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A Remarkable Eye for Out-of-the-Ordinary Mathematics

By Gilbert Strang

Maha is a very remarkable person. He studies physical phenomena that any of us might see (but he sees them more deeply). Three examples are the wrinkles in our skin, the flutter of a flag, and the folds in a skirt or sari or tablecloth. The physics of these "soft materials" combines geometry, mechanics, and optimization. When you crumple a piece of paper, all the hard geometric problems are there.

What controls the way a tablecloth hangs? A tablecloth is a thin sheet, pulled down by gravity where it hangs over the edges of the table. At the same time, it has bending energy from the folds that form at the corners. In the competition between elasticity and gravity, the shape with minimum energy has highly localized (and oscillatory) bending. Most of the surface is nearly flat, or cylindrical or conical, but the folds display high gradients (large strains).

We see a piece of fabric, a couturier beautifies it, and Maha analyzes it.

Maybe I should be more formal, but not for long. L. Mahadevan is a professor in the Division of Engineering and Applied Sciences at Harvard University, where he also has affiliate appointments in the Department of Organismic and Evolutionary Biology, and the Department of Systems Biology (in the Harvard Medical School). His path to Harvard began in Chennai, at the Indian Institute of Technology, and passed through Stanford (PhD with Joe Keller), followed by positions at MIT and Cambridge University. He describes working with Keller, which wasn't planned when he went to Stanford, as "the second-best thing that ever happened to me." Maha will give an invited lecture at this summer's SIAM Annual Meeting in New Orleans. And he is inspiring to talk to, which is the reason for this article.

Draping study for a seated figure, Leonardo da Vinci (1452–1519). For L. Mahadevan, an invited speaker at this summer's SIAM Annual Meeting in New Orleans ("Draping, Wrinkling, and Crumpling: Geometry and Physics"), the intricate folding so beautifully drawn by the artist provided an early research problem—one that combines geometry, mechanics, and optimization.

The tablecloth is a static example, but Maha's interests are increasingly dynamic and biological. Singularities—when nature focuses energy into rupture, or explosion, or sudden motion—are a key. Something happens that is out of the ordinary. The mathematics is not smooth.

The Venus flytrap is an amazing example of rapid motion. A plant catches bugs that are too quick for us! In a letter to *Nature* (January 27, 2005) based on the study of high-speed images, Maha and his co-authors explain how a snap-buckling instability causes the leaf to close in a tenth of a second. The plant varies its Gaussian curvature, the authors write, and this "nastic motion" suddenly transforms the leaf from convex to concave. "The macroscopic mechanism of closure is determined solely by leaf geometry." The bug ends up inside.

Maha looks for the connections between shape and purpose. How does structure affect function, and how does function determine structure? One metaphor is a lock and key: The pieces fit. For soft materials another metaphor is preferred: A glove shapes itself to the wearer's hand.

The geometry of a biological material is crucial. Membranes are thin, filaments are long, blobs are useless. The only way to have function is to have form. The shape evolves to achieve the purpose. A long slender structure can bend, twist, shake, shear, and stretch. Its form produces a stability problem in applied mathematics, as chemical energy is converted to mechanical energy, and vice versa.

Think of DNA, which resembles a thin rod that loops and twists dynamically as it interacts with proteins. This thin-rod approximation connects elasticity theory to key problems of biology. The DNA loop can stay at the continuum level, while the protein is studied by molecular dynamics. Computing the interaction of 700,000 atoms is vastly accelerated.

The deformation of a thin rod leads to familiar problems in computational science. Maha contributed to early results of Klaus Schulten's group at the University of Illinois, reported in SIAM's *Multiscale Modeling and Simulation*: www.siam.org/journals/mms/2-4/60478.html. The authors used spline collocation and continuation methods to understand how the DNA responds mechanically (they suppressed its molecular dynamics). The binding of the protein supplies the crucial boundary conditions at the ends of the "worm-like chain" of DNA.

In talking to Maha, I was struck by his successful but unspoken strategy in moving from mechanics and physics to biology and medicine. He hasn't really moved! He just "bent" a little, to change function. His problems remain macroscopic, not microscopic. They still begin with observation, not with computation. He had to learn about constraints from chemists and biologists (Harvard provided a lab). I think that his work offers a truly helpful model for an applied mathematician who knows mechanics and wants to work in biology.

My best suggestion is simple: Read one of Maha's papers (http://www.deas.harvard.edu/softmat/). And if you are in New Orleans at 9:15 on July 13, go to a good talk.

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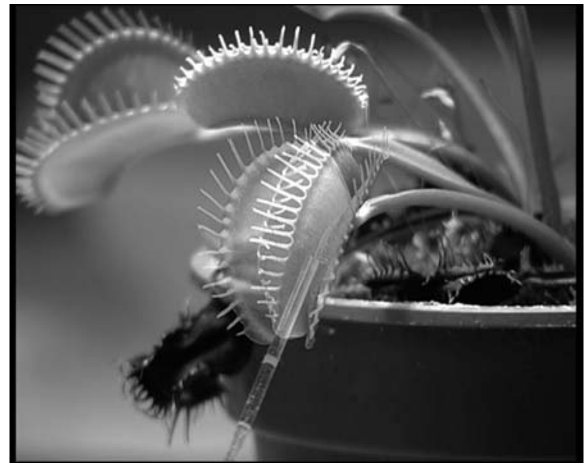
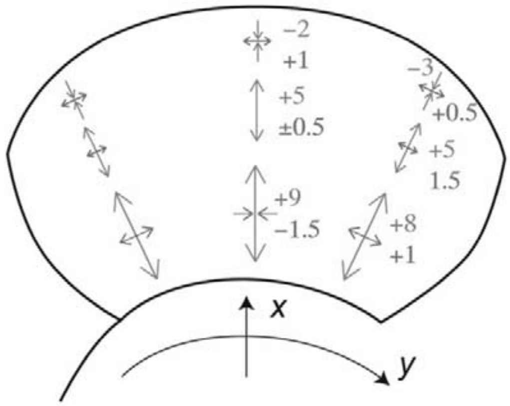
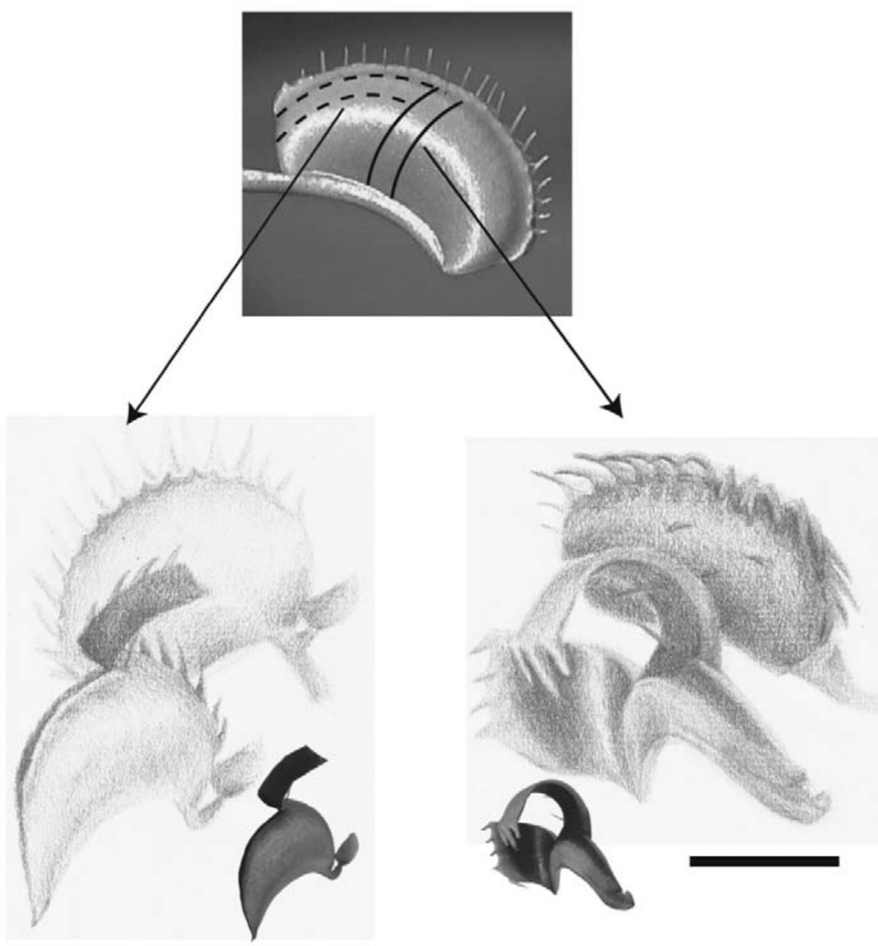


Superposition of open and closed leaves of the Venus flytrap, an example of Mahadevan's increasingly dynamic and biological interests. The glass needle in the foreground was used to trigger the closure. From Nature, *January 27, 2005; image courtesy of Yoël Forterre and L. Mahadevan.*





Illustrations of the Venus flytrap, showing the changes in shape associated with closure of the leaf. Cuts in a leaf, parallel and perpendicular to the midrib, show that the snapping is driven only by curvature changes in the perpendicular direction. Images created by and courtesy of Farrah Shindler.