Homework #1

Nonlinear dynamics and chaos

(A) (Strogatz 2.4)

Use linearized stability, to classify the fixed points of the following systems. if linearized stability fails, use graphical/ geometric approach:

$$\dot{x} = 1 - e^{-x^2} \tag{1}$$

$$\dot{x} = ax - x^3 \text{for all possible values of } a \tag{2}$$

$$\dot{x} = x(1-x)(2-x) \tag{3}$$

$$\dot{x} = x^2(6-x) \tag{4}$$

$$\dot{x} = \ln x \tag{5}$$

(B) Consider the quadratic map:

$$x_{n+1} = rx_n(1-x_n). \qquad x \in [0,1], \quad r \in \mathrm{R} \tag{6}$$

1. Find the fixed points and their stability (analyze the stability analytically using linearization, and graphically using cobweb plots).

2. Using your computer/ calculator find, describe and plot the behavior for large n, for: $r = 0.4,\ 2.,\ 2.9,\ 3.2,\ 2.8,\ 4.$ (Can use the sample Matlab program for the logistic map on the course home page for this purpose).

(C) An optional (easy) challenge problem: For $r < 1$, find analytically the asymptotic behavior for large n.