**Supplemental Material**

**Two-Path Solid-State Interferometry Using Ultra-Subwavelength Two-Dimensional Plasmonic Waves**

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1. **Materials and fabrication**

Before device fabrication, the GaAs/AlGaAs 2DEG at 4.2 K has a carrier density, *n*, of 1.54 × 1011 cm-2 and a mobility, *μ*, of 2.5 × 106 cm-2/Vs in the dark. Under illumination, *n* = 2.8 × 1011 cm-2 and *μ* = 3.9 × 106 cm-2/Vs. At room temperature, *n* = 3.76 × 1011 cm-2 and *μ* = 3.66 × 103 cm-2/Vs in the dark. We etch the GaAs/AlGaAs sample by > 80 nm from the top surface to define the two 2DEG mesa paths of the interferometer (above the 2DEG, there is a 48 nm undoped Al0.3Ga0.7As layer, a 26 nm Si-doped Al0.3Ga0.7As layer, and a 6 nm GaAs cap). The ohmic contacts are created by thermally evaporating layers of Ni (5 nm), Au (20 nm), Ge (25 nm), Au (10 nm), Ni (5 nm), and Au (40 nm), followed by annealing at 420ºC for 50 seconds. The CPWs and the top gate are deposited by thermal evaporation of Cr (8 nm) and Au (500 nm).

We use the Sonnet electromagnetic field solver to design the 50- CPWs10.The signal lines have a width of 24 μm and a length of 218 μm and are separated by 15 μm from the 123 μm-wide ground lines on each side. Each ohmic contact occupies an area of 6×24 μm2. The separations between the top gate and the CPW signal lines on the left and right sides of the interferometer are 7 μm [Fig. 1]. These ungated 2DEG paths are far shorter than the gated 2DEG paths. This is to minimize the nonlinear dispersion effect of the ungated 2DEG regions.

1. **Calibration and de-embedding in *s*-parameter measurements**

The measurements are done inside a Lakeshore cryogenic probe station. The Agilent E8364A vector network analyzer generates an *ac* signal up to 50 GHz with -45 dBm power reaching the device via ground-signal-ground microwave probes (100-μm pitch) and coaxial cables. The network analyzer, cables, and probes all have a characteristic impedance of 50 Ω. The network analyzer measures *s*-parameters. The delay and loss from the network analyzer up to the probe tips are calibrated out by using the NIST-style multi-line TRL calibration methodS1; this procedure involves measuring a set of *s*-parameters for CPWs of varying lengths fabricated on a separate GaAs substrate with no 2DEG presentS2, S3. This calibration leads to the raw *s*-parameters, which include the effects of the interferometer, the phase delays through the on-chip CPWs, and the direct parasitic coupling between the two on-chip CPWs, which bypass the interferometer. The phase delays of the electromagnetic waves traveling through the CPWs are far smaller than the phase delays of the much slower plasmonic waves traveling through the interferometer, so we safely ignore the CPW phase delays. The effect of the parasitic coupling is separately measured by applying a gate bias of -0.4 V, thus depleting the 2DEG to imitate an open circuit [Fig. S1]. These open-device *s*-parameters are then de-embedded from the interferometer’s raw *s*-parametersS4. Figure S1 juxtaposes the raw and the de-embedded *s*-parameters at an example bias at 0.55 V. All the *s*-parameters discussed in the main text, except those in Fig. 4, are the de-embedded *s*-parameters.

**Figa1_deembed.jpg**

**Fig. S1:** (a) Magnitudes of raw *s*21 (red, dashed), de-embedded *s*21 (black, solid), and open-device (blue semi-dashed) *s*21 at 4.2 K with a gate bias of 0.55 V. (b) Phases of the three *s*21 parameters.

1. **Extracted *A*2/*A*1 and **

Figure S2(a) shows the *A*2/*A*1 ratios extracted from the *s*21 magnitudes following the prescription given in the main text. Since, we can then extract the attenuation constant, *α* [Fig. S2(b)]. The median values of *α*are around 8000 m-1 at gate biases in excess of 0.4 V, where ohmic contact effects are less pronounced in the *s*21 magnitudes.

a ratios.tif

**Fig. S2:** (a) *A*2/*A*1 extracted from the *s*21 magnitude. (b) *α*extracted from *A*2/*A*1 of part (a).

**D. On characteristic impedance, *Z*, of the plasmonic transmission lines and on the factor 4*Z*0*Z*/(*Z*+2*Z*0)2 of Eq. (1)**

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**Fig. S4:** (a) Extracted *L*k. (b) Extracted *n*. (c) Extracted *R*. (d) Extracted **.

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**Fig. A3:** The circuit model of the gated 2D plasmonic transmission line.

Both gated 2D plasmonic transmission lines of the interferometer have an identical 2DEG width, *w*, thus the same characteristic impedance, *Z*. The gated 2D plasmonic line can be modeled as a distributed ladder network of kinetic inductors and capacitorsS2, S5 [Fig. S3]; *Lk* = *m\**/*ne2w* is the kinetic inductance per unit lengthS2, S3, S5, *C* = *κε0w*/*d* is the capacitance per unit length, and *d*z is an infinitesimal segment of the line. The ohmic resistance per unit length, *R* = 1*/*(*neµw*), in series with *Lk* stems from the electron scatterings. The characteristic impedance, *Z*, of the plasmonic line is then given by, where *Q* = *L*k/*R* = *ωτ* is the quality factor. To evaluate *Z*, we should first know the values for *L*k and *R* (on the other hand, *C* = *κε0w*/*d* ~ 1.14×10-8 F/m can be readily evaluated using the known geometric parameters). By using *v*p values extracted from the *s*21 phase [Fig. 3] in *v*p = 1/√(*L*k*C*), we extract *L*k values [Fig. S4(a)]. By noting thatS6 *Q* = *ωL*k/*R* on the one hand and *Q* ~ *β/*2*α =ω*/(2*v*p*α*) on the other hand, we can write *R* = 2*αv*p*L*k; then by using the extracted values of *α* [Fig. S2(b)], *v*p [Fig. 3], and *L*k [Fig. S4(a)] in this formula, we can extract *R* [Fig. S4(c)]. With the extracted *R* and *L*k values, we can evaluate. As *Q* = *ωL*k/*R* ~ 1 occurs below 5 GHz, the imaginary part of *Z* becomes increasingly small as frequency rises. Moreover, by substituting this *Z* into *F* ≡ 4*Z*0*Z*/(*Z*+2*Z*0)2 of Eq. (1), we find that *F* itself has a negligible imaginary part as compared to its almost constant real part over nearly all measurement frequency range [Fig. S5(a)]. That is, *F* has a negligible phase [Fig. S5(b)] and a constant magnitude. This is self-consistent with our *v*p extraction from *s*21 phase, where we ignored the phase of *F*.

F.tif

**Fig. S5:** (a) Real (black, solid) and imaginary (red, dashed) parts of *F*. (b) Phase of *F*. Bias: 0.55 V.

The extracted *L*k and *R* values also allow us to extract the electron density, *n*, and the mobility, *µ* Fig. S4(b) and Fig. S4(d)]. The median values of the extracted *µ* lie between 1 × 106 ~ 1.5 × 106 cm-2/Vs, which are comparable to, but justifiably smaller due to fabrication steps than the mobility of the pristine sample (2.5 × 106 cm-2/Vs).

1. **Derivation of Eq. (1)**

Figure S6(a) illustrates our interferometer consisting of two plasmonic transmission lines ‘1’ and ‘2’ along with the two on-chip CPWs ‘A’ and ‘B’. When an electromagnetic wave is launched onto CPW A, multiple transmissions and reflections will occur at the two junctions in the figure, and multiple waves will appear at CPW B through many different signal pathways. Superposition of these multiple waves represents the total transmitted wave. There are two 1st-order signal pathways from CPW A to CPW B that exhibit the lowest degree of loss: A→1→B and A→2→B. There are eight 2nd-order signal pathways from CPW A to CPW B that exhibit the second lowest degree of loss (in what follows, a path number with no prime signifies left-to-right propagation on that path, and a path number with prime signifies right-to-left propagation): A→1→1’→1→B, A→1→1’→2→B, A→1→2’→1→B, A→1→2’→2→B, A→2→1’→1→B, A→2→1’→2→B, A→2→2’→1→B, and A→2→2’→2→B. Similarly we can identify higher-order signal pathways.

We first calculate the contribution of the 1st-order signal pathways (A→1→B and A→2→B) to *s*21. Since *s*-parameters are defined in terms of power wavesS7, we start by calculating *local* transmission coefficients for A→1, 1→B, A→2, and 2→B for power waves. Figure S6(b) shows the left junction. The incoming power wave *a*A+ on CPW A produces, at the junction, transmitted power wave *a*1+ and *a*2+ on path 1 and 2 (as well as the reflected power wave *a*A-, which is not involved in the *s*21 calculation with the 1st-order signal pathways), whereS7:

 (S1)

Here *V*1+ and *V*2+ (=*V*1+ due to the shunt connections of path 1 and path 2 at the junction) are the voltages of the plasmonic waves transmitted onto path 1 and path 2, read at the junction; *I*1+ and *I*2+ (=*I*1+ because path 1 and path 2 have the same characteristic impedance, *Z*) are the currents of the plasmonic waves transmitted onto path 1 and path 2, read at the junction. Then the *local* transmission coefficientsS6 *t*A→1 = *a*1+/*a*A+ and *t*A→2 = *a*2+/*a*A+ are given by

 . (S2.1)

Similarly, we can calculate the *local* transmission coefficients *t*1→B and *t*2→B as:

 . (S2.2)

Then the overall A-to-B transmission, *s*21, through the 1st-order signal pathways is given by , that is:

 [1st order]. (S3)

**multiplert.jpg**

**Fig. S6** (a) Schematic showing the interferometer’s two plasmonic paths (1 and 2) along with the two on-chip CPWs (A and B). (b) Illustration of local transmissions and reflections for the incoming power wave from CPW A.

We now consider the contribution of the 2nd-order signal pathways to *s*21. Given that *α*8000 m-1 (Section C), traversing path 1 (*l*1~191 µm) once will reduce the amplitude by a factor of exp(-*αl*1)~0.22 and traversing path 2 (*l*2~120 µm) once will reduce the amplitude by a factor of exp(-*αl*2)~0.38. Since each of the eight 2nd-order signal pathways involves traversing path 1 and/or path 2 a total of three times, each pathway will suffer significant attenuation. In addition, each 2nd-order pathway involves two additional local reflections and/or transmissions, which further attenuates the signal. The eight substantially attenuated signals superpose in CPW B, but they have generally different phases, thus, their superposition does not help much in countering the attenuation. All in all, the contribution from the 2nd-order pathways to *s*21 is negligibly small, which is confirmed by the actual calculation of the 2nd-order contribution [Fig. S7]. In sum, it is sufficient to calculate *s*21 up to the 1st order as in Eq. (S3), which is Eq. (1) of the main text.

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**Fig. S7:** Contributions of the 1st (black, solid) and 2nd (blue, dashed) order signal pathways to the *s*21 magnitude and their sum (red, semi-dashed). Physical parameters used (*L*k, *R*, *α* and *β*) for this plot are those (median values) extracted at 0.55 V.

1. **Frequency scaling**

The plasmonic quality factor is given by *Q=ωτ*, as stated in the main text. *Q*/2 signifies the number of local oscillation cycles of electrons in a given plasmonic wave during a mean scattering time *τ*. A larger *Q* facilitates plasmonic wave observation; if *Q* becomes smaller than unity, scattering occurs before electrons undergo a fraction of one oscillation cycle, thus, the plasmonic dynamics is increasingly masked by the ohmic resistance (scattering). In this work, to ensure *Q=ωτ* in excess of 1 at the GHz frequencies, we increased *τ* by operating the plasmonic interferometer at 4.2 K thus by suppressing electron scatterings by phonons. With a higher frequency, ** can be made shorter (*i.e.*, operation temperature can be higher) while maintaining large enough *Q=ωτ*. In fact, 2D plasmons have been observed at THz frequencies at room temperature with both GaAs/AlGaAs 2DEGS10 and grapheneS8, S9. Design into these higher frequencies can be done by scaling down the device dimensions. For instance, to attain the first destructive dip [*Δφ* = *ω*(*l*1*-l*2)*/v*p =π] at 3 THz with *v*p ~*c*/300, the path length difference *l*1-*l*2 can be set at 170 nm.

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