-ring resonators produce negative magnetic permeability over a particular frequency range

•Wire elements produce negative electric permittivity in an overlapping frequency region

From: Shelby et al, Science, vol. 292, pp. 77-79 (2001)

Negative Index of Refraction at Microwave Frequencies

Transmitted Power vs Refraction Angle



- Measurements for Teflon & LHM samples

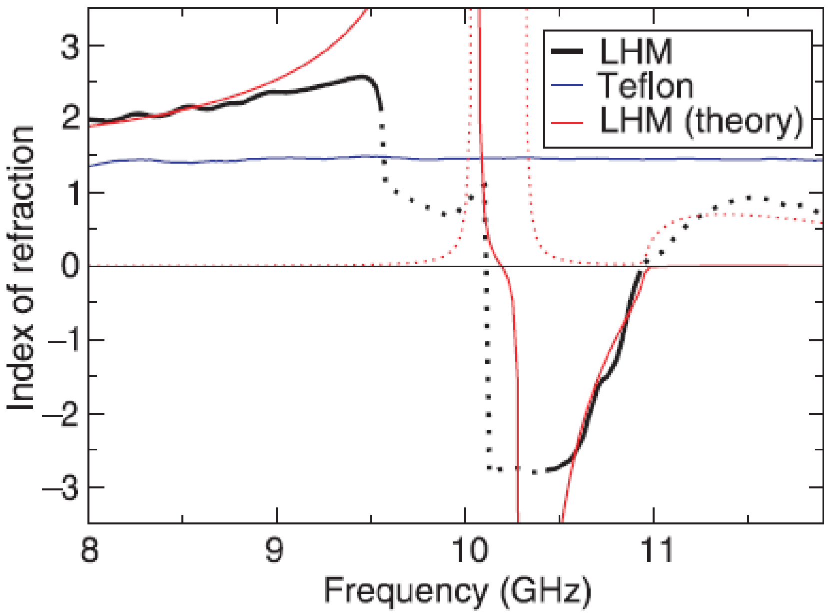
- Teflon: refracted power peak at $27°$, corresponding to index of 1.4

- LHM: refracted power peak at $-61°$, corresponding to index of –2.7

From: Shelby et al, Science, vol. 292, pp. 77-79 (2001)

Negative Index of Refraction at Microwave Frequencies

Index of Refraction vs Frequency



- •Blue: data from Teflon sample
- •Black: LHM data
- •Dotted curve: LHM data where index is either outside limit of detection (|*n*| > *3*) or dominated by imaginary component
- •Solid red curve: real component of refractive index (theory)
- •Dotted red curve: imaginary component of refractive index (theory)

From: Shelby et al, Science, vol. 292, pp. 77-79 (2001)