1 Control Flow Analysis

Consider the following lambda calculus program.

\((\lambda f. (f \ 76) + (f \ 77)) (\lambda a. a)\)

(a) Add labels to the program. That is, make it an expression in the labeled lambda calculus of Lecture 22, where every label is unique.

(b) Let \(e\) be your labeled lambda calculus program. Write out \(C[e]\), i.e., the set of constraints for the program \(e\). (Hint, you should have 20 constraints in total. In particular, for each of the 3 applications, you should have 4 constraints, 2 for each of the lambda terms in the program.)

(c) Find \(C^*\) and \(r^*\), the smallest functions that satisfy the constraints you generated in the question above.

(d) Check that your functions \(C^*\) and \(r^*\) make sense. That is, if an expression labeled \(l\) can evaluate to an expression labeled \(l'\), do you have \(l' \in C^*(l)\)?

(e) Consider adding the expression \((\text{let } x = e_1 \text{ in } e_2)^l\) to the language. Define \(C[(\text{let } x = e_1 \text{ in } e_2)^l]_e\). Try rewriting the program above using one or more let expressions, and make sure that the constraints you generate for the modified program produce the same solution \(C^*\) and \(r^*\).

2 Logic Programming

(a) Consider the following Prolog program (where [] is a constant representing the empty list, [t] is shorthand for \(\text{cons}(t, [])\) and \([t_1, t_2, t_3]\) is shorthand for \(\text{cons}(t_1, \text{cons}(t_2, t_3))\)).

\[
\text{foo}([], []).
\text{foo}([X], [X]).
\text{foo}([X, Y|S], [Y, X|T]) :- \text{foo}(S, T)
\]

For each of the following queries, compute the substitutions that Prolog will generate, if any. (Note that there is a difference between an empty substitution, and no substitution.) If the query evaluation will not terminate, explain why.

- \(\text{foo}([a, b], X)\).
- \(\text{foo}([a, b, c], X)\).
- \(\text{foo}([a, b], [a, b])\).
- \(\text{foo}(X, [a])\).
- \(\text{foo}(X, Y)\).
(b) Consider the following Datalog program.

\[
\begin{align*}
  & \text{bar}(a, b, c). \\
  & \text{bar}(X, Y, Z) :\neg \text{bar}(Y, X, Z). \\
  & \text{bar}(X, Y, Z) :\neg \text{bar}(Z, Y, X), \text{quux}(X, Z). \\
  & \text{quux}(b, c). \\
  & \text{quux}(c, d). \\
  & \text{quux}(X, Y) :\neg \text{quux}(Y, X). \\
  & \text{quux}(X, Z) :\neg \text{quux}(X, Y), \text{quux}(Y, Z).
\end{align*}
\]

Find all solutions to the query \text{bar}(X, Y, Z).