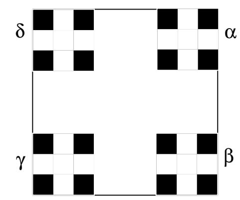
Homework #10
Nonlinear dynamics and chaos

1. **Dimension spectrum and multifractals:**

   (a) (Ott problem 3.7, page 103) Consider the fractal shown here:

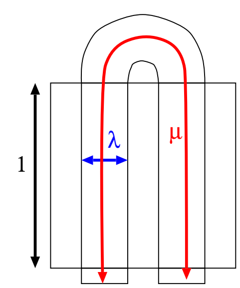
   

   Remember that the dimension spectrum D_q makes sense only when there is a non-uniform measure on the object being considered. Put a measure on this fractal as follows. Let $\alpha, \beta, \gamma, \delta$ be positive numbers such that $\alpha+\beta+\gamma+\delta = 1$. For the first stage of constructing the fractal, let the measure on the squares be as marked on the above figure. At the next stage, when each square is again divided into 4 smaller squares, let the total measure on the four smaller squares be equal to what it was in the previous iteration, and let this total be divided as in the previous iteration. That is, the fraction of the weight that is assigned to the upper right smaller box is α of the total assigned to the entire larger box in the previous iteration, etc. Calculate D_q and plot it (plot by assuming some values for $\alpha, \beta, \gamma, \delta$).

   (b) **Optional challenge problem:** read in Ott about the relation between D_q and $f(\alpha)$, and try to calculate and plot $f(\alpha)$ for this fractal.

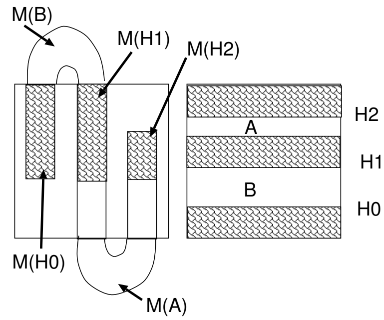
2. Horseshoe map:

   (a) Calculate the fractal dimension of the invariant set (intersect of horizontal and vertical stripes) of the horseshoe map with stretching and contraction factor μ, λ. See figure.

   (b) For stretching and contraction factors both equal to 3, plot the set $H_{ijk} \cap V_{ijk}$. Mark the squares containing the fixed points, and the location of the points belonging to the period-3 orbit $\overline{011}$. To what accuracy can you find them?

   (c) Limited shift Horseshoe (Ott, 1st ed, problem 1, p 148): Consider the horseshoe-type map shown in the following figure:



The map is equivalent to a shift operation over the three symbols 0, 1 and 2. Note, however, that region H_2 is mapped only to H_1, while H_0 and H_1 are mapped only to H_1 and H_2. This means that in a symbolic representation of the dynamics using bi-infinite symbols, 2 must be followed by 1, and 0 and 1 may be followed by either 1 or 2 (did I get this right…?). That is, the shift operation is limited to certain combinations of the symbols. Write all the possible periodic orbits up and including period 4. You may want to read the 1 page discussion of such "limited shift" maps in Ott (pp 113-114 in 1st ed).