Published in Advances in Fracture Research, Vol. 6, edited by D. Francois (Proceedings of the 5th International Conference on Fracture, Cannes, France, 29 March - 3 April, 1981), Pergamon Press, 1982.

CRACK-TIP SINGULARITY FIELDS IN NONLINEAR FRACTURE MECHANICS: A SURVEY OF CURRENT STATUS

John W. Hutchinson

Division of Applied Sciences, Harvard University
Cambridge, Massachusetts 02138, U.S.A.

ABSTRACT

Recent activity is surveyed in the analysis of crack-tip stress and strain fields for stationary and growing cracks in inelastic solids under monotonic loading. Some of the main obstacles to further progress are discussed.

KEYWORDS

Fracture; growing cracks; plastic deformation; creep cracking; steady-state crack growth.

INTRODUCTION

Singularity fields associated with near-tip behavior of solutions to crack problems have played a central role in the development of fracture mechanics. Linear elastic fracture mechanics employs the stress intensity factor K which measures the intensity of the singular stress and strain fields obtained from linear elasticity theory. For nearly all materials to which linear fracture mechanics is applied, nonlinear effects such as plasticity, creep or even nonlinear elasticity intervene near the tip to invalidate the assumptions of linear elasticity theory. Nevertheless, under certain restrictive conditions the stress intensity factor still uniquely measures, or controls, the near-tip behavior of the material, and this is the basis of linear elastic fracture mechanics. The effort to understand and document these restrictive conditions was an important part of the development of the subject.

More recent progress in nonlinear fracture mechanics has close parallels to earlier developments in the linear theory. The J-integral is the measure of the singularity fields from the small strain deformation theory of plasticity. Here, too, this description of an elastic-plastic material breaks down sufficiently near the tip due to effects not modeled by deformation theory such as strongly nonproportional plastic deformations, finite strain effects or micro-voiding and cracking. While use of J is not restricted to small scale yielding at the tip, use is subject to a number of restrictive conditions imposed to ensure that J controls near-tip behavior in a unique way. The process of detailing these restrictions is still underway, and we will outline the current status of this effort.

It has only recently become clear that under rather restrictive conditions, also reviewed here, J controls near-tip behavior under moderate and large scale yielding even in the presence of small amounts of crack growth. When these conditions are met, the resistance curve approach (based on an experimentally measured relation between J and crack advance) can be used to analyze the stability of limited amounts of crack growth. This extended use of J is of considerable practical significance since the materials for which nonlinear fracture mechanics is most applicable tend to display fairly substantial tearing resistance. In other words, appreciable increases in J above the value at which crack growth starts are possible with only small amounts of accompanying crack advance. Under these circumstances, the importance of the initiation of crack growth, per se, becomes secondary to the point at which a small amount of crack advance becomes unstable.

While it seems reasonably certain that the J-resistance curve approach will have an important range of applications, it is equally clear that fairly severe restrictions on its use will have to be invoked, including limitations to small amounts of crack growth. At the present moment these restrictions are poorly understood. This reason alone provides impetus for more fundamental studies of quasi-static crack growth employing near-tip stress and strain fields coupled with basic laws governing material separation. It is also expected that a more fundamental understanding of quasi-static crack growth will reveal how material parameters such as yield stress, hardening, inclusion size and spacing, among others, influence tearing resistance. In problem areas such as creep cracking, where even more variables must be taken into account, an understanding of near-tip behavior becomes almost essential.

The review which follows will concentrate on developments taking place since ICF4. It will start with a discussion of conditions for applicability of J to stationary and growing cracks under moderate and large scale yielding conditions. We will then turn to recent work on the characterization of near-tip stress and strain fields for growing cracks, including efforts to predict tearing resistance. The importance of strain hardening will be emphasized, and it will be noted that the status of near-tip fields for growing cracks in hardening materials is uncertain. The survey ends with a discussion of singular fields which have just been discovered for cracks growing in an elastic creeping material. This review will not cover recent advances in finite strain studies of near-tip fields of cracks in nonlinear elastic solids. Work in this area has been surveyed by Sternberg (1979).

CONDITIONS FOR J-DOMINANCE AND J-CONTROLLED GROWTH

Stationary Cracks under Monotonic Loading

The J-integral (Rice, 1968) is a measure of intensity of the field at the tip of a crack in a material modeled by the small strain deformation theory of plasticity. Specifically, consider a strain hardening material for which the nonlinear (plastic) part of the strain ε is given in terms of the uniaxial stress σ by the power-law relation

$$\varepsilon/\varepsilon_0 \sim \alpha(\sigma/\sigma_0)^n \quad \text{for} \quad \varepsilon >> \varepsilon_0 \tag{1}$$

where σ_0 is the yield stress and $\varepsilon_0 = \sigma_0/E$ where E is Young's modulus. In this case, the so-called HRR singularity field (Hutchinson, 1968; Rice and Rosengren, 1968) is specified by

$$\sigma_{ij} \sim \sigma_0 \left(\frac{J}{\alpha \sigma_0 \varepsilon_0 I_n r} \right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(\theta, n) \tag{2}$$

$$\varepsilon_{ij} \sim \alpha \varepsilon_0 \left(\frac{J}{\alpha \sigma_0 \varepsilon_0 I_n r} \right)^{\frac{n}{n+1}} \tilde{\varepsilon}_{ij}(\theta, n) \tag{3}$$

$$u_i - u_i^0 \sim \alpha \varepsilon_0 r \left(\frac{J}{\alpha \sigma_0 \varepsilon_0 I_n r} \right)^{\frac{n}{n+1}} \tilde{u}_i(\theta, n) \tag{4}$$

where r and θ are planar-polar coordinates centered at the tip with θ measured from the line ahead of the crack. The dimensionless θ-variations, $\tilde{\sigma}_{ij}$, $\tilde{\varepsilon}_{ij}$ and $\tilde{u}_i$, depend on the symmetry of the field and on whether plane strain or plane stress holds as the tip is approached, as does the normalizing constant I_n.

For a hardening material with n finite, (2) and (3) imply a unique relation between the stress and strain fields sufficiently near the tip and J within the context of the small strain deformation theory. Let R denote the size, or roughly the radius, of the region at the crack tip within which the singular fields, (2) and (3), provide a good approximation to the full solution based on the small strain deformation theory. Thus, R measures the zone of dominance of the singular fields. It depends on whether plane strain or plane stress is assumed, on load, on strain hardening and on configuration. Some estimates of R for specific examples will be given below.

For J to be a meaningful unique measure of near-tip behavior it is essential that the region of incipient material separation (the fracture process zone) and the region in which finite strain effects become important be contained well within the zone of dominance measured by R. In most ductile metals the fracture process zone is roughly the same size as the finite strain region near the blunted tip of the crack. Finite element solutions for plane strain, Mode I in small scale yielding by McMeeking (1977) have shown that finite strain effects are only important within a radius of about 2 or 3 times the crack tip opening displacement δ_t. Outside this radius there is little difference between the predictions of small strain theory and finite strain theory. Assuming the fracture process zone is also within this radius, it follows that for Mode I in plane strain the zone of dominance of the J-fields must satisfy (approximately)

$$R > 3\delta_t \tag{5}$$

if J is to be a unique measure of crack-tip behavior under monotonic loading.

The relationship between an effective definition for δ_t and J from the singularity field (4) has been provided by Shih (1979) for both plane strain and plane stress in the form of curves for $d(\alpha\varepsilon_0, n)$ where

$$\delta_t = d(\alpha\varepsilon_0, n) \frac{J}{\sigma_0} \tag{6}$$

In plane strain with light to moderate strain hardening

$$\delta_t \cong .6 \frac{J}{\sigma_0} \tag{7}$$

Assuming (7) holds, the condition (5) for J-dominance can be restated as (approximately)

$$R > 1.8 \frac{J}{\sigma_0} \tag{8}$$

Linear elastic fracture mechanics is premised on the assumption of small scale yielding, which can also be considered as the basic reference for nonlinear fracture mechanics. In small scale yielding under Mode I, plane strain conditions, the maximum extent of the plastic zone from the tip occurs at roughly ±70° from the line ahead of the crack and for materials with $n > 3$ is given approximately by $.15J/(\sigma_0\varepsilon_0)$. Thus, by comparison with (8), it can be seen that the maximum extent of the plastic zone in small scale yielding is more than 25 times the minimum necessary dominant zone size for a typical yield strain of $\varepsilon_0 = .003$. Numerical analyses of the small scale yielding problem (Tracey, 1976; McMeeking, 1977) do indeed verify that the J-dominance condition (8) for the HRR-fields is satisfied for all n, including $n \to \infty$ corresponding to the elastic-perfectly plastic limit.

Under large scale yielding conditions the size of the zone of dominance R is a strong function of geometric configuration and also of strain hardening in certain cases. In the limit of perfect plasticity $(n \to \infty)$ the near-tip stresses and strains are not uniquely related to J, or any other single parameter, independent of configuration when large scale yielding occurs as emphasized by McClintock (1971). At one extreme where plastic flow is sufficiently constrained, the stress field at the tip of a Mode I plane strain crack is the well known Prandtl field with its high level of triaxial tension ahead of the tip. This is the field that prevails at the tip in small scale yielding and it is also the limit of the HRR fields (2) as $n \to \infty$. For geometries for which this high triaxiality is maintained, there is good reason to assume that J should adequately correlate the near-tip behavior of one such configuration with another and with small scale yielding. There is numerical and experimental evidence that plane strain configurations such as the edge-cracked strip in bending, the deeply double edge-cracked strip in tension and the standard compact tension specimen each maintains this high triaxaility level at the tip even under fully plastic conditions whether the material strain hardens or not.

In the case of the deeply-notched strip in bending (or the compact tension specimen) under fully plastic yielding, one can argue that the zone of dominance R must be some fraction of the length of the yielded ligament b, independent of loading. Numerical studies of Shih and German (1979) indicate that (approximately)

$$R \cong .07\, b \tag{9}$$

for the fully yielded bend-type specimen. This result is not strongly dependent on strain hardening. Combining (9) with condition (8) for J-dominance gives

$$b > 25 \frac{J}{\sigma_0} \tag{10}$$

This constraint on ligament size has been previously established for this type of configuration by testing specimens with varying ligament size. Note that (10) can also be interpreted as the requirement that b exceed about 40 times the crack opening displacement. Since the crack opening displacement at the initiation of growth in a tough steel can be, typically, .2 mm, the minimum ligament size for such a material with this specimen-type is about 1 cm.

At the other extreme are configurations, such as the center-cracked strip in plane strain tension, which lose the high triaxial state of stress ahead of the crack tip under fully plastic yielding. Accordingly, the near-tip fracture environment cannot be correlated with high triaxiality cases. The zone of dominance R of the HRR field goes to zero with zero strain hardening $(n \to \infty)$ for the fully plastic center-cracked strip under tension. Even in the presence of moderate strain hardening R will tend to be very small. Numerical studies of the fully yielded center-cracked strip by McMeeking and Parks (1979) and Shih and German (1979) indicate that (very approximately) $R \cong .01\,b$ and

$$b > 200 \frac{J}{\sigma_0} \tag{11}$$

when $n = 10$, where b is again the uncracked ligament length. This constraint severely restricts the use of J to correlate the large scale yielding of this type of configuration with the others mentioned above.

Suggestions have been made that two parameters -- J and a measure of near-tip triaxiality -- might suffice to characterize the full range of near-tip fracture environments, but nothing substantive along these lines has yet appeared. Most effort has been invested in tests on configurations which maintain the high crack-tip triaxiality associated with J-dominance since these conditions appear to be the most critical in that they lead to crack initiation and advance at the lowest levels of J for a given material, although some unpublished experiments by F. A. McClintock indicate that this may not always be the case. Further discussion of the issues related to J-dominance can be found in a recent article by Parks (1980).

<u>Quasi-Statically Growing Cracks under Monotonic Loading</u>

Paris and coworkers (1979) and Turner (1979) have applied J to analyze small amounts of quasi-static crack growth and to determine the point at which the quasi-static advance becomes dynamically unstable. The approach is analogous to the K-resistance curve analysis developed many years ago and includes the K-based analysis as a special case in the limit of small scale yielding. The rationale for using the deformation theory J to analyze quasi-static crack growth relies on the substantial tearing resistance of most ductile materials. Once crack advance has been initiated, additional advance requires positive increments of J. The resistance of the material to tearing is determined experimentally in the form of the J-resistance curve, $J_R(\Delta a)$, where Δa is the amount of crack advance.

Some of the toughest pressure vessel steels require a doubling of J above the initiation value, J_{Ic}, to advance the crack one or two millimeters, and J-values as much as 10 times J_{Ic} have been measured with standard plane strain test specimens. In what follows, it is useful to introduce the material-based length quantity D corresponding to the amount of crack advance needed to double J above J_{IC}. Estimated from the initial slope of the resistance curve, D is given by

$$\frac{1}{D} = \frac{1}{J_{Ic}}\left(\frac{dJ_R}{da}\right)_c \tag{12}$$

Hutchinson and Paris (1979) have discussed conditions which must be met for J-controlled growth. The conditions for dominance for the stationary crack mentioned in the previous subsection still apply. Additional restrictions must also be satisfied to ensure that the J-field (e.g. the HRR field) dominates behavior outside the immediate vicinity of the crack tip where elastic unloading and strongly nonproportional plastic deformations occur. These latter effects are not modeled by the deformation theory on which J is based. One restriction is that the amount of crack advance should not exceed the zone of dominance of the crack tip singularity field, i.e.

$$\Delta a < R \tag{13}$$

The second condition is less obvious. It guarantees that the zone of nonproportional plastic deformation is smaller than R and it takes the form

$$D < R \tag{14}$$

For a fully yielded, bend-type configuration under plane strain for which (9) applies, conditions (13) and (14) become

$$\frac{\Delta a}{b} < .07 \tag{15}$$

and

$$\frac{b}{J_{Ic}}\left(\frac{dJ_R}{da}\right)_c > 14 \tag{16}$$

The above should only be regarded as estimates. Test results do indicate that J-controlled growth tends to break down when (15) is exceeded (Shih, Delorenzi and Andrews, 1979), while (16) may overly restrict b.

The ability to analyze the stability of small amounts of crack growth using the resistance curve approach is of considerable importance for tough materials since large increases of J above J_{Ic} are possible, as has already been emphasized. The method is semi-empirical in that it is based on experimentally measured resistance data. While this approach may be less fundamental than one might wish from a scientific standpoint, the use of experimentally determined resistance data is a strength of the method from the vantage point of engineering application. At the same time, however, the method is quite limited. A more fundamental approach using a near-tip failure criterion is needed to deal with more extensive crack growth and with materials and configurations for which J-controlled growth does not pertain.

GROWING CRACKS: NEAR-TIP FIELDS AND STABLE CRACK ADVANCE IN ELASTIC-PERFECTLY PLASTIC SOLIDS

The main progress in the analysis of near-tip fields of growing cracks has been for elastic-perfectly plastic materials and our discussion will start with this work, first for Mode III and then for Mode I in plane strain.

Mode III

McClintock (1958) and McClintock and Irwin (1965) were the first to demonstrate that the source of stable crack growth is due to the nature of plastic flow. They analyzed the quasi-static growth of a crack in an elastic-perfectly plastic material with elastic shear modulus G, shearing yield stress τ_0 and shearing yield strain $\gamma_0 = \tau_0/G$. They considered Mode III deformations (anti-plane shear) under small scale yielding conditions and took the yield surface to be the Mises surface, which is identical to the Tresca surface in anti-plane shear. By imposing a simple near-tip failure criterion on the solution, they were able to predict the tearing resistance curve of the material for small scale yielding.

In a general increment of crack advance involving both an increase in crack length da and an increase in the extent of the plastic zone ahead of the tip, dr_p , the increment in the shear strain $d\gamma$ a distance r directly ahead of the tip in the plastic zone is

$$d\gamma = \gamma_0 r^{-1} dr_p + \gamma_0 r^{-1}[1 + \ln(r_p/r)]da \tag{17}$$

When no crack advance has occurred the strain ahead of the tip in the plastic zone from (17) is

$$\gamma = \gamma_0(r_p/r) \tag{18}$$

where the plastic zone size is $r_p = (2/\pi)J/(\tau_0\gamma_0)$ in small scale yielding. After extensive crack advance (i.e. more than several times the plastic zone size) the crack tip field reaches a steady-state in which the field appears unchanging to an observer moving with the tip. In the steady-state limit $dr_p = 0$ and (17) can be integrated to give

$$\gamma = \gamma_0[1 + \ln(r_p/r) + \frac{1}{2}\ln^2(r_p/r)] \tag{19}$$

A numerical analysis of the steady-state problem by Chitaley and McClintock (1971) gives essentially the same plastic zone size $r_p \cong (2/\pi)J(\tau_0\gamma_0)$ as in the stationary problem. Thus, comparison of (18) and (19) reveals a much stronger strain singularity for the stationary crack than for the steadily growing crack. This is a consequence of the irreversible nature of plastic flow. More specifically, it is a result of resistance to plastic flow experienced by a material element when the deformation involves changing relative proportions of the stress components (nonproportional stressing), as occurs when the advancing tip passes over or under a material element.

McClintock and Irwin (1965) imposed the condition that the initiation of growth and continued advance of the crack requires attainment of a critical shear strain γ_c a distance r_c ahead of the tip in the plastic zone. The quantities γ_c and r_c should be regarded as parameters which (hopefully) can be chosen to fit experimental data rather than precise measures of actual failure parameters. With J_c denoting the value of J at initiation of growth and J_{ss} the value required for steady-state growth, one finds from (18) and (19)

$$\frac{J_{ss}}{J_c} = \frac{1}{\lambda}\exp[\sqrt{2\lambda-1} - 1] \tag{20}$$

where $\lambda = \gamma_c/\gamma_0$. For $\lambda = 1$, $J_{ss} = J_c$; for $\lambda = 10$, $J_{ss} \doteq 2.9J_c$; and for $\lambda = 30$, $J_{ss} = 26.6J_c$. The potential for stable crack growth in a ductile material is considerable.

Equation (17), together with the fracture criterion, can be used to predict the entire resistance curve in small scale yielding assuming $r_p \cong (2/\pi)J/(\tau_0\gamma_0)$. Here we will only record the initial slope of the resistance curve for the first increment of crack advance expressed in terms of Paris's nondimensional tearing modulus appropriate for Mode III shearing:

$$T \equiv \frac{G}{\tau_0^2}\left(\frac{dJ_R}{da}\right)_c = \frac{\pi}{2}(\lambda - 1 - \ell n\ \lambda) \tag{21}$$

giving $T=0$ for $\lambda=1$, $T=10.5$ for $\lambda=10$, and $T=40.2$ for $\lambda=30$. The material-based length parameter (12) corresponding to the crack advance for a doubling of J above J_c is given by

$$D = r_c \frac{\lambda}{\lambda - 1 - \ell n\ \lambda} \tag{22}$$

For large λ , D is essentially r_c and even for $\lambda=3$ D is only about twice r_c .

Mode I, Plane Strain

Rice and Sorensen (1978) and Rice, Drugan and Sham (1980) have approached the Mode I, plane strain crack growth problem in the same spirit as was described for the Mode III problem. The analysis of the Mode I problem is considerably more difficult, however, and more reliance on numerical work is necessary. Furthermore, a sensible near-tip fracture criterion is less obvious in Mode I plane strain since the most intense stress conditions are ahead of the tip while the straining is most intense above and below the tip according to the small strain formulation. Rice and coworkers used the crack opening displacement δ a given distance r_c behind the tip as an integrated measure of the near-tip intensity.

Another feature which somewhat complicates the elastic-perfectly plastic Mode I analysis is the fact that there is a difference between the stationary and growing near-tip stress fields in Mode I, while in Mode III the stress state in the important region ahead of the tip is the same in both cases. The near-tip field for the advancing crack in an elastic-perfectly plastic solid in Mode I, plane strain has been worked out independently by Slepyan (1974), by Rice, Drugan and Sham (1980), and by Gao (1980). Fortunately for fracture analysts, the near-tip stress field for the growing crack is not radically different from the Prandtl field at the tip of the stationary crack. The main difference is the existence of two wedge regions of elastic unloading extending from about 115° to 163° above and below the tip. Outside this elastic unloading wedge the stresses are within a few percent of those of the Prandtl field.

The approach of Rice and coworkers applies to large scale yielding as long as the high triaxiality of the Prandtl field and that of the modified Prandtl field is maintained. Here we will limit discussion to small scale yielding situations. The material is elastic-perfectly plastic with Young's modulus E , tensile yield stress σ_0 , Poisson's ratio of .3 and a Mises yield surface. The increment in crack opening displacement a small distance r behind the tip is related to simultaneous increments in J and a by

$$d\delta = \alpha \frac{dJ}{\sigma_0} + \beta \frac{\sigma_0}{E} da\ \ell n\left(\frac{r_0}{r}\right) \tag{23a}$$

where

$$\alpha = .65 \quad , \quad \beta = 5.08 \quad \text{and} \quad r_0 = .23\,\frac{EJ}{\sigma_0^2} \tag{23b}$$

A numerical solution for the growing crack was used to obtain α and the numerical coefficient .23 in the expression for r_0 by a fit to (23a), while β was derived from the modified Prandtl field. The length quantity r_0 is slightly larger than the small scale yielding plastic zone size.

For the stationary crack (23) gives

$$\delta = \alpha J/\sigma_0 \tag{24}$$

independent of sufficiently small r. At steady-state when $dJ = 0$, (23) integrates to

$$\delta = \beta\,\frac{\sigma_0}{E}\,r\left[1 + \ln\left(\frac{r_0}{r}\right)\right] \tag{25}$$

Imposition of the fracture condition $\delta = \delta_c$ at $r = r_c$ using (24) and (25) permits one to solve for the ratio

$$\frac{J_{ss}}{J_c} = \frac{1.04}{\lambda}\,\exp(.197\lambda) \tag{26a}$$

where now

$$\lambda = \frac{\delta_c}{r_c}\,\frac{E}{\sigma_0} \tag{26b}$$

As in the case of Mode III, (23) permits the calculation of the entire small scale yielding resistance curve. The initial nondimensional slope of the resistance curve is

$$T \equiv \frac{E}{\sigma_0^2}\left(\frac{dJ_R}{da}\right)_c = 1.53\lambda - 7.8\,\ln(.96\lambda) \tag{27}$$

and the amount of crack advance needed to double J above J_{1c} from (12) is

$$D = r_c\,\frac{\lambda}{\lambda - 5.08\,\ln(.96\lambda)} \tag{28}$$

If λ is regarded as a parameter to be chosen to fit experimental data for the initial tearing modulus (27), then for moderately tough to tough materials λ would fall in the range from about 20 to 150 corresponding to T-values between about 7 and 200 which have been tabulated by Paris and coworkers (1979). For this same range of λ, J_{ss}/J_{1c} from (26a) varies from about 3 to more than 10^{10}. For high T, $D \cong r_c$, as in Mode III.

From the above it is seen that extensive stable crack growth is to be expected under plane strain conditions in ductile metals. Rice and coworkers have also discussed the relation between growth under small scale yielding to that under large scale yielding for some special configurations. For elastic-perfectly plastic materials under large scale yielding they find that J-controlled growth is never strictly achieved. However, for sufficiently high tearing resistance they find rather minimal configurational dependence.

The possibility of the exceptionally large ratios J_{ss}/J_{Ic} from (26a) or from (20) as reflected by the exponential dependence on λ raises the question of the sensitivity of these predictions to the material model on which they are based. This exponential dependence comes about because of the logarithmic dependence on r in the terms associated with crack advance in (17) and (23a), and these terms, in turn, are tied to the elastic-perfectly plastic solid with the smooth Mises yield surface. Effects of strain hardening and yield surface corner formation on steady-state crack growth have been investigated using a specialized finite element procedure by Dean and Hutchinson (1980) and Parks, Lam and McMeeking (1981), and these effects will be discussed in the next section. Numerical studies of the steady-state problem have been carried out recently by a number of investigators. In addition to the two studies just mentioned, Nguyen and Rahimian (1980) considered a semi-infinite crack propagating steadily in an elastic-perfectly plastic strip of constant width, and Andersson (1974) analyzed the small scale yielding, Mode I plane stress problem, as well as the Mode III problem. Douglas, Freund and Parks (1981) considered dynamic steady-state growth in Mode III small scale yielding in an elastic-perfectly plastic material by taking into account inertial resistance to motion. They present results for the effect of crack tip speed on the plastic zone, the strains ahead of the tip, and the crack opening displacement. Aboudi and Achenbach (1980) investigated the transient approach to steady-state in Mode III for a crack running dynamically in a visco-plastic work-hardening material.

GROWING CRACKS: NEAR-TIP FIELDS AND STABLE CRACK ADVANCE IN HARDENING SOLIDS

Numerical Studies of Steady-State Crack Growth in Hardening Solids

Dean and Hutchinson (1980) considered Mode I, plane strain steady crack growth in a material with a Mises yield surface on a piecewise power-hardening tensile stress-strain curve of the form

$$\begin{aligned} \varepsilon/\varepsilon_0 &= (\sigma/\sigma_0) \qquad & \sigma \le \sigma_0 \\ &= (\sigma/\sigma_0)^n \qquad & \sigma > \sigma_0 \end{aligned} \tag{29}$$

where $\varepsilon_0 = \sigma_0/E$. They numerically calculated the crack opening displacement $\delta(r)$ behind the tip. By imposing the growth condition $\delta(r_c) = \delta_c$ on the steady-state solution and on the stationary problem (24), they calculated J_{ss}/J_{Ic} for $10 \le \lambda \le 35$ for $n = 3$, 10 and ∞ , where λ is defined by (26b). The results for the elastically-perfectly plastic solid $(n = \infty)$ are essentially identical to (26a), while the results for the hardening solids are greatly reduced below (26a). For example, for $\lambda = 30$, $J_{ss}/J_{Ic} = 13.5$ for $n = \infty$, while this same ratio is approximately 7 for $n = 10$ and 3.5 for $n = 3$. The trends indicate even greater divergence for larger λ , with similar trends in Mode III. Strain hardening appears to be of major importance in the prediction of crack advance using a near-tip criterion. Resistance curve predictions based on elastic-perfectly plastic calculations may well be highly unconservative.

In Mode III the effect of corner development on the yield surface of the solid did not appear to be nearly as influential as strain hardening. However, Parks, Lam and McMeeking (1981) found more noticeable differences in Mode I plane strain between the solution for the corner theory and the corresponding solution for the Mises solid. In particular, the level of triaxiality attained ahead of the tip for the corner theory is well below that associated with the smooth yield surface, i.e. with the Prandtl field. No attempt has yet been made to quantitatively assess the effect of this difference on stable crack growth.

The near-tip fracture criteria which have been used in the above mentioned studies are crude and, at best, can be expected to only approximately provide a measure of criticality of the complicated fracture processes taking place near the tip of the crack. The strain distribution at the tip of the growing crack is very different from that for the stationary crack, and this may mean that no such single criterion using a critical intensity such as γ_c or δ_c at r_c should be expected to give a unified criterion for both initiation and continuation of crack advance. A more direct coupling of details of the fracture process and the near-tip continuum theory stress and strain fields may be essential. Thus, it remains to be seen if it will be possible to reproduce experimentally determined resistance curves using a single criterion such as that described above. A first attempt along these lines by Hermann and Rice (1980) on a high strength steel having $J_{ss}/J_c \doteq 2$ does hold out promise for this approach.

Singular Fields for Steady-State Crack Growth in Hardening Solids

Further development of the approach surveyed above will require descriptions of the singularity fields for cracks growing in strain hardening materials. At present our knowledge of these fields is incomplete. Results as general as (17) or (23a) for the elastic-perfectly plastic transient problem are not available for strain hardening solids, although some results for the steady-state problem have been found and these will now be discussed.

For a linear hardening solid with Young's modulus E and tangent modulus E_t following yields, Amazigo and Hutchinson (1977) have shown that the dominant singularity fields at the tip of the steadily growing crack are of the form

$$\sigma_{ij} = A\sigma_0 r^{-q}\tilde{\sigma}_{ij}(\theta) \quad \text{and} \quad \dot{\varepsilon}_{ij} = A\varepsilon_0 r^{-q-1}\dot{\tilde{\varepsilon}}_{ij}(\theta) \tag{30}$$

where σ_0 and ε_0 are the yield stress and strain and A is a free amplitude factor. The strength of the singularity q and the θ-variations are functions of E_t/E and these have been obtained for Mode III and for Mode I in both plane strain and plane stress. The strength of the singularity q vanishes as $E_t/E \to 0$, corresponding to the elastic-perfectly plastic limit, and q increases sharply for small E_t/E, suggesting a strong dependence on strain hardening. Similar features have been found by Lo and Peirce (1980) in their investigation of the Mode III singularity fields for a linear hardening solid which develops a corner at the loading point of its yield surface.

In solving for the details of the fields (30), Amazigo and Hutchinson neglected plastic reloading along the flank behind the crack tip. The reloading zone is of little consequence in Mode III, but recent work mentioned above on the near-tip elastic-perfectly plastic behavior in Mode I indicates that the reloading zone is an important feature of the plane strain problem. Thus, the solutions for this case should be redone to account for reloading before they can be used in any application.

Achenbach and Kanninen (1978) have extended the analysis for the linear hardening material to include inertia. They find that in Mode III the dynamical effects have rather small influence on the strength of singularity in (30) or on the θ-variations of the fields. Further work on singularity fields for dynamic crack growth in elastic-plastic solids has been carried out by Achenbach and Dunayevsky (1980) and Achenbach, Kanninen and Popelar (1980).

The fields for a linear hardening material appear to be of limited usefulness for several reasons. Linear hardening is generally a fairly poor characterization of actual hardening behavior. Furthermore, in the limit as $E_t/E \rightarrow 0$, the fields (30) for the bilinear material do not appear to approach the solution for the elastic-perfectly plastic solid derived earlier, although this has not been established with certainty. Closely related is the question of the size of the region of validity, or dominance, of (30). It is possible, although again not certain, that the region of dominance of (30) vanishes in the limit of zero strain hardening.

Singularity fields for a crack growing in a more realistic strain hardening solid such as power-hardening (29) have been most elusive. However, in work to be reported at ICF5 Gao and Hwang (1981) propose a new form for the singularity fields in a power-hardening solid.

An essential feature of near-tip behavior of an advancing crack is a balance involving both elastic and plastic strain-rate as the tip is approached. The plastic strain-rates by themselves are incompatible without an elastic strain-rate contribution. This is the case for growth in elastic-perfectly plastic solids as emphasized by Rice (1973). It is also true for the bilinear hardening solutions and it is the key to the discovery by Hui and Riedel (1980) of the fields for creep crack growth discussed in the last section of this paper. By contrast, the elastic strains play no role in the determination of the dominant singularity behavior for the stationary crack, as, for example, in the HRR-field. A balance between elastic and plastic strain-rates also appears to be a feature of the singularity fields for a crack growing into a power-hardening material. Efforts to produce singularity fields under the assumption the elastic strain-rates are asymptotically negligible in the yielded zone have not been successful (Amazigo and Hutchinson, unpublished work; Gao and Hwang, 1981). With elastic strain-rates neglected in the yielded zone the equations governing near-tip behavior admit solutions of the form

$$\sigma_{ij} = Ar^{-q}\tilde{\sigma}_{ij}(\theta) \tag{31}$$

However, the resulting solutions do not permit one to construct the entire near-tip field.

Gao and Hwang (1981) considered another possible form for the singular fields which does involve an interaction between the elastic and plastic strain-rates in the actively yielding region as the tip is approached. They investigated Mode I, plane strain and gave most detail for an incompressible material. In their solution the near-tip expansion for the stresses is of the form

$$\sigma_{ij} = \left(\ln \frac{A}{r}\right)^{p} \left\{ \tilde{\sigma}_{ij}^{(0)}(\theta) + \left(\ln \frac{A}{r}\right)^{-1} \tilde{\sigma}_{ij}^{(1)}(\theta) + \ldots \right\} \tag{32}$$

where A is the amplitude factor with length dimensions. The exponent p can be chosen to give the correct balance of elastic and plastic strain-rates in the compatibility equation; in Mode I it is given by

$$p = \frac{1}{n-1} \quad (33)$$

Assuming the form of the solution to be correct, Gao and Hwang showed that the lowest order θ-variation, $\tilde{\sigma}_{ij}^{(0)}$, is necessarily precisely the same as for the elastic-perfectly plastic limit discussed earlier. Thus, the Gao-Hwang solution has the desirable feature that the dominant singular stress field approaches the elastic-perfectly plastic limit uniformly as n becomes large.

There are still some technical difficulties with the Gao-Hwang solution which will have to be resolved, or at least better understood, before the solution can be considered to be correct. One difficulty is that the plastic part of the strain-rate does not vanish as θ approaches the boundary between plastic loading and elastic unloading. While this is not necessary for the elastic-perfectly plastic problem, it is a requirement when hardening is present. Gao and Hwang suggest that this condition need not be met asymptotically as the tip is approached. If they are correct, this may mean that the domain of validity of (32) is very small. Hopefully, further clarification of this and other details of the Gao-Hwang solution will soon be forthcoming.

SINGULARITY FIELDS FOR CRACKS IN CREEPING SOLIDS

In uniaxial tension the strain-rate of a power-law creeping solid can be expressed as

$$\dot{\varepsilon} = \dot{\sigma}/E + \dot{\varepsilon}_0(\sigma/\sigma_0)^n \quad (34)$$

where σ_0 is a reference stress and $\dot{\varepsilon}_0$ is a reference strain-rate. The creeping part of (34) is customarily generalized to multiaxial stress states using the deviatoric stress and the Mises invariant. For a stationary crack in such an elastic-power-law creeping material, the near-tip fields are again given by the HRR-singularity (2)-(4) where strain and displacement quantities become strain-rates and velocities, respectively (with $\alpha = 1$). A body subject to a (quasi-static) step loading at $t = 0$ undergoes a transient period in which both elastic and creep strain-rates occur and in which the creep zone at the tip of the crack grows in size. For short times when the creep zone is small, the amplitude of the HRR field can be related to the elastic stress intensity factor. After a sufficiently long hold time, the body approaches a steady situation in which the stresses cease changing so that the entire straining is then due to creep. Under these limiting conditions a path independent, J-type integral, now designated by C^* , can again be defined. Landes and Begley (1976) and Nikbin, Webster and Turner (1976) have been instrumental in developing the experimental approach based on C^* . Estimates of C^* for power-law materials are being made available by Kumer and Shih (1980). Riedel and Rice (1980) and Bassani and McClintock (1980) have studied the transient problem for the stationary crack and have obtained estimates of the time needed for the near-tip fields to settle down to steady creep and for the amplitude of the HRR field in the transient period.

Hui and Riedel (1980) have determined the singularity fields for a growing crack in the elastic-power-law creeping solid (34). They have presented results for Mode III and Mode I in plane stress and plane strain for steady-state crack growth, and they have shown that these results apply sufficiently close to the tip under transient creep cracking. In arriving at their solution, Hui and Riedel show that when $n > 3$ there must be a balance between elastic and creep strain-rates as the tip is approached. If one assumes that the elastic strain-rates are negligible then the stresses are necessarily just those of the HRR field, but this leads to a contradiction since the elastic strain-rates derived from the HRR stress field are more singular than the creep strain-rates derived under this assumption. Conversely, if it is assumed that the creep strain-rates are negligible as the tip is approached, then the singular stress field is just the well known elastic singularity field with the $1/\sqrt{r}$ singularity in stresses. But this assumption also leads to a contradiction when $n > 3$ since it gives more singular creep-strain-rates than elastic strain-rates. Hui and Riedel show that equations for steady-state growth admit separated solutions of the form

$$\sigma_{ij} = c_n \left(\frac{\dot{a}}{r}\right)^{\frac{1}{n-1}} \tilde{\sigma}_{ij}(\theta,n) \qquad (35)$$

for $n > 3$ where $\dot{a}$ is the velocity of the crack tip and c_n is a dimensional constant which is fully determined by the singularity analysis. The exponent of the r-dependence in (35) is the only choice which leads to comparable singularities in the elastic and plastic strain-rates. For $n < 3$ the near tip stress field is that of the elastic singularity field and the elastic strain-rates do dominate the creep strain-rates, as has also been discussed by Hart (1980).

The crack tip velocity $\dot{a}$ plays the role of the amplitude factor of the near-tip fields (35) in creep crack growth. Hui and Riedel discuss the fact that in small scale yielding, for example, this velocity $\dot{a}$ can be prescribed independently of the far field stress intensity factor K in the theoretical problem. Only when some criterion for creep crack advancement is imposed on the solution do $\dot{a}$ and K become related. Work by Hui (unpublished) indicates that the singular fields are dominate over approximately one third of the nominal creep zone in Mode I plane strain small scale yielding.

ACKNOWLEDGEMENT

This work was supported in part by the National Science Foundation under Grant ENG78-10756, and by the Division of Applied Sciences, Harvard University.

REFERENCES

Aboudi, J. and J. D. Achenbach (1980). Rapid Mode III crack propagation in a strip of viscoplastic work-hardening material. To be published in Int. J. Solids Structures.

Achenbach, J. D. and M. F. Kanninen (1978). Crack tip plasticity in dynamic fracture mechanics. In N. Perrone, et al (Eds.), Fracture Mechanics, The University of West Virginia Press. pp. 649-670.

Achenbach, J. D. and V. Dunayevsky (1980). Fields near a rapidly propagating crack tip in an elastic-perfectly plastic material. Northwestern University Report N.U.-SML TR#80-1.

Achenbach, J. D., M. F. Kanninen and C. H. Popelar. Near-tip fields for fast fracture in an elastic-plastic material. To be published in J. Mech. Phys. Solids.

Amazigo, J. C. and J. W. Hutchinson (1977). J. Mech. Phys. Solids, 25, 81-97.

Anderssen, H. (1974). J. Mech. Phys. Solids, 22, 285-308.

Bassani, J. L. and F. A. McClintock (1980). Creep relaxation of stresses around a crack tip. To be published in Int. J. Solids Structures.

Chitaley, A. D. and F. A. McClintock (1971). J. Mech. Phys. Solids, 19, 147.

Dean, R. H. and J. W. Hutchinson (1980). In Fracture Mechanics, A.S.T.M. STP 700. pp. 383-405.

Douglas, A. S., L. B. Freund and D. M. Parks (1981). Dynamic steady antiplane shear crack growth in an elastic plastic material. To be presented at ICF5, Cannes.
Gao, Y-C. (1980). Acta Mechanica Sinica, 1.
Gao, Y-C. and K-C. Hwang (1981). Elastic-plastic fields in steady crack growth in a strain-hardening material. To be presented at ICF5, Cannes.
Hart, E. (1980). Int. J. Solids Structures, 16, 807.
Hermann, L. and J. R. Rice (1980). Comparison of experiment and theory for elastic-plastic plane strain crack growth. Brown University, Division of Engineering Report, February 1980.
Hui, C. Y. and H. Riedel (1980). The asymptotic stress and strain field near the tip of a growing crack under creep conditions. To be published in Int. J. of Fracture.
Hutchinson, J. W. (1968). J. Mech. Phys. Solids, 16, 13-31, 337-347.
Hutchinson, J. W. and P. C. Paris (1979). In Elastic-Plastic Fracture, A.S.T.M. STP 668. pp. 37-64.
Kumar, V. and C. F. Shih (1980). In Fracture Mechanics, A.S.T.M. STP 700, 406-438.
Landes, J. D. and J. A. Begley (1976). In Mechanics of Growth, A.S.T.M. STP 590, 128-148.
Lo, K. K. and D. Peirce (1981). The effect of yield surface vertex on crack tip fields in Mode III. To be published in J. Mech. Phys. Solids.
McClintock, F. A. (1958). J. Appl. Mech., 25, 581.
McClintock, F. A. and G. R. Irwin (1965). A.S.T.M. STP 381. pp. 84-113.
McClintock, F. A. (1971). In H. Liebowitz (Ed.), Fracture, Vol. 3, Academic Press, New York. pp. 47-225.
McMeeking, R. M. (1977). J. Mech. Phys. Solids, 25, 357-381.
McMeeking, R. M. and D. M. Parks (1979). In Elastic-Plastic Fracture, A.S.T.M. STP 668. pp. 175-194.
Nguyen, Q. S. and M. Rahimian (1980). Mouvement Permanent d'une Fissure. To appear in J. de Mecanique Appliquée.
Nikbin, K. M., G. A. Webster and C. E. Turner (1976). In Cracks and Fracture. A.S.T.M. STP 601. pp. 47-62.
Paris, P. C., H. Tada, Z. Zahoor and H. Ernst (1979). In Elastic-Plastic Fracture, A.S.T.M. STP 668. pp. 5-36, 251-265.
Parks, D. M. (1980). The dominance of the crack tip fields of inelastic continuum mechanics. Presented at 2nd Int. Conf. on Numerical Methods in Fracture Mechanics, Swansea, July 1980.
Parks, D. M., P. S. Lam and R. M. McMeeking (1981). Some effects of inelastic constitutive models on crack tip fields in steady quasi-static growth. To be presented at ICF5, Cannes.
Rice, J. R. (1968). J. Appl. Mech., 35, 379-386.
Rice, J. R. and G. Rosengren (1968). J. Mech. Phys. Solids, 16, 1-12.
Rice, J. R. (1973). In Mechanics and Mechanisms of Crack Growth, British Steel Corp.
Rice, J. R. and E. P. Sorensen (1978). J. Mech. Phys. Solids, 26, 163.
Rice, J. R., W. J. Drugan and T-L. Sham (1980). In Fracture Mechanics, A.S.T.M. STP 700. pp. 189-219.
Riedel, H. and J. R. Rice (1980). In Fracture Mechanics, A.S.T.M. STP 700. pp. 112-130.
Shih, C. F. (1979). Relationships between the J-integral and the crack opening displacement for stationary and growing cracks. General Electric Company TIS Report No. 79CRD075. To appear in J. Mech. Phys. Solids.
Shih, C. F., H. G. Delorenzi and W. R. Andrews (1979). In Elastic-Plastic Fracture, A.S.T.M. STP 668. pp. 65-120.
Shih, C. F. and M. D. German (1979). Requirements for a one parameter characterization of crack tip fields by the HRR singularity. General Electric Company TIS Report No. 79CRD076. To appear in Int. J. Fracture.
Slepyan, L. I. (1974). Izv. AN SSSR. Mekhanika Tverdogo Tela, 9, 57-67.

Sternberg, E. (1979). In Trends in Solid Mechanics 1979, Delft University Press and Sijthoff and Noordhoff, Delft. pp. 225-236.
Tracey, D. M. (1976). J. Eng. Mat. and Technology, 98, 146.
Turner, C. E. (1979). In D. G. H. Latzko (Ed.), Post Yield Fracture Mechanics, Chap. 2, Applied Science, London.