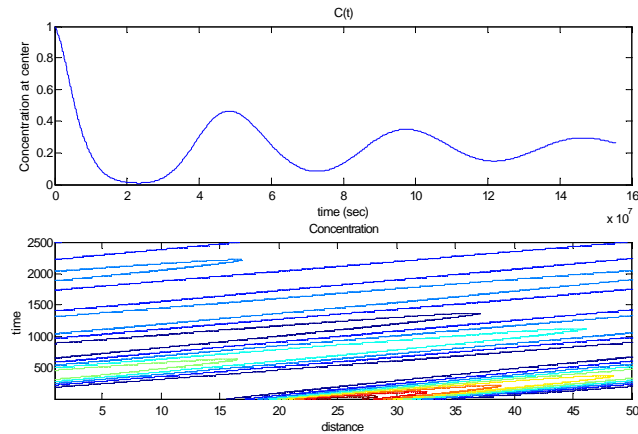Ian Eisenman

1. (a)   The program, exactly as it is on the website, produces the following plots. This is a simulation of fluid flowing around a looped pipe of length L. The concentration is initially a Gaussian centered at x=L/2 with width L/8. The region of high concentration travels around the pipe, advected by the background flow, spreading (from diffusion) as it goes.



(b)      For this value of the diffusion constant, the Robert filter doesn't make much difference until the advecting velocity gets to about u=10. Note that the program specifies that at both of the first two timesteps the concentration is a Gaussian centered at L/2. As discussed in class, the problem is overspecified by these two initial conditions. The faster the advection velocity is, the more wrong this overspecification becomes, and hence the more significant the Robert filter becomes. The solution is plotted below with and without the Robert Filter for u=20.
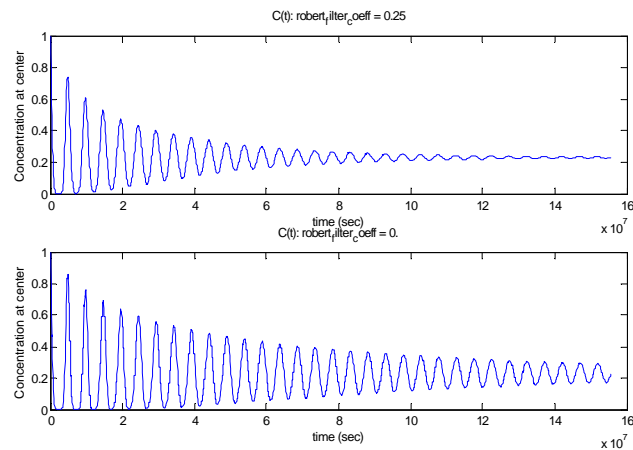
1. (c)   With the current parameters, we can reduce kappa to about 1/5 its initial value before the program begins to crash (makes the Gaussian concentration distribution become asymmetric after some time). Reducing kappa causes the Gaussian concentration to spread out less as it is advected along the pipe.

(d)      With dt=0.02*year, the program crashes. Halving this leads to a nice solution, halving it again makes little difference, and halving it once more makes virtually no difference at all. These diminishing effects as the time step is reduced imply that we may be converging on the theoretical solution to the differential equation (which has infinitesimal time steps).

2.      To add the radioactive decay term, I inserted
```
lambda=0*1e-8;
```
near the top of the program and added this correction to the leapfrog solution:
```
C(i,3)=C(i,1)+2*dt*(-u*C_x+kappa*C_xx-lambda*C(i,2));
```
Unlike the diffusion term that spreads the concentration out (approaching a uniform distribution after a long time), this term diminishes the concentration without affecting its structure (approaching zero concentration everywhere after a long time). The solution with kappa=6e5 is plotted below.
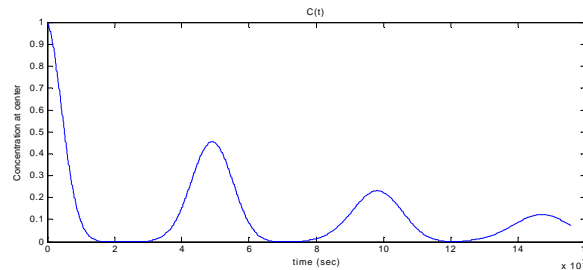


3.      Here's how I changed the initial condition to a sine wave:

```
num_waves=3;   %number of peaks in sine wave initial cond'n
% Initial conditions:
for i=1:nx
    for n=1:3
        C(i,n)= 1/2*(1-cos(num_waves*i*2*pi/nx));
    end
end
```

When lambda=0 but there is diffusion, the concentration should spread out, approaching C=1/2 everwhere. The rate of spreading goes like the second spatial derivative, so the more wiggly solution should spread more quickly, and it should take far longer for the longer wavelength sine wave to reach 1/3 its initial amplitude. On the other hand, when kappa=0 but there is radioactive decay, the concentration should reduce toward C=0 everywhere. The rate of reduction goes like C, so the decay shouldn't care how wiggly the distribution is, and both sine waves should reduce to a third their amplitude in about the same time (the error related to our finite difference approximation is actually similar to spatial diffusion, so the more wiggly wave still dies out a bit faster). In the plots below, the blue curve is the initial condition and the green curve is the concentration after t time steps. The value of t is indicated in the title of each plot.

## 3. cont'd

C(t): λ=1e-8, κ=0., t=600

C(t): λ=0, κ=6e5, t=70

C(t): λ=1e-8, κ=0., t=900

C(t): λ=0, κ=6e5, t=900