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Abstract. A three-dimensional extension of the recently demonstrated generalization of the laws of refraction and reflection was investigated for both flat and curved metasurfaces. We found that out-of-plane refraction occurs for a metasurface that imparts a wavevector out of the plane of incidence onto the incident light beam. Metasurfaces provide arbitrary control over the direction of refraction, and yield new critical angles for both reflection and refraction. A spherical meta-surface with phase discontinuities leads to unconventional light bending compared to standard refractive lenses. © 2012 Society of Photo-Optical Instrumentation Engineers (SPIE). [DOI: 10.1117/1.JNP.6.063532]

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1 Introduction

The formulation of laws of reflection and refraction dates to the 10th century when Alhazen conducted experiments on the movement of light passing through different media,1 and Ibn Sahl deduced the law of refraction.2 These laws have been reported in a simple mathematical formulation in the works of Thomas Herriot, Willebrord Snell, and Rene Descartes.3 Eventually, Pierre de Fermat generalized these laws to determine the path of a ray of light traveling in media with an arbitrary distribution of refractive indices.4 Recently, recalling a concept previously introduced by Veselago,5 Pendry suggested using negative refractive index materials to modify the refraction of light.6 Some years later, the concept of transformation optics was introduced,7 which is a method to control light propagation based on a complicated distribution of the optical refractive index of a heterogeneous material or metamaterial. These concepts led to the experimental demonstration of intriguing effects, such as negative refraction, subwavelength-focusing, and optical cloaking.8–10 However, in all of these demonstrations the redirecting of a light beam is a result of gradual phase accumulation along the beam path through the material or metamaterial.

Recently Snell’s law has been revisited and generalized by using metasurfaces that introduce abrupt phase shifts along the optical path.11 The bending of the refracted beam is not only determined by the refractive index jump at the interface but also by a distribution of phase discontinuities imposed by a metasurface comprising an array of ultrathin and subwavelength-spaced optical antennas. This leads to anomalous reflection and refraction of light11,12 that can be even directed out of the plane the incidence. This effect has been experimentally observed for the
reflection and refraction of an optical beam from a flat surface with a component of the phase gradient pointing out of the plane of incidence. The concept of phase discontinuities has also been applied to demonstrate other effects, such as the generation of vortex beams and focusing free from on-axis aberration. In this letter, we study refraction and reflection in a more general geometry, in which a constant gradient of phase discontinuities is imposed on curved interfaces. In particular, we derive the generalized Snell’s law for flat and spherical interfaces.

In its original formulation, Fermat’s principle states that the trajectory taken between two points, \( P_i \) and \( P_r \), by a ray of light is that of the least optical path, i.e., the physical length multiplied by the refractive index. A more general form of the principle, known as the principle of stationary phase, states that the derivative of the phase accumulated along the actual light path, is zero with respect to infinitesimal variations of the path.

Now, let us consider a ray of light with wave vector \( k_i \) incident on a curved interface that introduces a constant phase gradient \( \nabla \Phi \) oriented along an arbitrary direction along the interface, and separates two media of indices \( n_i \) and \( n_t \), respectively. The direction of the refracted ray with wave vector \( k_t \) can be derived using the principle of stationary phase; the difference of phase accumulated by light travelling along two different paths infinitesimally close to each other has to be zero (Fig. 1):

\[
\delta \int \varphi \, dr = \delta \left[ \int k_i \cdot dr + \nabla \Phi \cdot s + \int k_t \cdot dr \right] = (k_i - k_t + \nabla \Phi) \cdot ds = 0,
\]

where \( r \) and \( s \) are the position vector in space and its projection on the interface, respectively. Equation (1) is valid for all \( ds \) on the interface; therefore, the vector \( (k_i - k_t + \nabla \Phi) \) should be perpendicular to the tangent to the surface at the point of incidence. Hence we have,

\[
\hat{n} \times (k_t - k_i) = \hat{n} \times \nabla \Phi.
\]  

Accordingly, a similar relation can be derived for the reflected beam of light:

\[
\hat{n} \times (k_r - k_i) = \hat{n} \times \nabla \Phi.
\]

To illustrate the generality of the proposed relations, the reflection and refraction of a light beam are studied at flat and spherical interfaces.

2 Flat Interface

Figure 2 shows reflection and refraction of a ray of light impinging on a flat surface with an angle \( \theta_i \) with respect to the \( z \)-axis. Without loss of generality, the plane of incidence is the \( yz \)-plane and the interface is along the \( xy \)-plane. Therefore the unit vector \( \hat{n} = \hat{z} \) and the incident vector \( k_i \) has two non-zero components such that \( k_i = (0, k_{y,i}, k_{z,i}) \). The flat plane imparts an abrupt phase

![Diagram](image-url)
jump to the incident wave, characterized by a constant gradient $\nabla \Phi$ oriented along an arbitrary direction with respect to the plane of incidence. We can work out Eq. (2) for this geometry, obtaining:

$$\begin{align*}
 k_{x,t} &= \frac{d\Phi}{dx} \\
 k_{y,t} &= k_{y,i} + \frac{d\Phi}{dy} 
\end{align*}$$

Due to the lack of translational invariance along the interface, the tangential wavevector of light is not conserved; the interface contributes additional terms $d\Phi/dx$ and $d\Phi/dy$, which allow one to control the refracted beam along both $x$ and $y$ directions. Hence, the refracted ray of light does not necessarily lie in the plane of incidence.

We use angles $\theta_i$ and $\phi_t$ to describe the direction of the refracted beam (see Fig. 2) and obtain the following generalized law of refraction in three-dimensions:

$$\begin{align*}
 \cos \theta_t \sin \phi_t &= \frac{1}{n_t k_0} \frac{d\Phi}{dx} \\
 n_t \sin \theta_i - n_i \sin \theta_t &= \frac{1}{k_0} \frac{d\Phi}{dy}
\end{align*}$$

Similarly, using Eq. (3) we obtain the generalized law of reflection in three dimensions (Fig. 2).

$$\begin{align*}
 \cos \theta_r \sin \phi_r &= \frac{1}{n_i k_0} \frac{d\Phi}{dx} \\
 \sin \theta_r - \sin \theta_i &= \frac{1}{n_i k_0} \frac{d\Phi}{dy}
\end{align*}$$

Note that when the phase gradient is oriented along the plane of incidence ($d\Phi/dx = 0$) the reflected and refracted beams are also in this plane, and if both components of the phase gradient are zero, the conventional law of refraction is restored.

In conventional refraction, the critical angle is the angle above which total internal reflection occurs. An interface with a phase gradient arbitrarily oriented with respect to the plane of incidence alters the relations between the tangential components of the incident, reflected and refracted $k$-vector and consequently gives rise to new critical angles for both reflection and refraction. The new critical angles are reached when the refracted or reflected beams become evanescent, which means that the respective wave vectors are imaginary in the propagation direction. This condition is found by equating the tangential component of $k_{i(r)}$ to the modulus of the $k$-vector in the hosting medium:
From Eqs. (4), (5), and (7) we can derive the following expressions for the critical angles for transmission:

$$\theta_{c,t} = \sin^{-1} \left[ \pm \frac{1}{n_i} \sqrt{n_i^2 - \left( \frac{1}{k_0 n_i} \frac{d\Phi}{dx} \right)^2} - \frac{1}{n_i k_0} \frac{d\Phi}{dy} \right].$$

As already mentioned, the presence of the phase gradient breaks the translational invariance, and therefore the critical angles are not symmetric with respect to the $xz$-plane.

Equation (6) exhibits a nonlinear relationship between the angle of reflection and the incident angle, which yields two critical angles for reflection:

$$\theta_{c,r} = \sin^{-1} \left[ \pm \sqrt{1 - \left( \frac{1}{n_i k_0} \frac{d\Phi}{dx} \right)^2} - \frac{1}{n_i k_0} \frac{d\Phi}{dy} \right].$$

Figure 3 shows the critical angles versus the normalized gradients $\frac{d\Phi}{dx} / \frac{2\pi}{\lambda_0}$ and $\frac{d\Phi}{dy} / \frac{2\pi}{\lambda_0}$. The gray shaded areas are regions at which transmission/reflection exist for all incident angles. Unlike conventional refraction and reflection, we can design interfaces with no critical angles, or interfaces featuring critical angles only in reflection while preserving the transmission for all incident angles and vice versa (see Fig. 3).

A demonstration of the generalized refraction law in three dimensions, and out-of-plane refraction from a flat interface, is presented using an interface patterned with an array of optical antennas. By suitably designing their geometry one can control the phase and amplitude of

![Fig. 3](http://nanophotonics.spiedigitallibrary.org/)
light scattered by each antenna to generate the desired wavefront.\textsuperscript{16,17} The sample is created by periodically translating in the $x$–$y$ plane the unit cell consisting of 8 subwavelength $V$-shaped antennas [colored in yellow in the inset scanning electron microscope image of Fig. 4(a)]. The array imposes a phase gradient on the interface in the direction $s$ that forms an angle $\alpha$ with the $y$-axis as shown in the inset. When $\alpha$ is not zero, a component of the phase gradient points out of the plane of incidence, resulting in an out-of-the-plane scattered beam. The antennas were designed such that they need to be excited with a linear polarization at an angle of 45 deg, with respect to the antenna symmetry axes.\textsuperscript{11,16} To maintain this configuration when the sample is rotated, we simultaneously rotated the incident polarization.

As described in Ref. 11 part of the light is reflected and refracted according to the conventional laws in the same polarization as the incident beam. Anomalously reflected and refracted beams are also generated, as described by Eqs. (5) and (6), which are cross-polarized with respect to the incident beam.

To validate the predictions of the 3-D law of refraction [Eq. (5)], we performed an experimental study of anomalous refraction for various incidence angles $\vartheta_i$ and phase gradient orientations ($\alpha$ angles). The magnitude of the phase gradient is fixed to $|\nabla \Phi| = 2\pi/15 \mu m$ for all the experiments. The results are summarized in Fig. 4(b), which unambiguously show out-of-plane refraction with $\varphi_t \neq 0$. Additionally, the phase gradient orientation changes the out-of-plane refraction angles in good agreement with Eq. (5). It is worth noting that regardless of the different orientations of the phase gradient, the ordinarily refracted beam always lies in the plane of incidence, at angles predicted by the conventional Snell’s law. Our experimental setup did not allow us to monitor the reflected beam.

3 Spherical Interface

Figure 5 shows the schematic of refraction of a light beam at a spherical interface with radius $R$, which introduces a constant gradient of phase shift $\nabla \Phi$ to the incident light. The incident beam lies in the $yz$-plane and the interface is at $\rho = R$ in the spherical reference system. Therefore we have $\hat{n} = -\hat{\rho}$ and $(k_{\rho,i}, k_{\alpha,i}, 0)$.
From Eq. (2) we obtain:

\[
\begin{align*}
\mathbf{k}_x &= \frac{d\Phi}{d\phi} \\
\mathbf{k}_\theta &= \mathbf{k}_{\theta,i} + \frac{d\Phi}{d\theta}
\end{align*}
\]

By imposing these equalities for the component of the \( k \)-vector tangential to the interface, one can easily obtain the direction of the refracted beam, which in general does not lie in the plane of incidence. Similar equations can be obtained for reflection.

To see the effect of a constant phase gradient on a spherical interface, we consider a pencil of rays parallel to the \( z \)-axis and assume \( n_i > n_t \). We simplify the geometry by taking a cross section along the \( yz \)-plane, as shown in Fig. 5(b). In the absence of a phase gradient the rays will be refracted towards a point located on the \( z \)-axis at a distance from the interface that depends on the ratio of the refractive indices of the two media. The refraction from a spherical interface is commonly used in lenses and it can be easily shown that in the limit of the paraxial approximation, all the rays will converge to the focal point.3 Given this approximation, the distance \( f \) at which a ray will intersect the \( z \)-axis is:

\[
f = R \sin \theta_i \theta_t - \sin \theta_i \theta_t
\]

Using the approximately linear relation between the angle of incidence and the angle of refraction (\( n_i \theta_t \approx n_i \theta_t \)) we can obtain:

\[
f = R \frac{n_i}{n_t} - \sin \theta_i \theta_t
\]

which is the common expression derivable from the Lensmaker’s Formula4 for a plano-convex lens.

If now we consider an interface with a gradient of phase discontinuities, from Eq. (10) we can derive a relation between the angle of incidence and the angle of refraction. Assuming paraxial approximation and small phase gradient pointing in the \( \theta \) direction \( \mathbf{\nabla} \Phi = \frac{\partial \Phi}{\partial \theta} \hat{\theta} \), we obtain:

\[
n_i \theta_i = n_t \theta_t + \frac{1}{k_0} \frac{d\Phi}{d\theta}
\]

By substituting Eq. (11) in the expression of \( f = R \frac{n_i}{n_t} \), one can see that the latter is still a function of the incident angle. This means that a spherical interface with a phase gradient does not refract a paraxial beam to a single point and therefore cannot be used to focus light.

4 Concluding Remarks

We derived generalized laws of reflection and refraction for interfaces with phase discontinuities. These laws are general and can be adapted to various geometries. To illustrate the generality of the generalized laws, the reflection and refraction of a light beam are studied at both flat and spherical interfaces.

From the conservation of the tangential component of the \( k \)-vector, which originates from Fermat’s principle, we derived the expressions for the angles describing the out-of-plane
reflection and refraction. New critical angles follow from these nonlinear expressions for both reflected and refracted beams.

Finally, we showed that a spherical interface separating two media does not focus light to a single point in the presence of a constant phase gradient, even in the limit of paraxial approximation. Nonetheless, a flat interface with phase gradient can be used to focus light if a nonlinear phase distribution is used. This design has been recently demonstrated, showing the capability of light focusing without spherical aberrations.

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