Homework #10
Nonlinear dynamics and chaos

1. **Reconstructed phase space and quasi-periodicity:** Often in experimental systems, the equations are not known, only a time series of a certain measurement is available, but we still would like to plot things in a "reconstructed" phase space based on the available time series. There are several standard ways to do this, two of which are demonstrated here. Consider a time series of a "quasi-periodic" process $x(t) = a\sin(\omega_1 t) + b\sin(\omega_2 t)$ in a reconstructed 3d phase space and show that it produces a torus. (A quasi-periodic process is one with two frequencies whose ratio does not form a rational number). Do this in the two following ways to see that they produce topologically equivalent (i.e. two pictures that look similar) phase space portraits.

   (a) Use "delay coordinates" reconstructed phase space, such that each point in phase space is given by $\mathbf{x}(t) = (x(t), x(t-\tau), x(t-2\tau))$ for some delay τ.

   (b) Use a phase space based on $\mathbf{x}(t) = (x(t), \frac{d}{dt}x(t), \frac{d^2}{dt^2}x(t))$

   Try various choices for the parameters $a, b, \omega_1, \omega_2, \tau$ to get a nice looking torus as in the figure below (the Matlab command `plot3` may be useful). Explore cases in which $\omega_1/\omega_2 = p/q$ with p, q integers, for both $p > q$ and $p < q$, as well as cases in which ω_1/ω_2 is irrational.

   Explain why is it that while the time series in a case where ω_1/ω_2 is irrational never repeats itself, it is still not chaotic.