1 Dynamic types and contracts

(a) To make sure you understand the operational semantics of dynamic types and exceptions, show the execution of the following program under the semantics of Section 1 of the Lecture 22 notes.

\[
\begin{align*}
\text{let } f &= \lambda x. 42 + x \text{ in} \\
\text{let } g &= \lambda y. (y \text{ true}) + 42 \text{ in} \\
g \ f
\end{align*}
\]

(b) Modify the program from question (a) by adding appropriate error handlers (i.e., expressions of the form \(\text{try } e_1 \text{ catch } x. e_2\)) to catch the type error and return the integer 42 as the final result of the program. There are multiple places in the program where you can insert an error handler to achieve the desired result. Show three variations and their executions. (Note that the semantics for the execution of your programs is from Section 2 of the Lecture 22 notes.)

(c) Modify the program from question (a) by adding appropriate dynamic type checks to raise the error as early as possible. When does your program detect the error?

(d) Modify the program from question (a) by adding contracts that specify the types of the input and output of \(f\) and \(g\). Show the execution of the modified program.

(e) To make sure you understand the operational semantics of contracts, show the execution of the following program.

\[
\begin{align*}
\text{let } f &= \text{monitor} (\lambda x. x + 1, \text{is\_int? } \mapsto \text{is\_int?} ) \text{ in} \\
\text{let } g &= \text{monitor} (\lambda x. x (42) + 42, \text{is\_int? } \mapsto \text{is\_int?}) \mapsto \text{is\_int?} ) \text{ in} \\
\text{let } h &= \text{monitor} (\lambda x. \lambda y. 42, \text{is\_int? } \mapsto ((\text{is\_int? } \mapsto \text{is\_bool?} ) \mapsto \text{is\_bool?} )) \text{ in} \\
h \ (g\ f) \ (\lambda z. z + 42)
\end{align*}
\]

2 Lambda calculus machines

Consider the following program in the language from the lecture notes (Lecture 23):

\[
(((\lambda f. \lambda g. \lambda x. \lambda y. f\ x + g\ y)\ \lambda a. 0)\ \lambda b. 42)\ 43)\ 44
\]

Show its evaluation trace under CC, SCC, CK and CEK semantics.