The design of long range quantum electrodynamical forces and torques between macroscopic bodies

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Abstract

The interaction between electrically neutral surfaces at sub-micron separation is dominated by the force arising from quantum fluctuations of the electromagnetic field, known as the Casimir force. This effect has been witnessing a renewed interest because of its potential impact in micro- and nanotechnology. Most recent literature has focused on the study of the attraction between bulk-like metallic surfaces in vacuum. Because electromagnetic fluctuations depend on the dielectric function of the surfaces, the use of different materials might reveal new aspects of the Casimir force and suggest novel solutions for the design of micro- and nanofabricated devices. Following this approach, we have measured the Casimir force using Hydrogen Switchable Mirrors—a metallic mirror that switches from highly reflective to transparent when exposed to hydrogen. The comparison of the results obtained in air and in hydrogen sheds light on the relative contribution of visible and infrared wavelengths to the Casimir interaction. We have also studied the dependence of the Casimir force on the metallic film thickness and have shown the effect of the skin-depth. The final section of the paper discusses the torque induced by quantum fluctuations on two birefringent plates and describes an experiment that should allow us to observe this phenomenon.

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1. Introduction

According to quantum electrodynamics, quantum fluctuations of the electromagnetic field give rise to a zero-point energy that never vanishes, even in empty space \cite{1}. In 1948, Casimir \cite{2} showed that, as a consequence, two parallel plates, made out of ideal metal, should attract each other in vacuum even if they are electrically neutral, a phenomenon known as the Casimir effect. Because only the electromagnetic modes that have nodes on both walls can exist within the cavity, the zero-point energy depends on the separation between the plates, giving rise to an attractive force.

Lifshitz, Dzyaloshinskii, and Pitaevskii generalized Casimir’s theory to dielectrics \cite{3,4}. In their theory, the force between two uncharged parallel plates can be derived according to an analytical formula that relates the zero-point energy to the dielectric functions of the interacting surfaces and of the medium in which they are immersed. At very short distances, Lifshitz’s theory provides a complete description of the non-retarded van der Waals force \cite{5,6}.
At larger separations, retardation effects give rise to a long range interaction that in the case of two ideal metals in vacuum reduces to Casimir’s result.

Following the pioneering experiments of Spaarnay [7], which were not able to unambiguously confirm the existence of the Casimir force, strong experimental evidence for the effect was presented by van Blokland and Overbeek in 1978 [8]. Final decisive verification is due to Lamoureaux, who in 1997 reported the first high precision measurements of the Casimir force using a torsional pendulum [9]. This was followed by several experimental studies, which have produced further convincing confirmation [10–18]. In some of these studies, the authors claimed agreement with theory at the 1% level, a result that has been challenged in some of the literature. Our group, for example, has critically analyzed these claims and found most of them wanting due to the strong non-linear dependence of the force on distance, which limits the precision in the absolute determination of the latter [14,27].

Uncertainties in the knowledge of the dielectric functions of the thin metallic films used in the experiments and in the models of surface roughness used to correct the Lifshitz theory can also give rise to errors larger than 1% in the calculation of the expected force (for example, Refs. [14,19-22] for a reply to some of these criticisms).

Apart from its intrinsic theoretical interest, the Casimir interaction has recently received considerable attention for its possible technological consequences. The Casimir force, which rapidly increases as the surface separation decreases, is the dominant interaction mechanism between neutral objects at sub-micron distances. In light of the miniaturization process that is moving modern technology towards smaller electromechanical devices, it is reasonable to ask what role the zero-point energy might play in the future development of micro- and nanoelectromechanical systems (MEMS and NEMS).

The first experimental works related to this topic were performed at Bell Labs by one of the authors (FC), Chan, and their collaborators [23,24]. The goal of their first experiment [23] was to design a micro-machined torsional device that could be actuated solely by the Casimir force. The results not only demonstrated that this is indeed possible, but also provided one of the most sensitive measurement of the Casimir force between metallized surfaces. In their second experiment [24], the group showed that the Casimir attraction can also influence the dynamical properties of a micromachined device, changing its resonance frequency, and giving rise to hysteretic behavior and bistability in its frequency response to an AC excitation, as expected for a non-linear oscillator. They proposed that this device could serve as a position sensor.

The activity of our group currently focuses on investigating the dependence of the Casimir force on the boundary conditions for the electromagnetic fields. In particular, it is interesting to note that vacuum fluctuations strongly depend on the dielectric functions of the materials deposited on the interacting surfaces. One can thus imagine to tailor the Casimir force by a suitable choice of appropriately designed materials [25].

In order to investigate this possibility, we have assembled an apparatus similar to the one used in previous experiments [23], and we have implemented a data acquisition technique that addresses one of the main sources of systematic error in the measurements (Section 2). Coating one of the surfaces with suitably engineered materials, we have studied the influence of visible and infrared virtual photons in the Casimir interaction and the role of the skin-depth effect in Casimir forces (Section 3). Finally, we have designed a new set-up for the measurement of the torque induced by quantum fluctuations on two parallel birefringent plates (Section 4). In this paper we report the results of these first steps towards engineering quantum fluctuations.

2. Experimental set-up and data acquisition technique for Casimir force measurements

Our experimental apparatus [26] (Fig. 1) is designed to measure the force between a sphere and a plate at sub-micron distances with a force sensitivity on the order of 10 pN. The measurement is carried out by positioning the sphere on top of a micro-machined torsional balance (MTB) and by measuring its rotation induced by the Casimir...
attraction with the sphere as a function of the separation of the surfaces (see inset (b) of Fig. 1). The MTB is similar to a microscopic seesaw (see inset (a) of Fig. 1). Two thin torsional rods keep a gold-coated polysilicon plate (500×500 µm²) suspended over two polysilicon electrodes symmetrically located on each side of the pivot axis. The capacitance between the top plate and each bottom electrode depends on the tilting angle θ. When an external force \( F \) induces a rotation of the top plate, one of the two capacitances increases by \( \delta C \propto \theta \propto F \), while the other decreases by the same amount. An electronic circuit allows measurements of \( \delta C \) with a sensitivity of the order of \( 10^{-6} \) pF, corresponding to \( \theta \approx 10^{-7} \) rad. Because the spring constant of the seesaw \( k_s \) is about \( 10^{-8} \) Nm/rad, the sensitivity in the torque measurement is approximately equal to \( k_s \theta \approx 10^{-15} \) Nm, which corresponds to a force of 10 pN in our experiment [25].

The MTB is glued to a chip package and mounted inside a chamber that can be pumped down to \( 10^{-3} \) mTorr. A 100 µm radius polystyrene sphere, mounted on the end of a rigid support and coated with a metallic layer, is clamped to a manipulator that can bring the sphere close to the top plate of the MTB and controls the distance between the two surfaces. The manipulator consists of a triaxial stage for rough positioning and a piezoelectric translator (calibrated with an optical profiler) for fine tuning of the distance.

In the Bell Labs experiment [23], the authors measured the tilting angle of the MTB as a function of the voltage applied to the piezoelectric stage, converted the data into a force-versus-distance \( (F \text{ vs. } d) \) plot, and compared the results with the Lifshitz theory. In order to convert the raw data into a \( F \text{ vs. } d \) plot, it is necessary to calibrate the MTB and to determine the distance \( d_0 \) between the sphere and the plate corresponding to the situation in which the piezoelectric stage is fully retracted. When a voltage \( V_{pz} \) is applied to the piezoelectric stage, the distance between the sphere and the plate is given by \( d = d_0 - d_{pz} \), where \( d_{pz} \propto V_{pz} \) is known with sub-nanometric precision, while \( d_0 \) is a priori unknown. The calibration of the MTB can be performed using electrostatic forces. The measurement of \( d_0 \) is more problematic, and is considered one of the main sources of error in all modern experiments [17,27]. To avoid this problem, \( F \text{ vs. } d \) data were fitted directly with the theoretical curve using \( d_0 \) as a free parameter, giving good agreement with theory.

In most of our experiments, the goal is to use materials for which the Casimir force cannot be accurately calculated. Therefore, it is not possible to determine \( d_0 \) with the same technique described above. Furthermore, even when the theory can be calculated with high accuracy, a fitting procedure for \( d_0 \) can result in incorrect interpretation of data or in fortuitous agreements [14,27]. We have thus developed a different method for the measurement of the Casimir force as a function of distance [25,26]. After the chamber is evacuated, the piezoelectric stage is extended towards the MTB to reduce the separation between the sphere and the plate until the distance is only a few nanometers larger than the jump-to-contact point (i.e. the distance at which the restoring torque of the seesaw is not sufficient to overcome the external torque induced by the Casimir force, causing the plate to come into contact with the sphere). The output of the capacitance bridge \( A \) is then recorded as a function of the voltage applied to the sphere \( V_{bias} \), which is scanned a few hundred millivolts around the so-called residual voltage \( V_0 \) (=200 mV), i.e. the electrostatic potential drop arising from the difference of the work functions of the two films [8] plus the potential difference generated by the metallic contacts of the electronics [9]. The read-out system is designed so that \( A \) is proportional to \( \delta C \) and, therefore, to \( F \):

\[
A = c_1 F = c_1 \varepsilon_0 \pi R (V_{bias} + V_0)^2 \frac{(d_0 - d_{pz})}{(d_0 - d_{pz})^2} + c_1 |F_C|
\]

where \( \varepsilon_0 \) is the permittivity of vacuum, \( R \) is the radius of the sphere, and \( F_C \) is the Casimir force.

The measurement of \( A \) as a function of \( V_{bias} \) is then repeated for different values of \( d \), which is changed by sequentially retracting the piezoelectric stage by a few nanometers.

For each value of \( d \), data are interpolated with a generic quadratic equation \( y = \alpha (x + x_0)^2 + \beta \), where \( \alpha, \beta, \) and \( x_0 \) are free parameters. Note that

\[
\alpha = c_1 \frac{\varepsilon_0 \pi R}{(d_0 - d_{pz})}
\]

By fitting \( \alpha \) as a function of \( d_{pz} \) it is thus possible to determine \( d_0 \) and \( c_1 \). Once \( c_1 \) is known, \( F_C \) can be calculated by means of

\[
|F_C| = \frac{\beta}{c_1}
\]

Because \( d_0 \) has also been determined, one can finally plot \( F_C \) as a function of the distance between the sphere and the plate, \( d = d_0 - d_{pz} \) [28].

3. Tailoring the Casimir force at sub-micron distances

In an attempt to demonstrate that the Casimir interaction can be tailored by appropriate choice of the dielectric properties of the interacting surfaces, we investigated the effect of hydrogen switchable mirrors (HSMs) on the Casimir force [25]. HSMs [29] are shiny metals in their as deposited state. However, when they are exposed to a hydrogen-rich atmosphere, they become optically transparent. Because fluctuations of the electromagnetic field depend on the optical properties of the surfaces, the attraction between two HSMs in air should be different than the attraction between the same HSMs immersed in a hydrogen-rich atmosphere. In particular on intuitive grounds one expects that the Casimir force will be much weaker when the HSM is in the transparent state rather than in the reflective state.
Using an experimental set-up similar to the one described above [30], we have measured the Casimir force between a gold-coated plate and a sphere coated with a HSM for separations in the ≈70 to ≈400 nm range. The HSMs were obtained by repeating seven consecutive evaporations of alternate layers of magnesium (100 Å) and nickel (20 Å), followed by an evaporation of a thin film of palladium (50 Å). In the inset of Fig. 2, we show a glass slide coated according to this procedure, both in its as deposited state, and in its hydrogenated state. It is evident that the optical properties of the film are very different in the two situations. We have measured the transparency of the film over a wavelength range between 0.5 and 3 μm, and its reflectivity at ≈660 nm, keeping the sample in air and in an argon–hydrogen atmosphere (4% hydrogen). The results are in good agreement with the values reported in [31].

The results of Casimir force measurements obtained in air and in a hydrogen-rich atmosphere are shown in Fig. 2. It is evident that the force does not change significantly upon hydrogenation of the HSM.

In order to explain this result, we first note that the dielectric properties of the HSMs used in this experiment are known only in a limited range of wavelengths λ, spanning from approximately 0.3–2.5 μm [31]. However, because the separation between the sphere and the plate in our experiment is in the ≈100 nm range, one expects that it is not necessary to know the dielectric function for λ ≫ 2.5 μm, because those modes should not give rise to large contributions to the force. We have thus performed a mathematical exercise to see if this intuitive argument is correct. The Casimir attraction in vacuum between a sphere of radius R and a plate with dielectric function ɛ(ω) is given by [3]:

\[
F_C = \frac{\hbar R}{2\pi c^2} \left( \int_0^\infty \frac{d\xi}{\sinh \xi} \ln \left[ 1 + \frac{(s - p)^2}{(s + p)^2} \frac{e^{-\xi}}{(s + p)^2} \right] \right) d\omega
\]

(4)

Where s = \sqrt{\epsilon - 1 + p^2}, \epsilon is the dielectric function of the material of the two surfaces calculated at imaginary frequency iξ, and c and h are the usual fundamental constants. Using Eq. (4), we have calculated the Casimir force for a material with hypothetical dielectric function equal to the dielectric function of gold [32] at all λ, except for a wavelength interval spanning from λ_{min} = 0.2 μm to λ_{max}, where we have set the imaginary part of the dielectric function equal to zero. In Fig. 3 we report the decrease of the force expected with respect to the gold–gold interaction as a function of λ_{max} for different values of d. As these results show, it is necessary to change the dielectric function over a wide window in order to have large variations of the Casimir force.

If we assume that the HSM layers deposited on the sphere change their dielectric properties only in a limited wavelength range, which is not in contradiction with the experimental data available in literature, then the decrease of the force upon hydrogenation could be too small to be observable.

It is interesting to note that the dependence of the force from the dielectric properties of the materials, as described by Eq. (4), is mathematically related to the dielectric function calculated on the imaginary axis of frequencies, which can be determined from the equation [10]:

![Fig. 2. Casimir force between a gold-coated plate and a sphere coated with a hydrogen switchable mirror as a function of the distance, in air (green dots) and in argon–hydrogen (red dots). Inset: a hydrogen switchable mirror in air (a) and in argon–hydrogen (b). A similar mirror was deposited on the sphere of our experimental apparatus (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article).](image)

![Fig. 3. Calculated change of the Casimir force for a system in which we have set the imaginary part of the dielectric function \( \varepsilon'' \) to be zero over a spectral region of increasing width. \( F_{Au} \) is the calculated force between a sphere and a plate made of gold kept at distance d. \( F_{mod} \) is the force calculated for the same d if one assumes that the sphere and the plate are made of a material with \( \varepsilon'' \) equal to that of gold, except for a wavelength interval spanning from \( \lambda_{min} = 200 \) nm to \( \lambda_{max} \), where \( \varepsilon'' = 0 \). The graph represents the change (in %) as a function of \( \lambda_{max} \).](image)
\[\epsilon(i\omega) = 1 + \frac{2}{\pi} \int_0^\infty \frac{\omega \epsilon''(\omega)}{\omega^2 + \xi^2} d\omega \]  

where \(\epsilon''\) is the imaginary part of the dielectric function. The integral in Eq. (5) runs over all real frequencies, with non-negligible contributions arising from a very wide range of frequencies, as already noted in [33]. This explains the results illustrated in Fig. 3.

In our analysis, we have not accounted for the presence of the thin palladium layer on top of the HSM. The thickness of this layer, in fact, is of the order of 50 Å. The skin-depth of ultraviolet, visible, and infrared radiation is \(\geq 100\) Å. Therefore, the palladium layer can probably be considered transparent at all the wavelengths relevant to the calculation of the Casimir force. Nevertheless, a paper by Tan and Anderson [34] showed that, if the dispersion relation of two-dimensional plasmons is included in the calculation of the non-retarded van der Waals forces between low-dimensional metals such as graphite or polyacetylene, one obtains a dependence on distance \(d\) significantly different from the \(1/d^4\) theoretical prediction. Similar effects cannot be ruled out a priori in the case of the Casimir force between ultrathin metallic films.

Motivated by such considerations, we have studied the effect of thin film thickness on the Casimir force. We have measured the Casimir force between the top plate of the MTB (covered with a \(\approx 2000\) Å gold layer) and a polystyrene sphere covered with a \(29 \pm 2\) Å titanium adhesion film followed by a \(92 \pm 3\) Å palladium layer. We have compared these results to those obtained after evaporating a thicker layer of palladium (\(\approx 2000\) Å) on the same sphere [26].

This experiment requires careful measurements of the topology of the interacting surfaces. Surface roughness plays an important role in the Casimir force at short distances, where even small corrugations (a few tenths of the average distance) can give rise to relevant contributions to the force (\(\gg 10\%\)) [10]. After the evaporation of the first thin metallic layer (titanium and palladium), the sphere was imaged with an optical profiler and mounted inside our experimental apparatus. After completion of the Casimir force measurements, the sphere was removed from the experimental apparatus, coated with an additional 2000 Å of palladium, analyzed with the optical profiler, and mounted back inside the vacuum chamber for another set of measurements. The surface roughness measured before and after the deposition of the thicker palladium layer was the same within a few percent (see inset of Fig. 4) [35].

The results of this experiment are reported in Fig. 4 and compared with the force expected for the case of the thick film. The theoretical curve was obtained from the following equation:

\[F_C^{(p)}(d) = \sum_i v_i \delta_i^{(sp)} \delta_j^{(pl)} F_C(d - (\delta_j^{(sp)} + \delta_j^{(pl)})) \]  

where \(v_i\) is the probability that the surface of the sphere (superscript sp) or of the plate (superscript pl) is displaced by an amount \(\delta_i\) with respect to the ideally smooth surface (as obtained from the optical profiler images), and \(F_C\) is the Casimir force calculated according to Eq. (4) using the dielectric functions obtained from references [36–38]. It is evident that the data obtained with the thick film are in agreement with the theoretical prediction. It is also evident that the attraction measured using the thin film is significantly smaller than that obtained with the thick layer. One can thus conclude that it is possible to reduce the Casimir force by means of the skin-depth effect [26], and that the results obtained with HSMs are not ascribable to the presence of the thin palladium layer on the sphere.

4. Tailoring quantum fluctuations with birefringent plates

During the 70s, it was noted that quantum fluctuations of the electromagnetic field should give rise to a torque between two parallel birefringent slabs, causing them to spontaneously rotate [39,40]. This effect can be qualitatively understood by noting that the relative rotation of the two plates will result in a modification of the zero-point energy, because the reflection, transmission, and absorption coefficients of these materials depend on the polarization of the virtual photons, responsible for the zero-point energy, with
respect to the optical axis. The anisotropy of the zero-point energy between the plates then generates the torque that makes them rotate toward configurations of smaller energy.

Even in the simplified case of in plane optical anisotropy (see inset (a) of Fig. 5), the torque is given by a cumbersome equation that, for the sake of brevity, we will omit. We refer the reader to [40,41] for further mathematical details. Here, we will only discuss its main features and the results that we have obtained from the theory.

The torque \( M \) depends on the dielectric function of the two plates (along both the optical axis and the other in plane principal axis) and on the dielectric function of the medium in which they are immersed. The effect is proportional to the surface area of the plates and to \( \sin(2\theta) \), where \( \theta \) is the angle between the two optical axes. Furthermore, as expected on intuitive grounds, it increases as the separation between the plates \( d \) decreases. However, it is not possible to extract a unique power-law dependence of \( M \) on \( d \) from the general formula. In order to understand if it is feasible to design an experiment for the observation of the rotation induced by quantum fluctuations, we have performed calculation for a specific case: a 40 \( \mu \)m diameter, 20 \( \mu \)m thick disk, made of either calcite or quartz, kept parallel to a barium titanate plate.

In the inset (b) of Fig. 5 we show the expected value of \( M \) as a function of \( \theta \) for \( d = 100 \) nm in the case of calcite and barium titanate. Similar results were obtained for quartz, although the magnitude of the effect is smaller by an order of magnitude [42]. As anticipated, the torque is proportional to \( \sin(2\theta) \).

We have also calculated the value of \( M \) as a function of \( d \) for \( \theta = \pi/4 \). The results for the case of calcite are reported in Fig. 5. For a quartz disk we have again obtained values that are approximately one order of magnitude smaller. For \( d = 100 \) nm, the maximum torque (in the case of calcite) is approximately equal to \( 7 \times 10^{-19} \) Nm. In 1936, Beth performed an experiment where a torsional balance was used to measure the rotation of a macroscopic quartz disk induced by the transfer of angular momentum of light [43]. He achieved a sensitivity of \( \approx 10^{-17} \) Nm. It is thus reasonable to ask if a similar set-up with today’s improved technology could be used to observe the rotation between the 40 \( \mu \)m diameter disk and the barium titanate plate induced by virtual photons associated with the zero-point energy. The main difficulty of this experiment is that it is necessary to keep the disk freely suspended just above the other plate at separations where the two surfaces would tend to come into contact. From Fig. 5, we can estimate that in order to observe the effect the two surfaces should be kept at least at sub-micron distances. Because the torque is proportional to the area of the interacting surfaces, one could use a plate with a much larger diameter. For example, for a 1 cm diameter quartz disk kept parallel to a barium titanate plate, the torque is larger than \( 10^{-17} \) Nm for \( d \leq 1 \) \( \mu \)m. However, other problems would arise in this case. The surface roughness and the curvature of the two birefringent plates should be much smaller than 1 \( \mu \)m over an area of several cm\(^2\). This area should also be completely free from dust particles with diameter larger than a few hundreds of nanometers. Furthermore, one should still design a mechanical set-up to keep the two slabs parallel without compromising the sensitivity of the instrument. We conclude that the use of a torsional balance for the measurement of the torque induced by quantum fluctuations would present several major technical problems.

Instead of discussing how to solve these technical problems, we propose a simpler approach [41]. The idea is to levitate the disk over the plate at \( d = 100 \) nm by means of a repulsive Casimir-Lifshitz force. According to the Lifshitz theory [4], two plates made out of the same material always attract, regardless of the choice of the intervening medium. However, for slabs of different materials the sign of the force depends on the dielectric properties of the medium in which they are immersed. In particular the force between two plates with dielectric functions \( \varepsilon_1 \) and \( \varepsilon_2 \) immersed in a medium with dielectric function \( \varepsilon_3 \) should be repulsive if, for imaginary frequencies, \( \varepsilon_1 < \varepsilon_3 < \varepsilon_2 \) or \( \varepsilon_2 < \varepsilon_3 < \varepsilon_1 \), and should be attractive in all other cases [5,6]. Using the data reported in the literature [44,45], it is possible to show that \( \varepsilon_{\text{calcite}} \) and \( \varepsilon_{\text{quartz}} \) at imaginary frequencies are much smaller than \( \varepsilon_{\text{BaTiO}_3} \), and that the dielectric function of liquid ethanol [46] satisfies the conditions \( \varepsilon_{\text{calcite}} < \varepsilon_{\text{ethanol}} < \varepsilon_{\text{BaTiO}_3} \) and \( \varepsilon_{\text{quartz}} < \varepsilon_{\text{ethanol}} < \varepsilon_{\text{BaTiO}_3} \). Therefore, if the two birefringent slabs considered above are immersed in liquid ethanol, the disk should float parallel to the plate at a distance where its weight is counterbalanced by the repulsive Casimir-Lifshitz force. It can be shown that, for the geometry that we have chosen, the equilibrium distance would be approximately equal to 100 nm [41]. The static friction between the two

![Fig. 5](image-url)
birefringent plates should be virtually zero, and the disk should be free to rotate suspended in bulk liquid. Further calculations of $M$ as a function of $d$ and $\theta$ shows that the torque in liquid ethanol does not significantly change with respect to the case of vacuum [41]. This approach should thus allow us to observe the rotation of the disk induced by the fluctuations of the electromagnetic field in a reasonably straightforward experiment.

It is interesting to stress that repulsive Casimir-Lifshitz forces can be obtained with many other low cost and easily micro-machinable materials (such as silicon or gold over teflon or silica immersed in ethanol, bromobenzene, or cyclohexane), and that this technique could be used to develop ultra-sensitive force and torque sensors (like, for example, a nano-compass sensitive to very small static magnetic fields) [48].

In Fig. 6, we show a schematic view of the experimental set-up that we are implementing in our laboratory [41]. A 40 $\mu$m diameter, 20 $\mu$m thick disk made out of either calcite or quartz is placed on top of a barium titanate plate immersed in ethanol. The optical axes of the birefringent crystals are oriented as shown in inset (a) of Fig. 5. A 100 mW polarized laser beam can be collimated onto the disk to rotate it by the transfer of angular momentum of light. A shutter can then block the beam to stop the light-induced rotation. The position of the disk can be monitored by means of a microscope objective coupled to a CCD-camera for imaging.

Using the laser, one can rotate the disk until $\theta = \pi/4$. Once the laser beam is shuttered, the disk is free to rotate back towards the configuration of minimum energy following the behavior reported in the inset of Fig. 6 [41]. For the calcite disk, rotations should be easily measured within a few minutes after closing the laser beam shutter. The quartz disk should rotate much slower, and it is questionable whether its rotation can be detected [47]. However, it is worth stressing that the set-up presented above is not yet optimized. Disks with different dimensions and geometries might rotate faster. Suitably engineered samples could also result in more favorable experimental configurations. For example, a thick layer of lead could be deposited on a portion of the disk to make it heavier: the disk would then float at a smaller distance, where the magnitude of the driving torque would be larger. Furthermore, the use of a different liquid with optical properties similar to ethanol but with a smaller viscosity would significantly increase the angular velocity of the disk. Finally, a more sophisticated optical set-up could be implemented for the measurement of small rotations.

One could argue that it might be difficult to distinguish the cause of the rotation of the disk from other effects that could mimic the phenomenon under investigation and that the accumulation of electric charges on the slabs might be a relevant technical problem. It is thus worth stressing that long range electrostatic interactions can be screened out by changing the concentration of ions in the liquid, so that the Debye screening length is much smaller than the separation between the plates [6]. Furthermore, the torque induced by quantum fluctuations has periodicity $\pi$, a characteristic that should help the experimenter rule out spurious effects. Finally, the experiment could be repeated by placing the disk over a non-birefringent plate with $\varepsilon_{\text{liquid}} > \varepsilon_{\text{ethanol}}$. The Casimir force would still be repulsive, but there would be no torque induced by quantum fluctuations.

**5. Conclusions**

We have reviewed recent experiments of our group on quantum electrodynamical forces and torques between macroscopic bodies.

Using an experimental apparatus that relies on a micromachined force sensor, we have measured the Casimir attraction between a gold-coated plate and a sphere coated with a HSM, a highly reflective mirror that can be switched to a transparent window by exposure to a hydrogen-rich atmosphere. In spite of such a dramatic change in the optical properties of the sphere, we did not observe any significant decrease of the Casimir force after filling the experimental apparatus with hydrogen. This experiment has demonstrated an interesting property of the Lifshitz theory: in order to significantly change the Casimir force between surfaces at separations on the order of 100 nm it is not sufficient to change their optical (visible and infrared) reflectivity, as intuition would suggest, but it is necessary to modify their dielectric functions over a much wider spectral range.

We have also performed comparative measurements of the Casimir force between a metallic plate and a transparent sphere coated with metallic films of different thicknesses. We have observed that, if the thickness of the coating is less than the skin-depth of all the electromagnetic modes that contribute to the interaction, the force is significantly

![Fig. 6. A sketch of the experimental set-up proposed for the observation of the torque between birefringent plates induced by zero-point fluctuations of the electromagnetic field. Inset: calculated value of the angle between the optical axes of the two birefringent crystals as a function of time.](image-url)
smaller than that measured with a thick bulk-like film. Our results represents the first direct study of the role of the skin-depth effect on the Casimir force between metallic surfaces.

Finally, we have discussed the possibility of observing the torque induced by quantum fluctuations of the electromagnetic field on two parallel birefringent plates. We have carried out detailed numerical calculations of the mechanical torque between a micromachined birefringent disk, made of quartz or calcite, and a barium titanate birefringent plate. We have shown that a demonstration of the effect could be readily obtained if the birefringent slabs were immersed in liquid ethanol. In this case the disk would float on top of the plate at a distance where the repulsive Casimir force balances gravity, giving rise to a mechanical bearing with virtually zero static friction. The disk, initially set in motion via transfer of angular momentum of light from a laser beam, would return to its equilibrium position solely driven by the torque arising from quantum fluctuations.

It is a pleasure to dedicate this paper to Prof. Elias Burstein, a towering figure in experimental condensed matter physics during the last 50 years. His wide-ranging contributions to the understanding of the optical properties of solids have opened up new areas of research. One of us (FC) has had the privilege of knowing him and receiving encouragement from him in his research for many years. His highly interactive style, his youthful enthusiasm for new developments in science, his dedication to the scientific community and in particular to younger colleagues and last but not least his uncompromising scientific integrity have been a source of inspiration and set very high standards for others to follow.

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References

[10] B. Barash, L. Levitov, A.H. Stone are grateful for
[18] It is interesting to note that other groups have reported many measurements of retarded van der Waals forces between dielectrics in liquids. The precision of these experiments appears to be less accurate than the one obtained with metals in vacuum. See for example Ref. [6], and L. Suresh and J.Y. Waltz, J. Colloid Interface Sci. 196, 177 (1997), M.A. Bevan, and D.C. Prieve, Langmuir 15, 7925 (1999), S. Lee, and W.M. Sigmund, J. Colloid Interface Sci. 243, 365 (2001), and references therein.
[28] In our experiments, we ignore the fact that, because of the rotation of the MTB, $d$ is actually smaller than $d_0 - d_{vac}$. This effect is usually kept into account in modern experiments with a procedure that requires the sphere to be brought into contact with the plate. This procedure can damage the sensor and where surface damages can significantly alter the measurement. However, it is possible to estimate that the separation between the sphere and the plate varies by less than 1 nm for a plate rotation of 5 rad induced by the Casimir force. This affects any discrepancy between theory and experiment by $\approx 1\%$ and is therefore negligible in the context of this paper. This effect of course also affects the calibration
of the set up with an electrostatic force. One can show that keeping it into account would affect the data in the opposite direction with respect to the above correction.


[30] We have actually used a slightly different set-up. The experiment, however, was carried out by using the data acquisition technique described in Section 2.


[32] For the sake of simplicity, we have used the Drude model for the dielectric function of gold (see Ref. [10]).


[35] Optical measurements do not allow us to distinguish topological details whose typical horizontal dimensions are smaller than \( \approx 500 \) nm. We have thus imaged the surface of the sphere used for thick film measurement using an AFM. The rms value obtained over a \( 1 \times 1 \) \( \mu \)m\(^2\) (\( \approx 12 \) nm) is slightly smaller than what was obtained with the optical profiler (\( \approx 15 \) nm) over a larger area. Therefore, ignoring topological details not accessible to optical profiler measurements does not affect the conclusions of our experiment.


[42] The sign of the torque expected in the case of a quartz disk is opposite to the one obtained with calcite. We refer the reader to [41] for further details.


[47] It can be shown that Brownian motion should not play an important role in the rotation. We refer the reader to [41] for a detailed analysis of this topic.