Curry-Howard Isomorphism; Existential Types
CS 152 (Spring 2020)

Harvard University

Thursday, March 26, 2020
Today, we will learn about

- Curry-Howard Isomorphism
- Existential types
Curry-Howard Isomorphism
Conjunction = Product
Disjunction = Sum
Function Types?
Parametric Polymorphism
What about False?
Example 2: From Type to Formula
Negation and Continuations
Existential Types
Syntax

e ::= x | \lambda x : \tau. e | e_1 e_2 | n | e_1 + e_2
    | \{ l_1 = e_1, \ldots, l_n = e_n \} | e.l
    | \text{pack} \{ \tau_1, e \} \text{ as } \exists X. \tau_2
    | \text{unpack} \{ X, x \} = e_1 \text{ in } e_2

v ::= n | \lambda x : \tau. e | \{ l_1 = v_1, \ldots, l_n = v_n \}
    | \text{pack} \{ \tau_1, v \} \text{ as } \exists X. \tau_2

\tau ::= \text{int} | \tau_1 \rightarrow \tau_2 | \{ l_1 : \tau_1, \ldots, l_n : \tau_n \} | X | \exists X. \tau
Example: Counter ADT
Example: Counter ADT, ctd
Operational Semantics

\[ E ::= \cdots \mid \text{pack } \{ \tau_1, E \} \text{ as } \exists X. \tau_2 \\
\mid \text{unpack } \{ X, x \} = E \text{ in } e \]

\[
\text{unpack } \{ X, x \} = (\text{pack } \{ \tau_1, v \} \text{ as } \exists Y. \tau_2) \text{ in } e \rightarrow e\{v/x\}\{\tau_1/X\}
\]
Typing rules

\[
\Delta, \Gamma \vdash e : \tau_2\{\tau_1/X\}
\]

\[
\Delta, \Gamma \vdash \text{pack } \{\tau_1, e\} \text{ as } \exists X. \tau_2 : \exists X. \tau_2
\]

\[
\Delta, \Gamma \vdash e_1 : \exists X. \tau_1 \quad X \notin \Delta
\]

\[
\Delta \cup \{X\}, \Gamma, x : \tau_1 \vdash e_2 : \tau_2 \quad \Delta \vdash \tau_2 \text{ ok}
\]

\[
\Delta, \Gamma \vdash \text{unpack } \{X, x\} = e_1 \text{ in } e_2 : \tau_2
\]

\[
\Delta \vdash \tau \text{ ok}
\]

\[
\Delta \vdash \exists X. \tau \text{ ok}
\]