HARVARD
John A. Paulson
School of Engineering
and Applied Sciences

CS153: Compilers
Lecture 11: LR Parsing

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https://www.seas.harvard.edu/courses/cs153

Contains content from lecture notes by Greg Morrisett and Steve Zdancewic

Announcements

- Reminder: CS Nights, Tuesdays 8pm
  - With pizza!
- HW3 LLVMlite out
  - Due Tuesday Oct 15 (1 week)
- HW4 Oat v1 will be released today
  - Due Tuesday Oct 29 (3 weeks)
  - Simple C-like Imperative Language
    - supports 64-bit integers, arrays, strings
    - top-level, mutually recursive procedures
    - scoped local, imperative variables
  - Compile to LLVMlite

Today

- Oat overview
- LR Parsing
  - Constructing a DFA and LR parsing table
  - Using Menhir

HW4: Oat v1

- Oat is a simple C-like imperative language
  - supports 64-bit integers, arrays, strings
  - top-level, mutually recursive procedures
  - scoped local, imperative variables
- See examples in hw04/at1programs directory
- You will:
  - Finish implementing lexer and parser
  - Compile from Oat v1 to LLVMlite
    - You can use your `backend.ml` from HW3 to compile from LLVMlite to X86!
- HW5 will extend Oat with more features...

LR(*k*)

Left-to-right parse

Rightmost derivation

Derivation expands the rightmost non-terminal

(Constructs derivation in reverse order!)

k-symbol lookahead

LR(k)

- Basic idea: LR parser has a stack and input
  - Given contents of stack and k tokens look-ahead parser does one of following operations:
    - Shift: move first input token to top of stack
    - Reduce: top of stack matches rule, e.g., $X \rightarrow A\ B\ C$
      - Pop C, pop B, pop A, and push X

Example

$$E \rightarrow \texttt{int}$$
$$E \rightarrow (E)$$
$$E \rightarrow E + E$$

Stack	**Input**
	`( 3 + 4 ) + ( 5 + 6 )`

Shift `(` on to stack

Example

$E \rightarrow$ `int`
$E \rightarrow$ `(` E `)`
$E \rightarrow E + E$

Stack	**Input**
(	3+4)+(5+6)

Shift (on to stack
Shift 3 on to stack

Example

$E \rightarrow$ `int`
$E \rightarrow$ `(` E `)`
$E \rightarrow E$ `+` E

Stack	**Input**
`( 3`	`+ 4 ) + ( 5 + 6 )`

Shift `(` on to stack
Shift `3` on to stack
Reduce using rule $E \rightarrow$ `int`

Example

$E \rightarrow$ `int`
$E \rightarrow$ `(` E `)`
$E \rightarrow E$ `+` E

Stack	**Input**
`(` E	`+4)+(5+6)`

Shift `(` on to stack
Shift `3` on to stack
Reduce using rule $E \rightarrow$ `int`
Shift `+` on to stack

Example

$E \to$ `int`

$E \to$ (E)

$E \to E + E$

Stack	**Input**
(E +	4) + (5 + 6)

Shift (on to stack

Shift 3 on to stack

Reduce using rule $E \to$ `int`

Shift + on to stack

Shift 4 on to stack

Example

$$E \rightarrow \texttt{int}$$
$$E \rightarrow (E)$$
$$E \rightarrow E + E$$

Stack	**Input**
(E + 4	) + (5 + 6)

Shift (on to stack
Shift 3 on to stack
Reduce using rule $E \rightarrow \texttt{int}$
Shift + on to stack
Shift 4 on to stack
Reduce using rule $E \rightarrow \texttt{int}$

Example

$E \rightarrow$ `int`
$E \rightarrow$ `(` E `)`
$E \rightarrow E + E$

Stack	**Input**
`(` $E + E$	`)+(5+6)`

Shift `(` on to stack
Shift `3` on to stack
Reduce using rule $E \rightarrow$ `int`
Shift `+` on to stack
Shift `4` on to stack
Reduce using rule $E \rightarrow$ `int`
Reduce using rule $E \rightarrow E + E$

Example

$E \rightarrow$ `int`

$E \rightarrow$ (E)

$E \rightarrow E + E$

Stack	**Input**
(E	) + (5 + 6)

Reduce using rule $E \rightarrow E + E$

Shift) on to stack

Example

$E \rightarrow$ `int`

$E \rightarrow$ `(` E `)`

$E \rightarrow E + E$

Stack	**Input**
`(` E `)`	`+(5+6)`

Reduce using rule $E \rightarrow E + E$

Shift `)` on to stack

Reduce using rule $E \rightarrow$ `(` E `)`

Example

$E \rightarrow$ `int`

$E \rightarrow$ (E)

$E \rightarrow E + E$

Stack **Input**

E + (5 + 6)

Reduce using rule $E \rightarrow E + E$

Shift) on to stack

Reduce using rule $E \rightarrow$ (E)

Shift + on to stack

Example

$E \rightarrow$ `int`

$E \rightarrow$ `(` E `)`

$E \rightarrow E + E$

Stack	**Input**
E +	(5 + 6)

Reduce using rule $E \rightarrow E + E$

Shift `)` on to stack

Reduce using rule $E \rightarrow$ `(` E `)`

Shift + on to stack

... and so on ...

Example

$E \rightarrow$ `int`

$E \rightarrow$ `(` E `)`

$E \rightarrow E$ `+` E

Stack	**Input**
E `+ (` E	`+ 6 )`

Reduce using rule $E \rightarrow E + E$

Shift `)` on to stack

Reduce using rule $E \rightarrow$ `(` E `)`

Shift `+` on to stack

… and so on …

Example

$E \to$ `int`

$E \to$ (E)

$E \to E + E$

Stack	**Input**
E + (E + E	)

Reduce using rule $E \to E + E$

Shift) on to stack

Reduce using rule $E \to$ (E)

Shift + on to stack

... and so on ...

Example

$E \to$ `int`

$E \to (E)$

$E \to E + E$

Stack	**Input**
$E + (E$	$)$

Reduce using rule $E \to E + E$

Shift $)$ on to stack

Reduce using rule $E \to (E)$

Shift $+$ on to stack

... and so on ...

Example

$E \rightarrow$ `int`

$E \rightarrow$ `(` E `)`

$E \rightarrow E + E$

Stack **Input**

$E + E$

Reduce using rule $E \rightarrow E + E$

Shift `)` on to stack

Reduce using rule $E \rightarrow$ `(` E `)`

Shift + on to stack

... and so on ...

Example

$E \rightarrow$ `int`
$E \rightarrow$ `(` E `)`
$E \rightarrow E + E$

Stack	**Input**
E	

Reduce using rule $E \rightarrow E + E$

Shift `)` on to stack

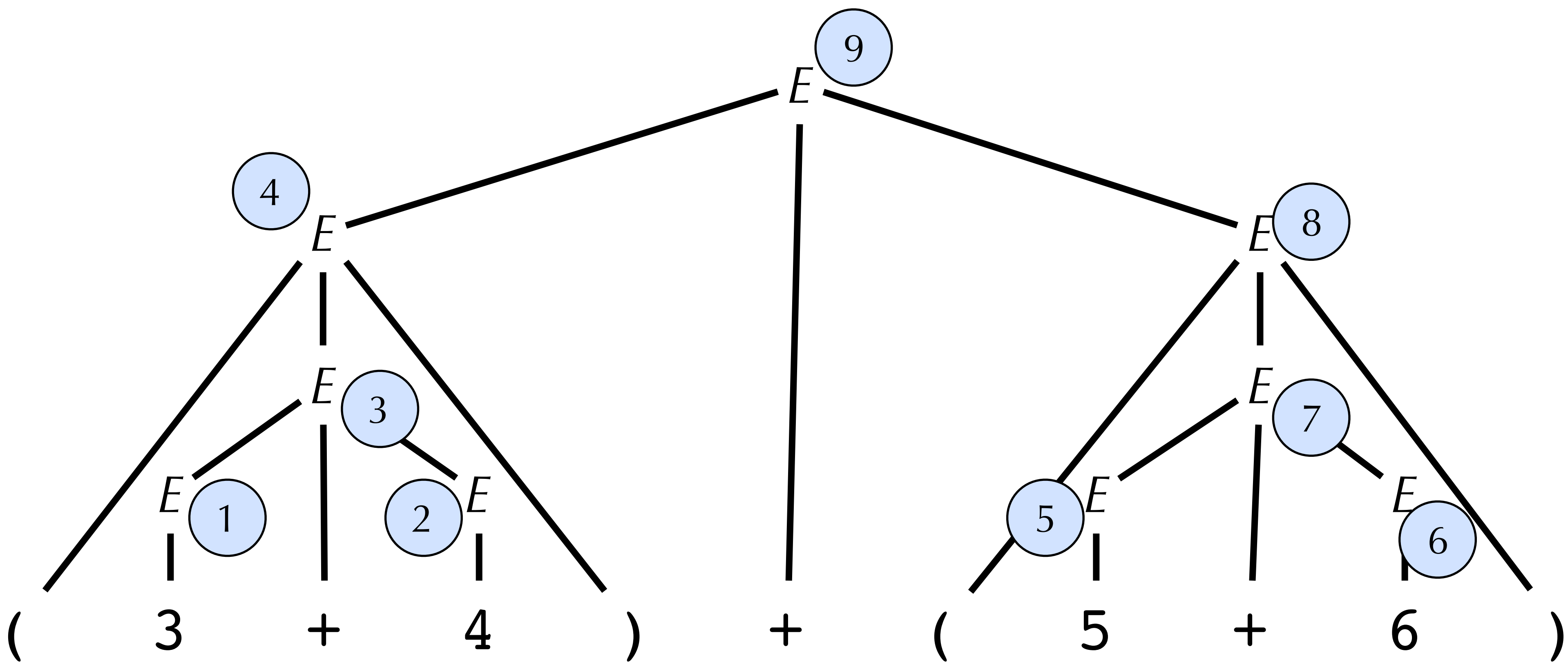
Reduce using rule $E \rightarrow$ `(` E `)`

Shift + on to stack

… and so on …

Rightmost derivation

- LR parsers produce a rightmost derivation



- But do reductions in reverse order

What Action to Take?

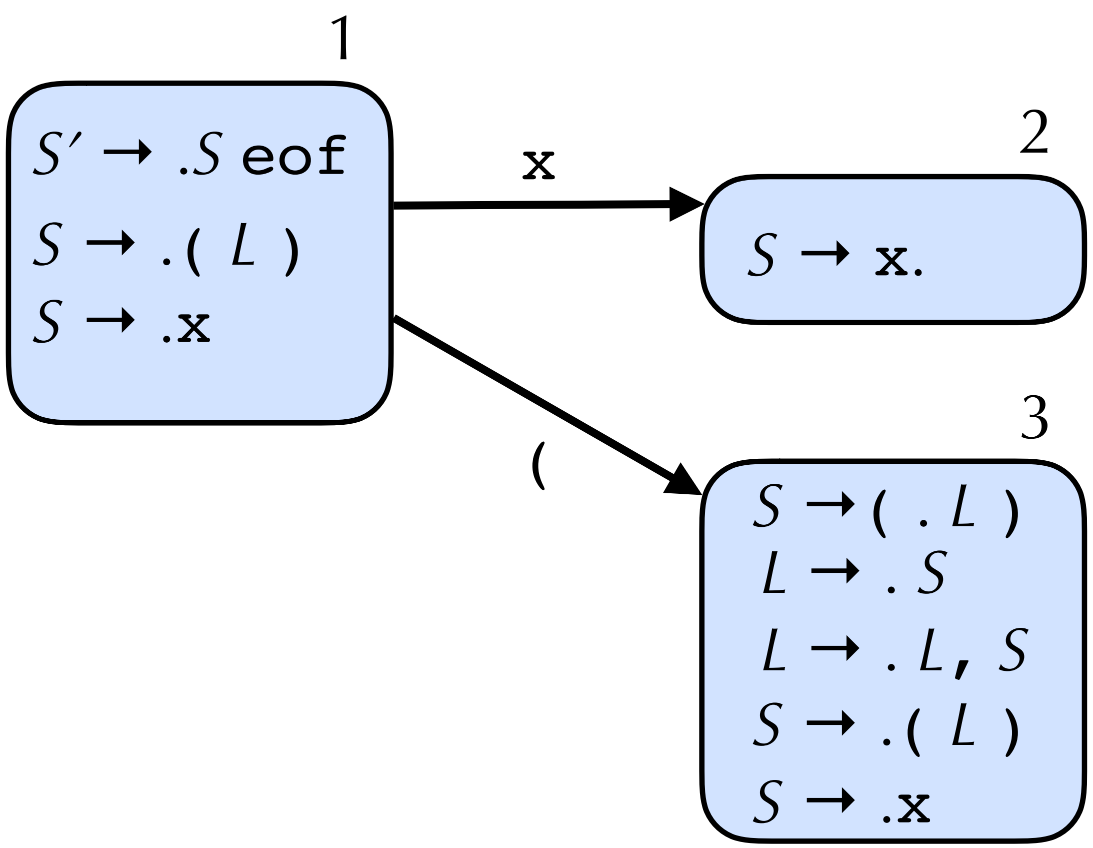
- How does the LR(k) parser know when to shift and to reduce?
- Uses a DFA
  - At each step, parser runs DFA using symbols on stack as input
    - Input is sequence of terminals and non-terminals from bottom to top
  - Current state of DFA plus next *k* tokens indicate whether to shift or reduce

Building the DFA for LR parsing

- Sketch only. For details, see Appel
- States of DFA are sets of **items**
  - An **item** is a production with an indication of current position of parser
  - E.g., Item $E \rightarrow E\ .\ +\ E$ means that for production $E \rightarrow E + E$, we have parsed first expression E have yet to parse + token
  - In general, item $X \rightarrow \gamma\ .\ \delta$ means γ is at the top of the stack, and at the head of the input there is a string derivable from δ

Example: LR(0)

Add new start symbol with production to indicate end-of-file



$S' \to S$ **eof**

$S \to$ **(** L **)**

$S \to$ **x**

$L \to S$

$L \to L$ **,** S

First item of first state: at the start of input

State 1: item is about to parse *S*: add productions for *S*

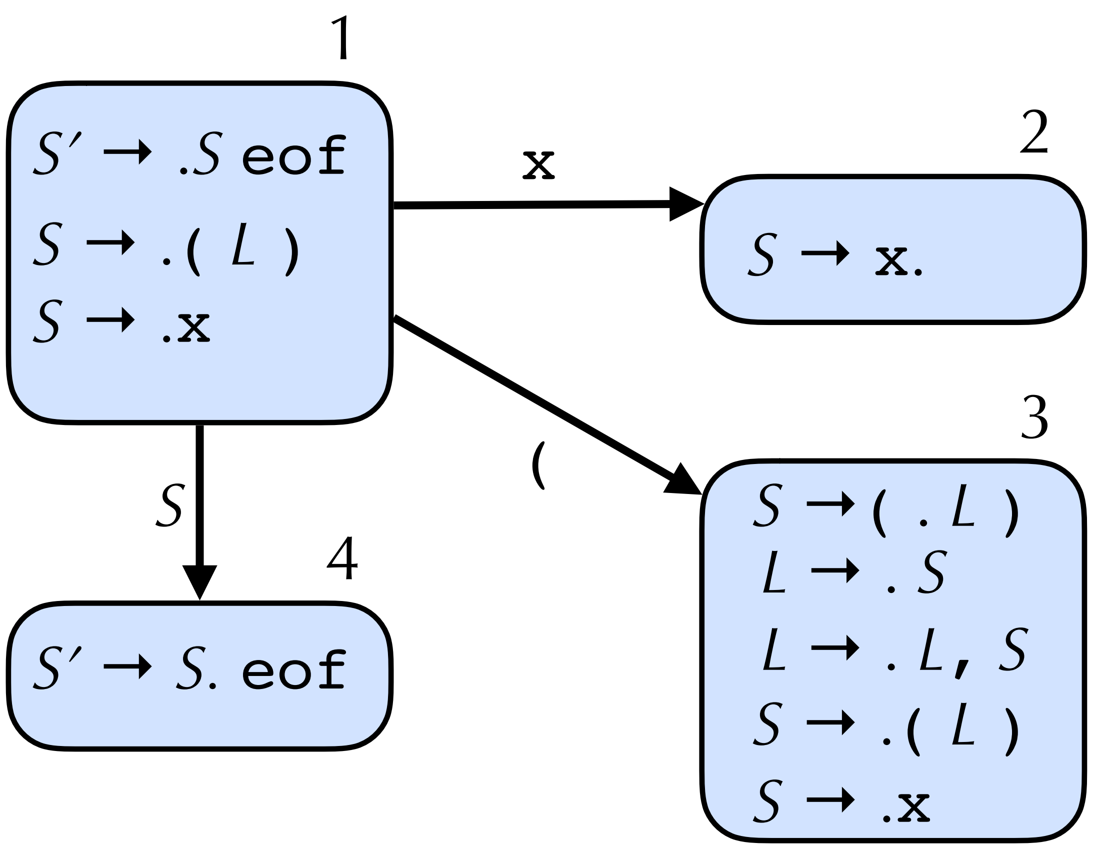
From state 1, can take **x**, moving us to state 2

From state 1, can take **(**, moving us to state 3

State 3: item is about to parse *L*: add productions for *L*

State 3: item is about to parse *S*: add productions for *S*

Example: LR(0)



$S' \to S$ eof

$S \to$ (L)

$S \to$ x

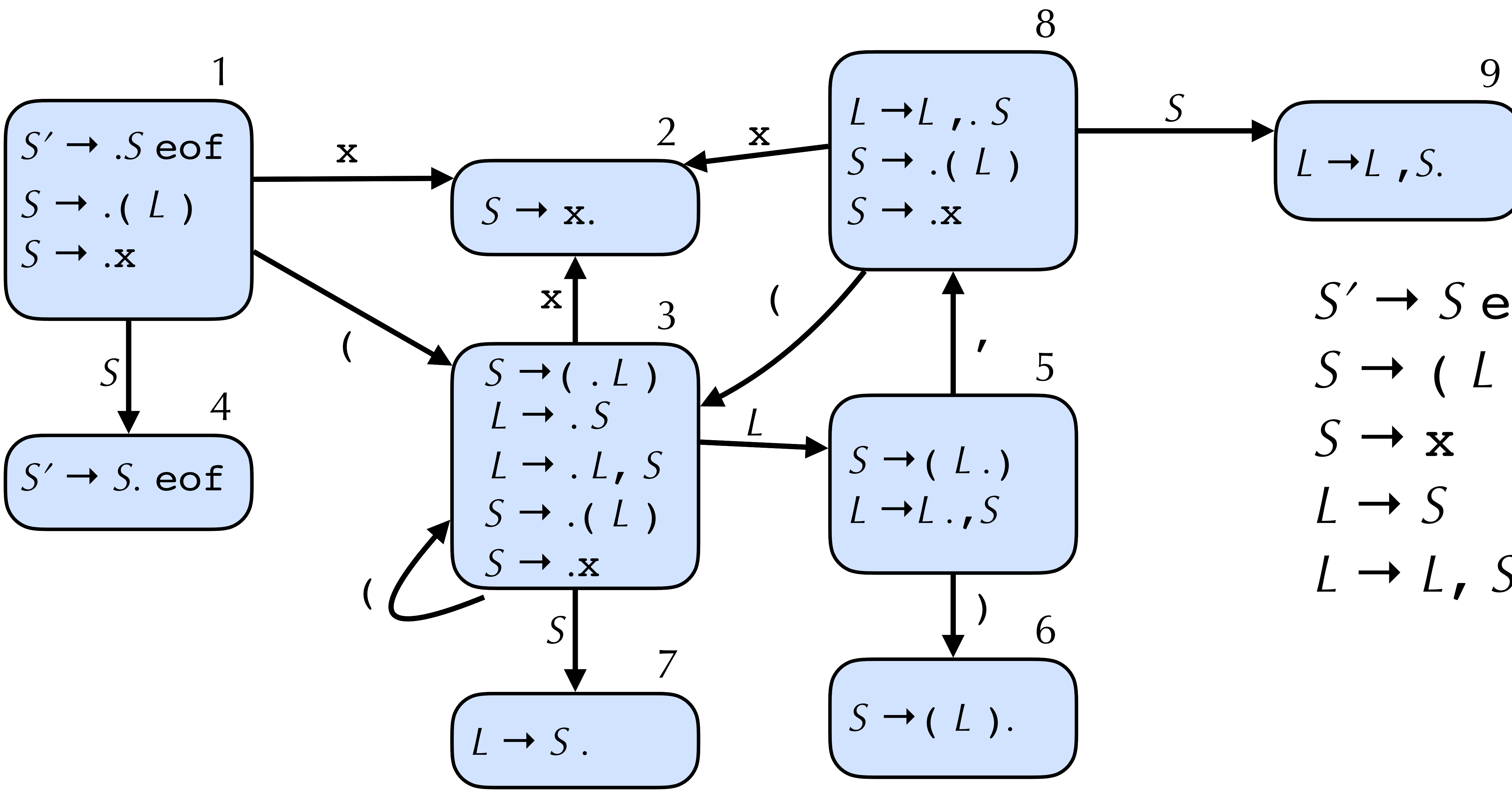
$L \to S$

$L \to L$, S

State 1: can take S, moving us to state 4

State 4 is an accepting state (if at end of input)

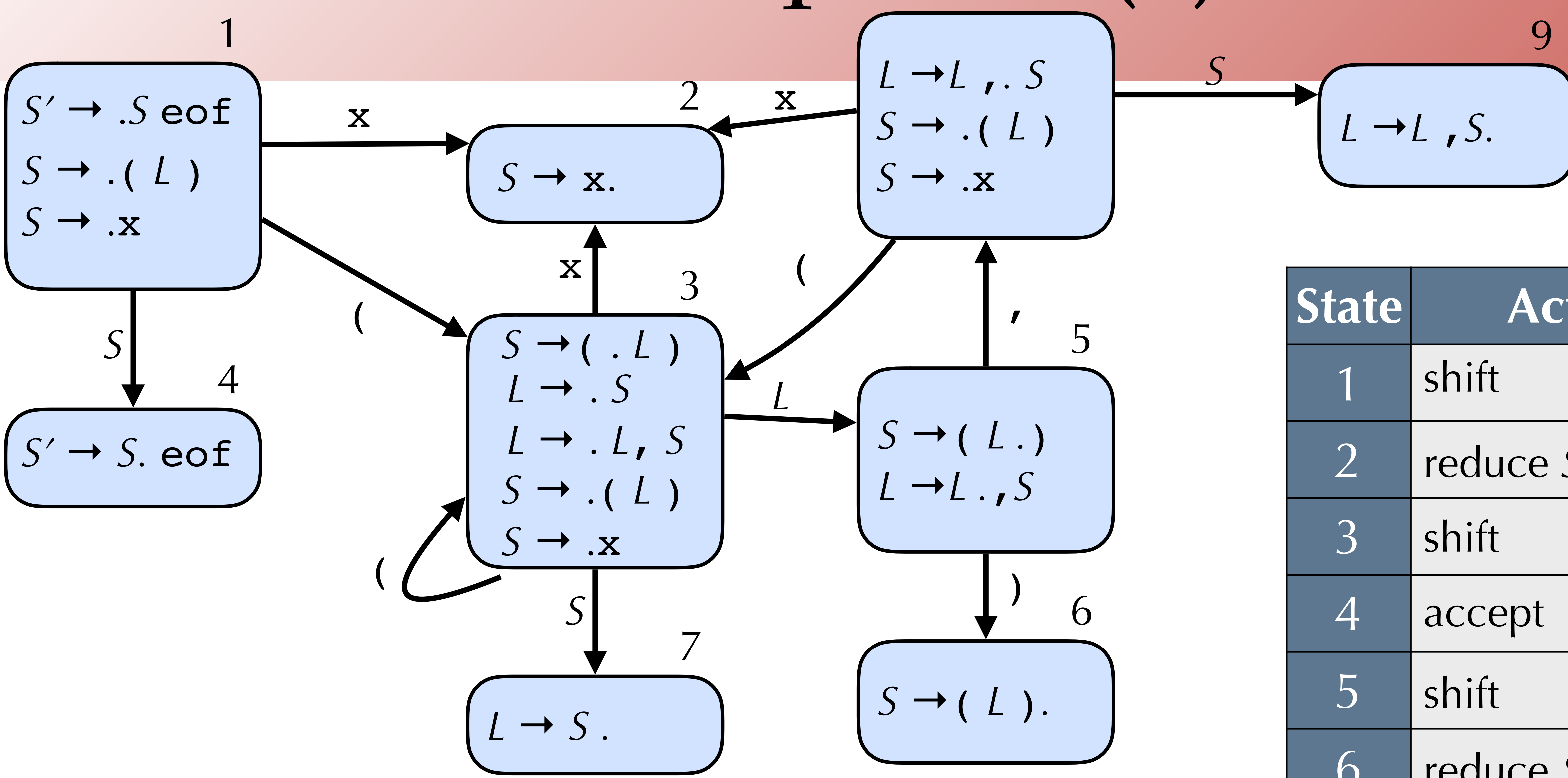
Example: LR(0)



$$
\begin{aligned}
S' &\rightarrow S\ \texttt{eof}\\
S &\rightarrow (\ L\ )\\
S &\rightarrow \texttt{x}\\
L &\rightarrow S\\
L &\rightarrow L,\ S
\end{aligned}
$$

Continue to add states based on next symbol in item

Example LR(0)



State	Action
1	shift
2	reduce $S \rightarrow$ x
3	shift
4	accept
5	shift
6	reduce $S \rightarrow (L)$
7	reduce $L \rightarrow S$
8	shift
9	reduce $L \rightarrow L , S$

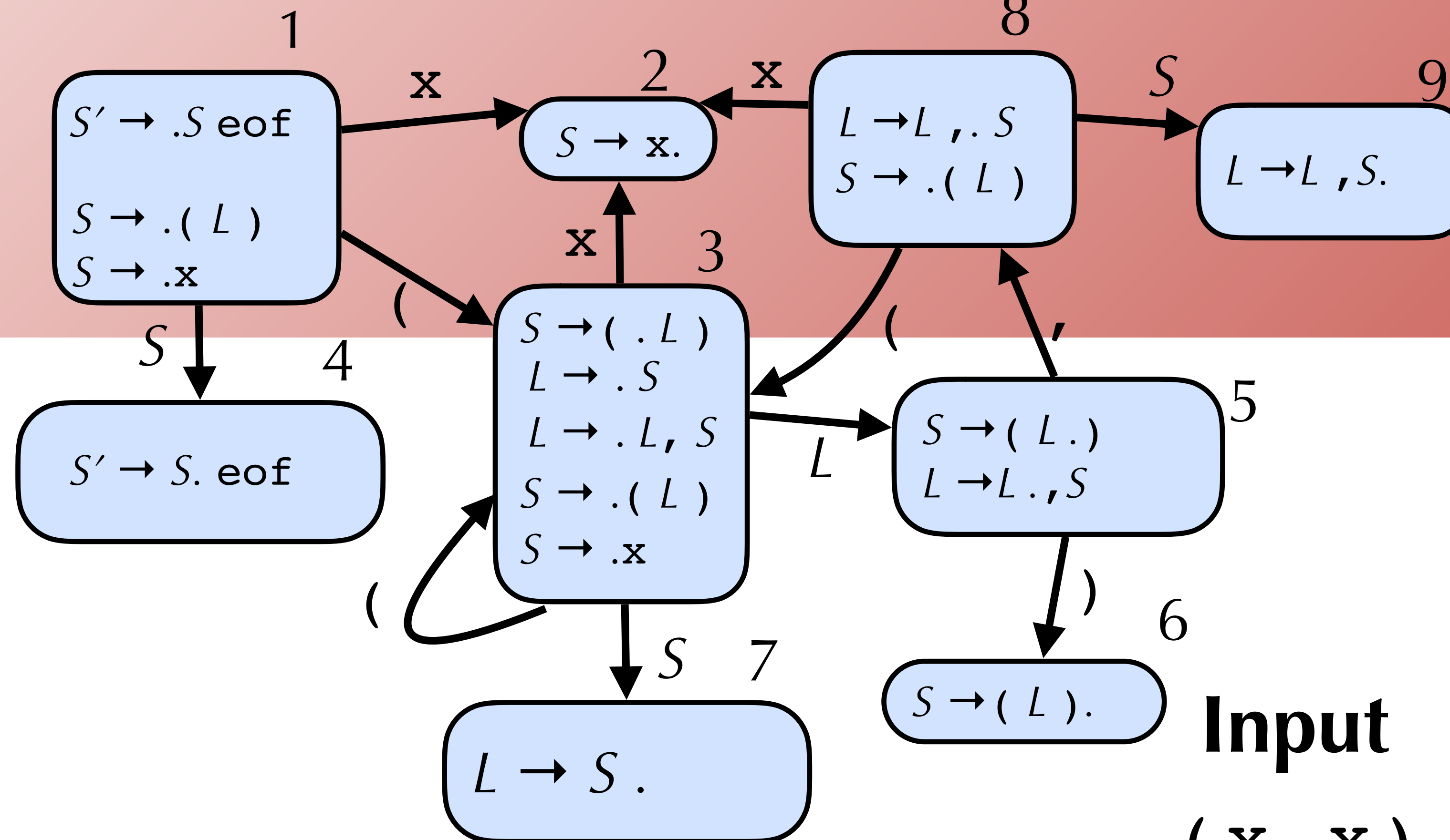
- Build action table
- If state contains item $X \rightarrow \gamma$.eof then **accept**
- If state contains item $X \rightarrow \gamma$. then **reduce** $X \rightarrow \gamma$
- If state *i* has edge to *j* with terminal then **shift**

Using the DFA & Action Table

- At each step, parser runs DFA using symbols on stack as input
  - Input is sequence of terminals and non-terminals from bottom to top
  - Current state of DFA and action table indicate whether to shift or reduce

Example Revisited

$S' \to S$ eof
$S \to (L)$
$S \to$ x
$L \to S$
$L \to L , S$



Stack

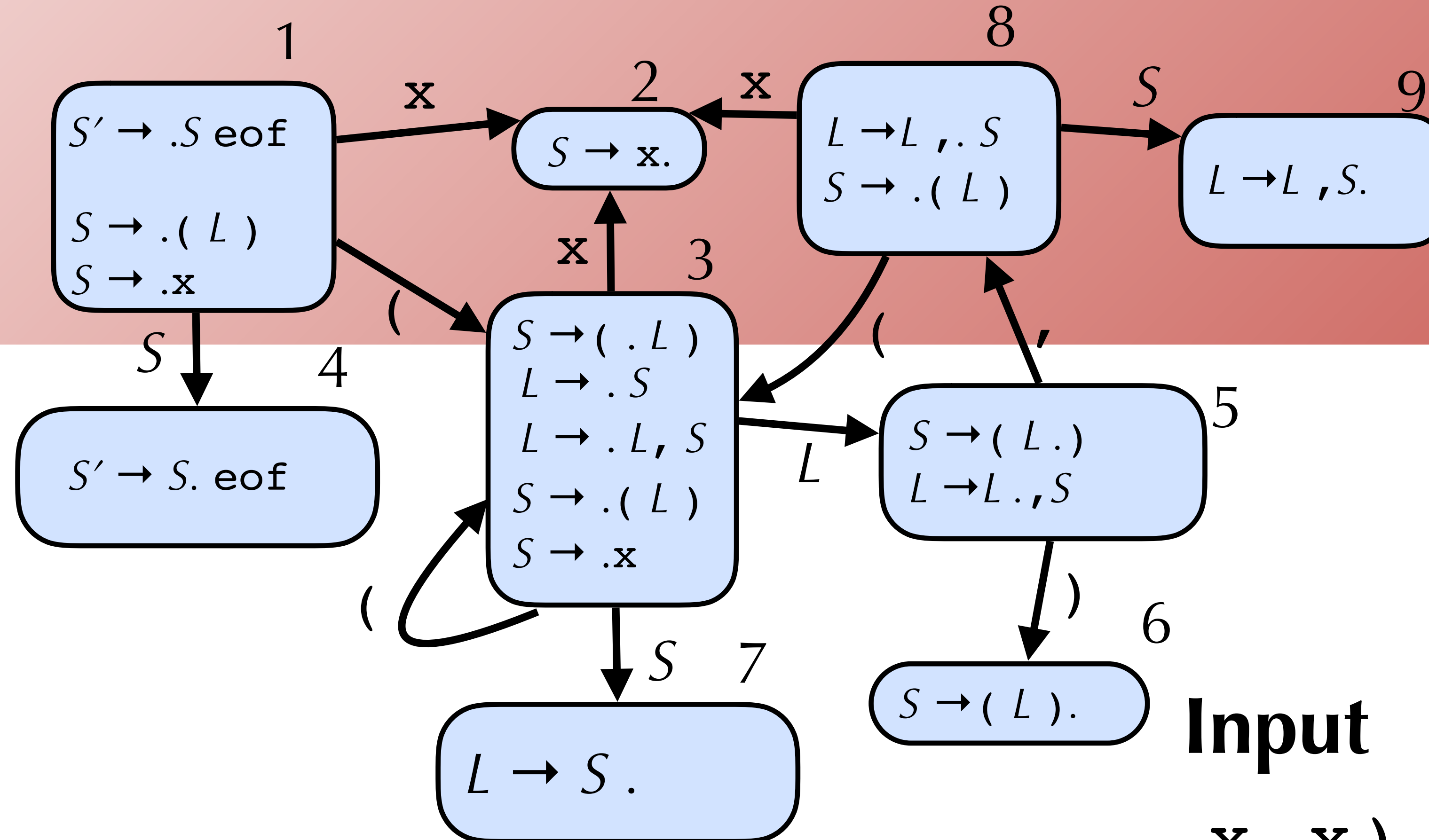
Input

(x , x)

Shift (on to stack

State	Action
1	shift
2	reduce $S \to$ x
3	shift
4	accept
5	shift
6	reduce $S \to (L)$
7	reduce $L \to S$
8	shift
9	reduce $L \to L , S$

Example Revisited



$S' \rightarrow S$ eof

$S \rightarrow (L)$

$S \rightarrow$ x

$L \rightarrow S$

$L \rightarrow L, S$

Stack

(

Input

x , x)

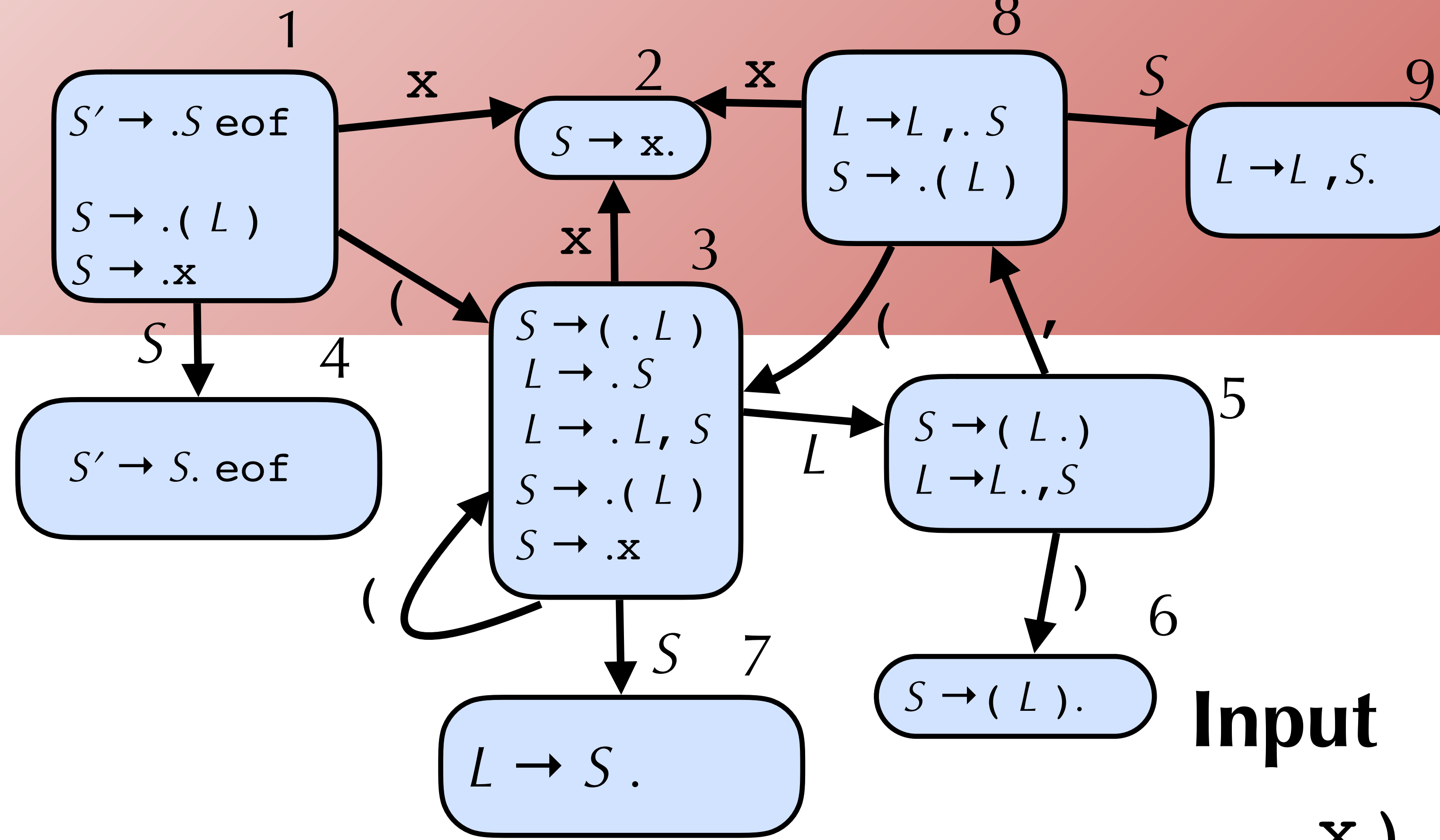
Shift (on to stack

Shift x on to stack

State	Action
1	shift
2	reduce $S \rightarrow$ x
3	shift
4	accept
5	shift
6	reduce $S \rightarrow (L)$
7	reduce $L \rightarrow S$
8	shift
9	reduce $L \rightarrow L , S$

Example Revisited

$S' \to S$ eof
$S \to ($ L $)$
$S \to$ x
$L \to S$
$L \to L, S$



Stack

(x

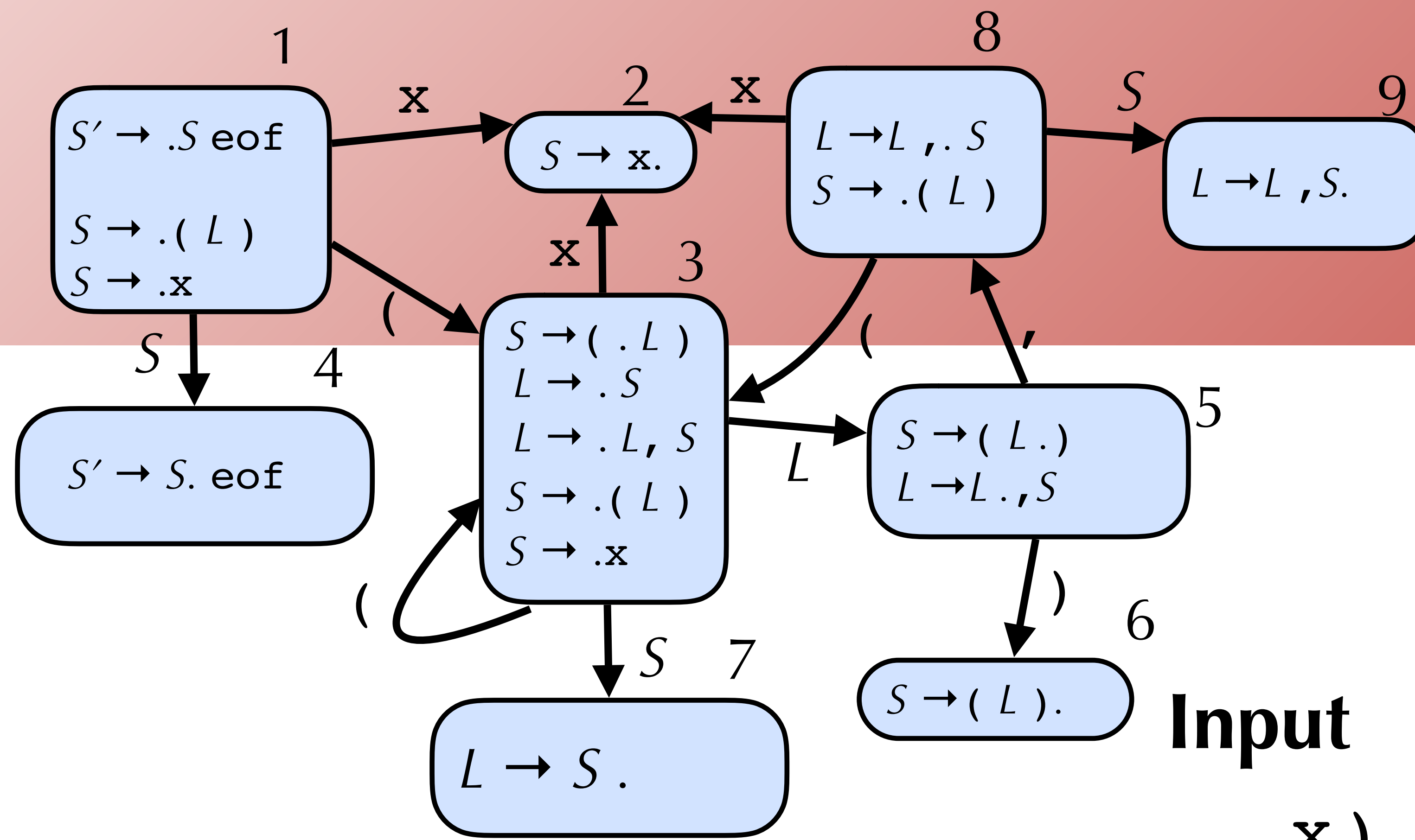
Input

, x)

Shift (on to stack
Shift x on to stack
Reduce $S \to$ x

State	Action
1	shift
2	reduce $S \to$ x
3	shift
4	accept
5	shift
6	reduce $S \to ($ L $)$
7	reduce $L \to S$
8	shift
9	reduce $L \to L$, S

Example Revisited



$S' \to S$ eof

$S \to (L)$

$S \to$ x

$L \to S$

$L \to L, S$

Stack

(S

Input

, x)

Shift (on to stack

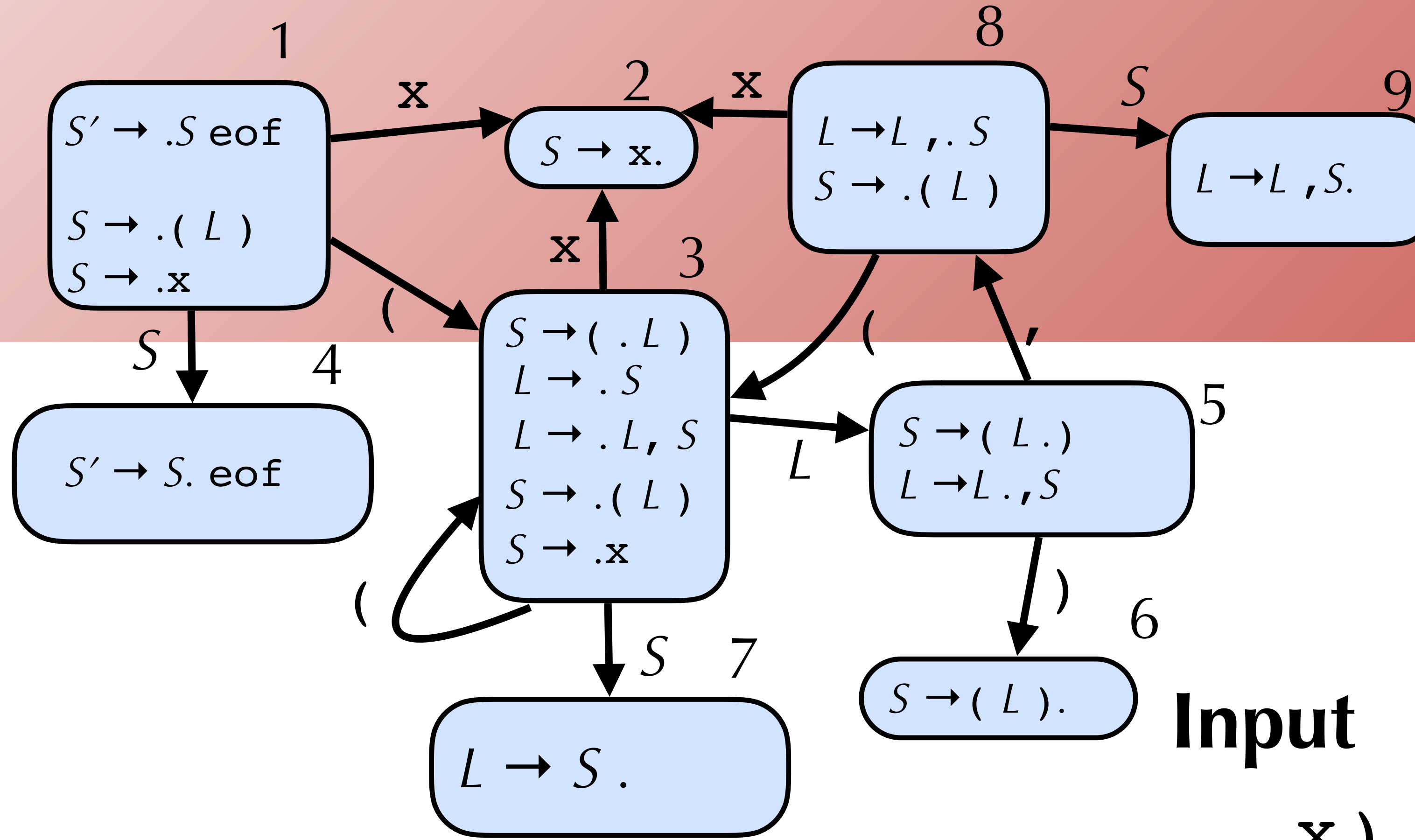
Shift **x** on to stack

Reduce $S \to$ **x**

Reduce $L \to S$

State	Action
1	shift
2	reduce $S \to$ x
3	shift
4	accept
5	shift
6	reduce $S \to (L)$
7	reduce $L \to S$
8	shift
9	reduce $L \to L , S$

Example Revisited



$S' \rightarrow S$ eof
$S \rightarrow$ (L)
$S \rightarrow$ x
$L \rightarrow S$
$L \rightarrow L$, S

Stack

(L

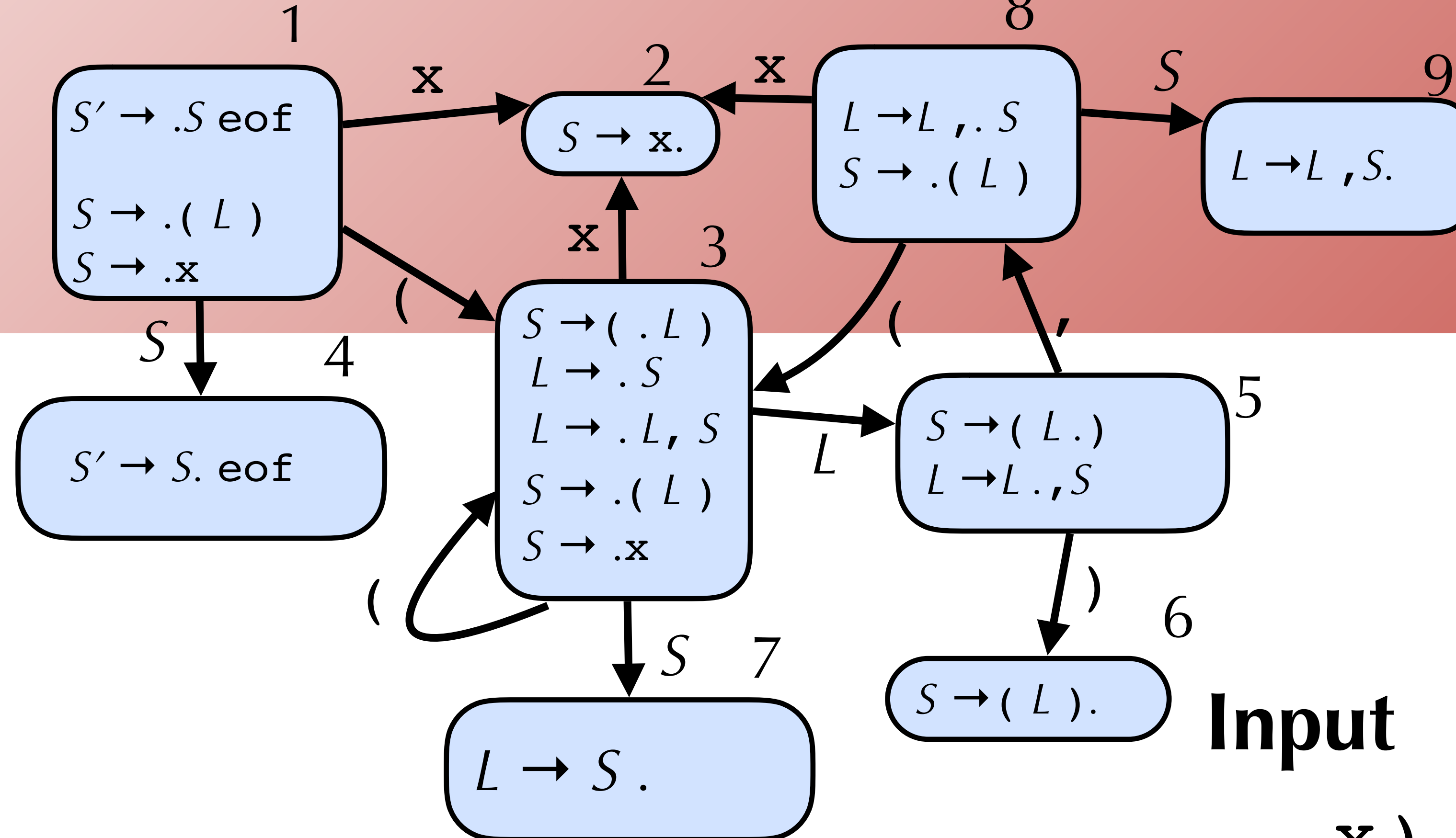
Input

, x)

Shift (on to stack
Shift x on to stack
Reduce $S \rightarrow$ x
Reduce $L \rightarrow S$
Shift , on to stack

State	Action
1	shift
2	reduce $S \rightarrow$ x
3	shift
4	accept
5	shift
6	reduce $S \rightarrow$ (L)
7	reduce $L \rightarrow S$
8	shift
9	reduce $L \rightarrow L$, S

Example Revisited



$S' \to S$ eof
$S \to (L)$
$S \to$ x
$L \to S$
$L \to L , S$

Stack

(L ,

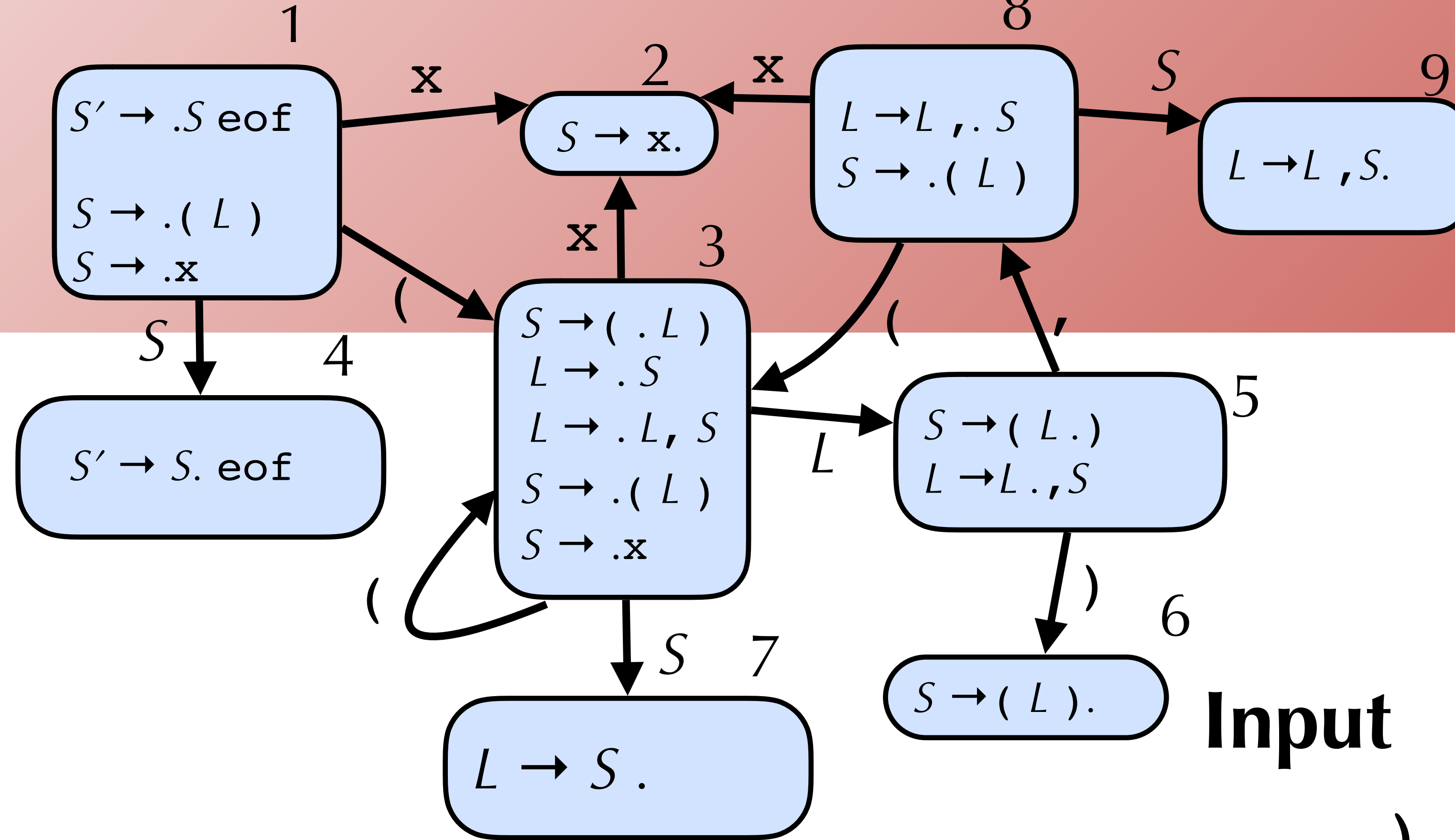
Input

x)

Shift (on to stack
Shift x on to stack
Reduce $S \to$ x
Reduce $L \to S$
Shift , on to stack
Shift x on to stack

State	Action
1	shift
2	reduce $S \to$ x
3	shift
4	accept
5	shift
6	reduce $S \to (L)$
7	reduce $L \to S$
8	shift
9	reduce $L \to L , S$

Example Revisited



$S' \to S$ eof

$S \to$ (L)

$S \to$ x

$L \to S$

$L \to L$, S

Stack

(L , x

Input

)

Shift (on to stack

Shift x on to stack

Reduce $S \to$ x

Reduce $L \to S$

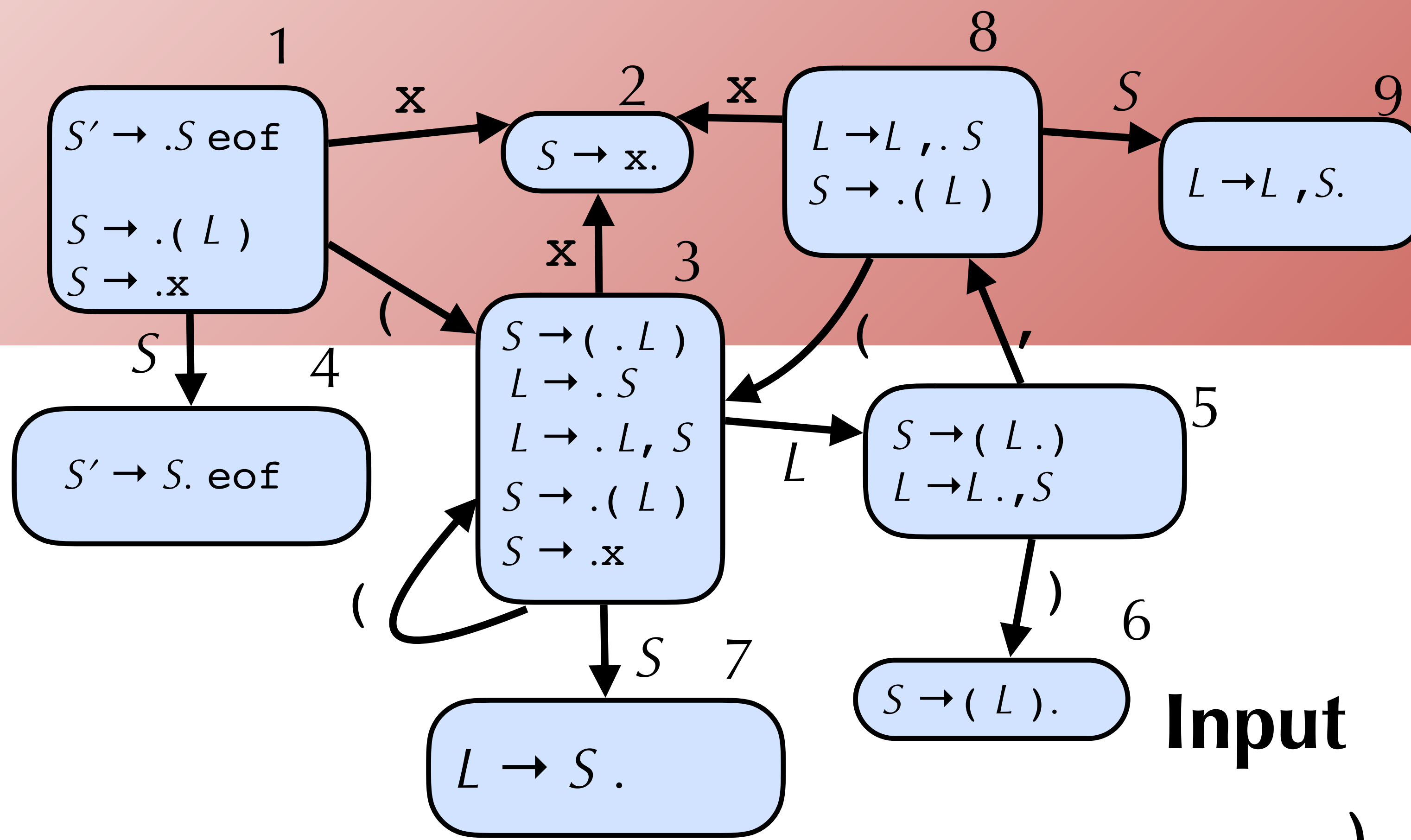
Shift , on to stack

Shift x on to stack

Reduce $S \to$ x

State	Action
1	shift
2	reduce $S \to$ x
3	shift
4	accept
5	shift
6	reduce $S \to$ (L)
7	reduce $L \to S$
8	shift
9	reduce $L \to L$, S

Example Revisited



$S' \to S$ eof
$S \to (L)$
$S \to$ x
$L \to S$
$L \to L , S$

Stack

(L , S

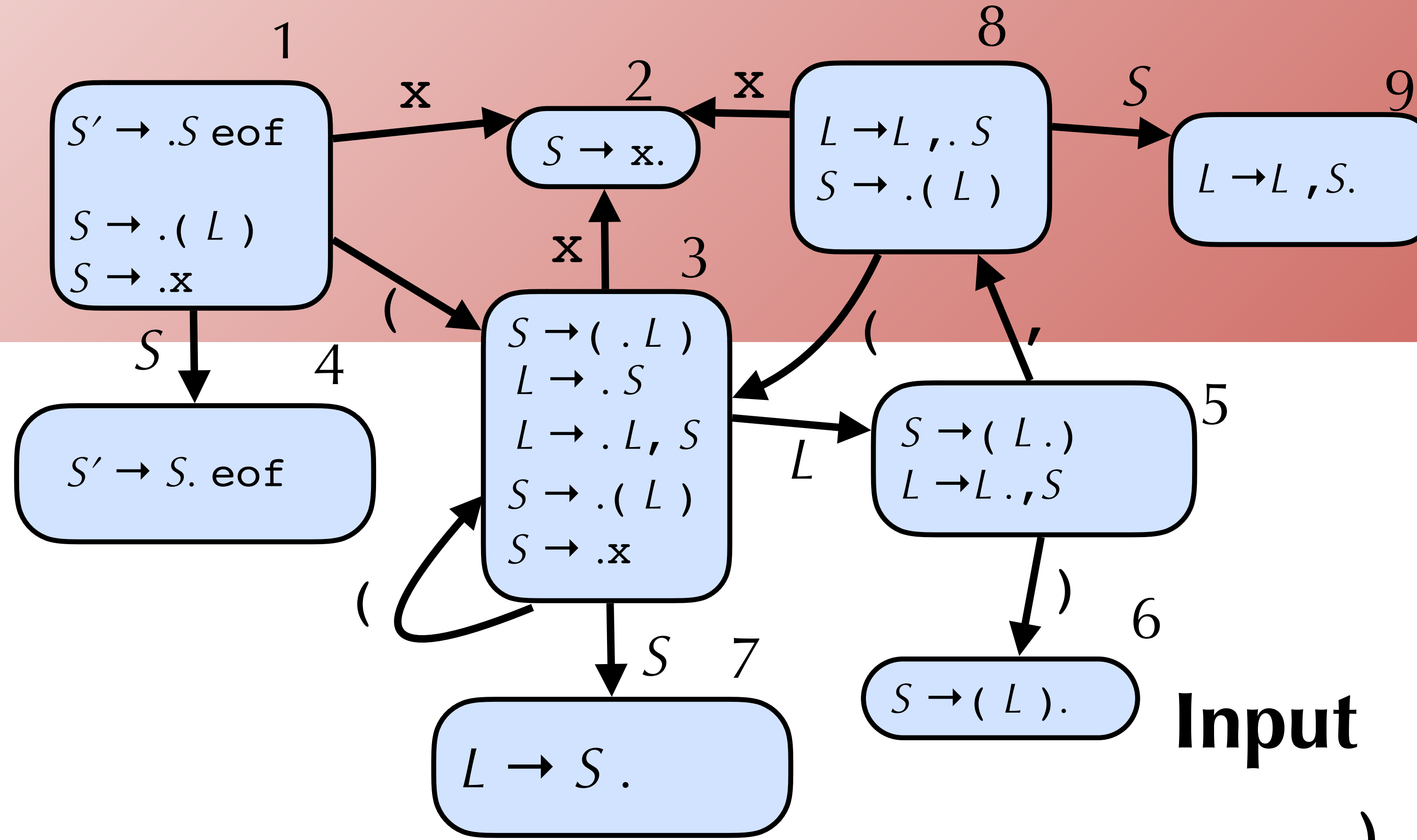
Input

)

Shift (on to stack
Shift x on to stack
Reduce $S \to$ x
Reduce $L \to S$
Shift , on to stack
Shift x on to stack
Reduce $S \to$ x

State	Action
1	shift
2	reduce $S \to$ x
3	shift
4	accept
5	shift
6	reduce $S \to (L)$
7	reduce $L \to S$
8	shift
9	reduce $L \to L , S$

Example Revisited



$S' \to S$ eof
$S \to (L)$
$S \to$ x
$L \to S$
$L \to L , S$

Stack

(L , S

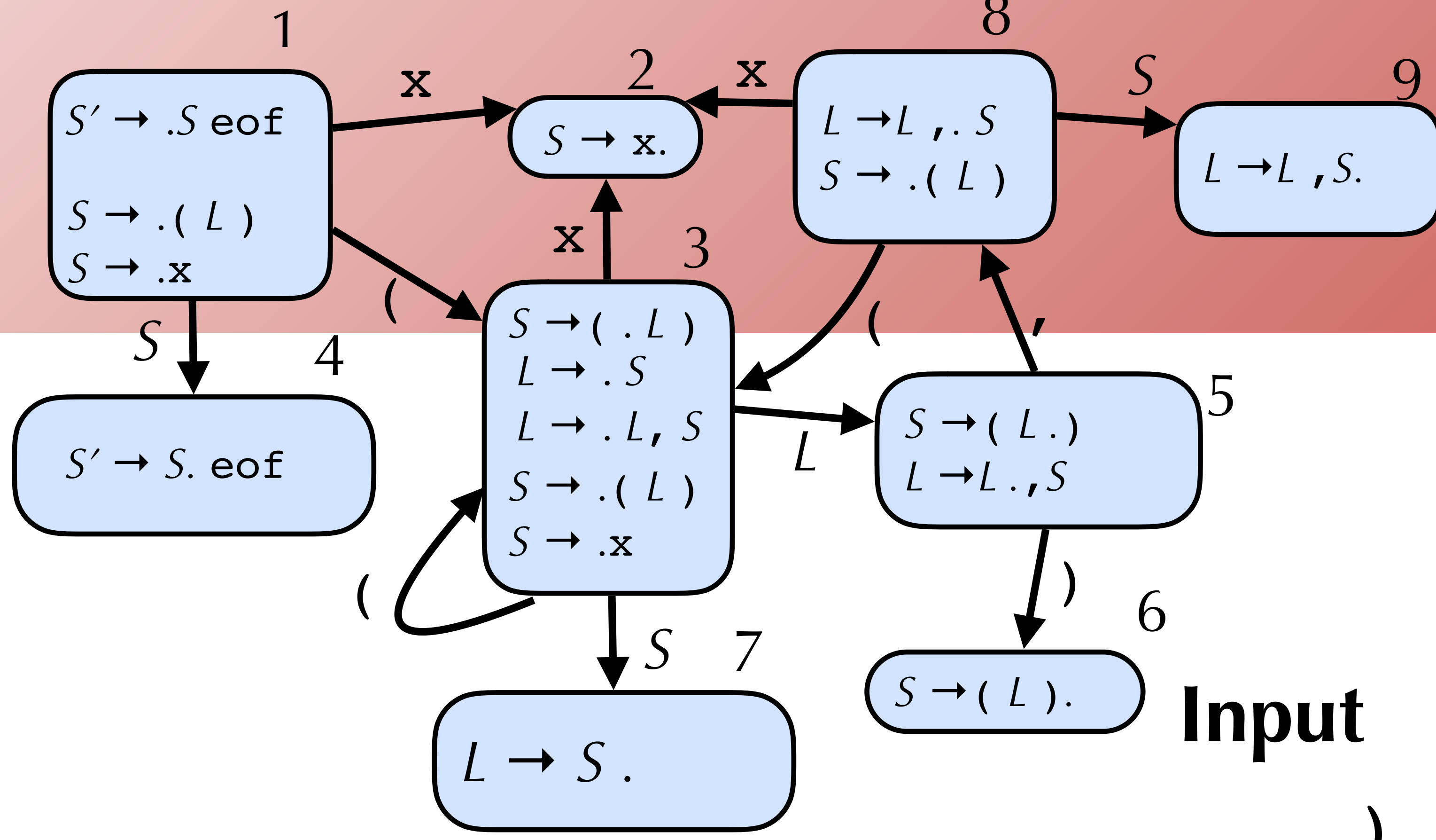
Input

)

Reduce $S \to$ x
Reduce $L \to L , S$

State	Action
1	shift
2	reduce $S \to$ x
3	shift
4	accept
5	shift
6	reduce $S \to (L)$
7	reduce $L \to S$
8	shift
9	reduce $L \to L , S$

Example Revisited



$S' \to S$ eof
$S \to$ (L)
$S \to$ x
$L \to S$
$L \to L$, S

Stack

(L

Input

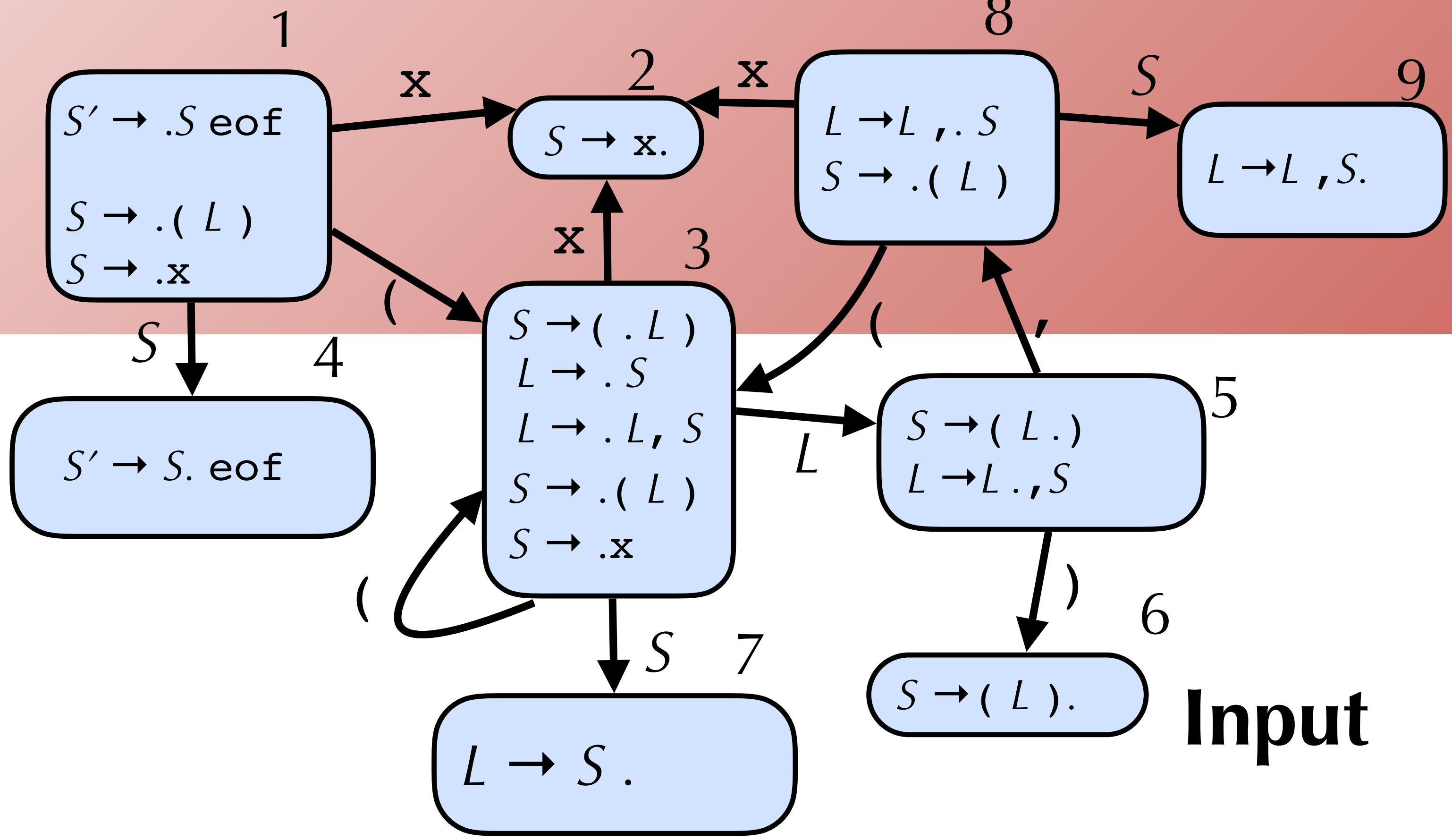
)

Reduce $S \to$ **x**
Reduce $L \to L$, S
Shift) on to stack

State	Action
1	shift
2	reduce $S \to$ x
3	shift
4	accept
5	shift
6	reduce $S \to$ (L)
7	reduce $L \to S$
8	shift
9	reduce $L \to L$, S

Example Revisited

$S' \to S$ eof
$S \to (L)$
$S \to$ x
$L \to S$
$L \to L, S$



Stack

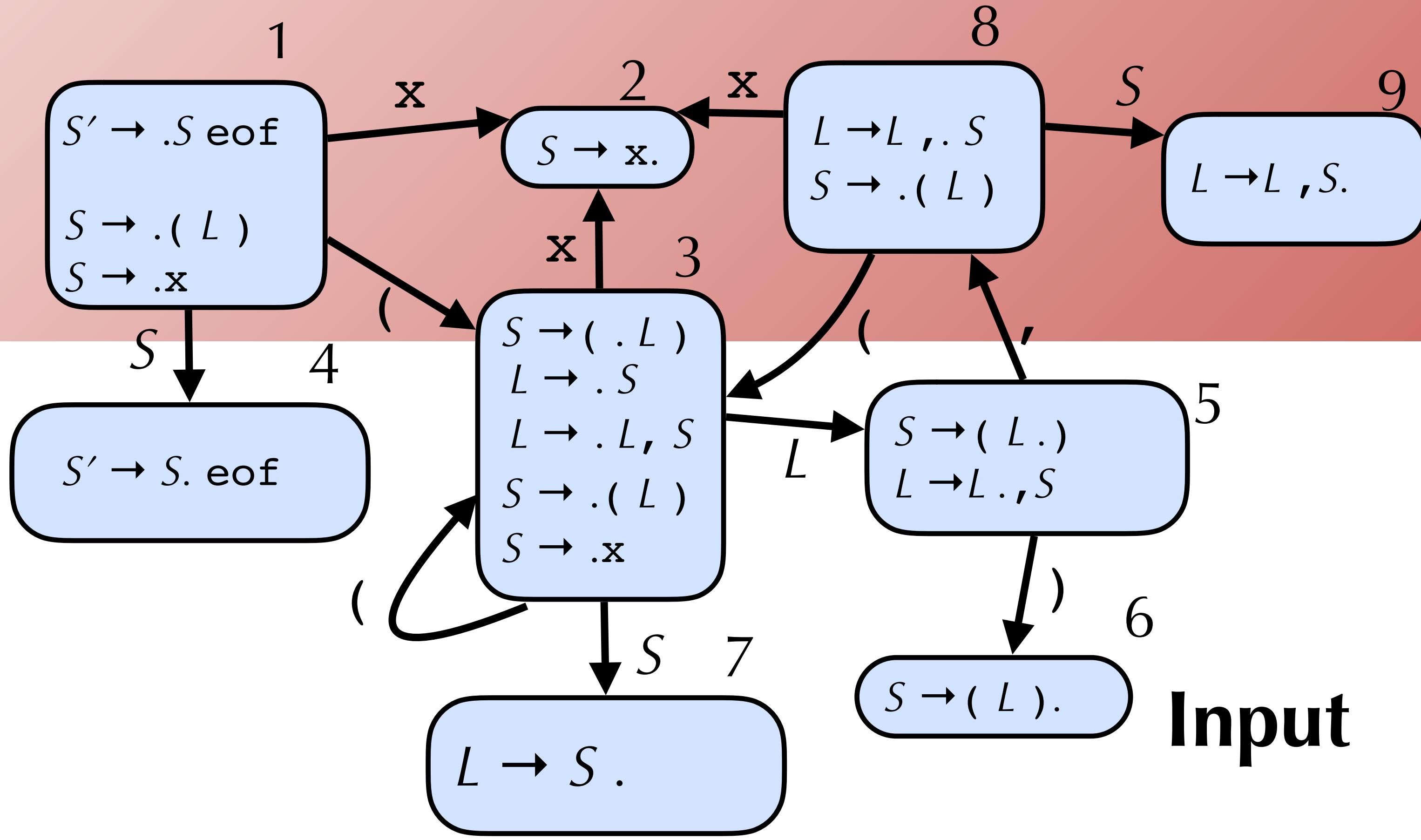
(L)

Input

Reduce $S \to$ x
Reduce $L \to L, S$
Shift) on to stack
Reduce $S \to (L)$

State	Action
1	shift
2	reduce $S \to$ x
3	shift
4	accept
5	shift
6	reduce $S \to (L)$
7	reduce $L \to S$
8	shift
9	reduce $L \to L , S$

Example Revisited



$S' \to S$ eof
$S \to (L)$
$S \to$ x
$L \to S$
$L \to L, S$

Stack

S

Input

Reduce $S \to$ **x**
Reduce $L \to L, S$
Shift) on to stack
Reduce $S \to (L)$

Accept!

State	Action
1	shift
2	reduce $S \to$ x
3	shift
4	accept
5	shift
6	reduce $S \to (L)$
7	reduce $L \to S$
8	shift
9	reduce $L \to L , S$

Implementation Details

- Optimization: no need to run DFA from start state each time
  - Use stack to also record information about which DFA state corresponds to it
- Combine DFA and action table into single lookup table

LR(0) Limitations

- An LR(0) machine only works if states with reduce actions have a single reduce action.
  - In such states, the machine always reduces (ignoring lookahead)
- With more complex grammars, the DFA construction will yield states with shift/reduce and reduce/reduce conflicts:

OK	Shift/reduce conflict	Reduce/reduce conflict
$S \rightarrow (\ L\ ).$	$S \rightarrow (\ L\ ).$ $L \rightarrow .L\ ,\ S$	$S \rightarrow L , S.$ $S \rightarrow , S.$

- Such conflicts can often be resolved by using a look-ahead symbol: LR(1)

LR(1)

- In practice, LR(1) is used for LR parsing
  - not LR(0) or LR(*k*) for *k*>1
- Item is now pair ($X \rightarrow \gamma\ .\ \delta$, x)
  - Indicates that γ is at the top of the stack, and at the head of the input there is a string derivable from δx (where x is terminal)
  - Algorithm for constructing state transition table and action table adapted. See Appel for details.
    - Closure operation when constructing states uses FIRST(), incorporating lookahead token
    - Action table columns now terminals (i.e., 1-token lookahead)
    - Note: state transition relation and action table typically combined into single table, called **parsing table**

LR(0) Conflicts

- Consider the left associative and right associative "sum" grammars:

left	right
$S \to S + E$	$S \to E + S$
$S \to E$	$S \to E$
$E \to \texttt{num}$	$E \to \texttt{num}$
$E \to (S)$	$E \to (S)$

- One is LR(0) the other isn't… which is which and why?
- What kind of conflict do you get? Shift/reduce or Reduce/reduce?
- Right associative gives a Shift/reduce conflict
  - Between items $S \to E. + S$ and $S \to E.$
- Ambiguities in associativity/precedence usually lead to shift/reduce conflicts

Dangling Else Problem

- Many language have productions such as

  $S \rightarrow$ `if` E `then` S `else` S

  $S \rightarrow$ `if` E `then` S

  $S \rightarrow$ …

- Program `if a then if b then s1 else s2` could be

  either `if a then { if b then s1 } else s2`

  or `if a then {if b then s1 else s2 }`

- In LR parsing table there will be a shift-reduce conflict
  - $S \rightarrow$ `if` E `then` S . with lookahead `else`: reduce
  - $S \rightarrow$ `if` E `then` S . `else` S with any lookahead: shift
  - Which action corresponds to which interpretation of

    `if a then if b then s1 else s2` ?

Resolving Ambiguity

- Could rewrite grammar to avoid ambiguity
  - E.g.,

$$S \rightarrow O$$
$$O \rightarrow V \texttt{ := } E$$
$$O \rightarrow \texttt{if } E \texttt{ then } O$$
$$O \rightarrow \texttt{if } E \texttt{ then } C \texttt{ else } O$$
$$C \rightarrow V \texttt{ := } E$$
$$C \rightarrow \texttt{if } E \texttt{ then } C \texttt{ else } C$$

Resolving Ambiguity

- Or tolerate conflicts, indicating how to resolve conflict
  - E.g., for dangling else, prefer shift to reduce.
    - i.e., for `if a then if b then s1 else s2`
      prefer `if a then {if b then s1 else s2 }`
      over `if a then { if b then s1 } else s2`
    - i.e., `else` binds to closest `if`
  - Expression grammars can express operator-precedence by resolution of conflicts
- Use sparingly! Only in well-understood cases
  - Most conflicts are indicative of ill-specified grammars

YACC and Menhir

- **Y**et **A**nother **C**ompiler-**C**ompiler
  - Originally developed in early 1970s
  - Various versions/reimplimentations
    - Berkeley Yacc, Bison, Ocamlyacc, ...
  - From a suitable grammar, constructs an LALR(1) parser
    - A kind of LR parser, not as powerful as LR(1)
    - Most practical LR(1) grammars will be LALR(1) grammars
- Menhir
  - "90% compatible with ocamlyacc"
  - Adds some additional features including better explanations of conflicts

Menhir

- Usage: `menhir` *`options grammar`*`.mly`
- Produces output files
  - *`grammar`*`.ml`: OCaml code for a parser
  - *`grammar`*`.mli`: interface for parser

Structure of Menhir File

```
%{
  header
%}
  declarations
%%
  rules
%%
  trailer
```

- Header and trailer are arbitrary OCaml code, copied to the output file
- Declarations of tokens, start symbols, OCaml types of symbols, associativity and precedence of operators
- Rules are productions for non-terminals, with **semantic actions** (OCaml expressions that are executed with production is reduced, to produce value for symbol)

Menhir example

- See `parser-eg.mll`
  and output files `parser-eg.ml`
  and `parser-eg.mli`
- Typically, the `.mly` declares the tokens, and the lexer opens the parser module
- You can get verbose ocamlyacc debugging information by doing:
  - `menhir --explain` ...
  - or, if using ocamlbuild:
    `ocamlbuild –use-menhir -yaccflag -–explain` ...
  - The result is a <basename>.conflicts file that contains a description of the error
  - The parser items of each state use the '.' just as described above
  - The flag `--dump` generates a full description of the automaton
  - Example: see `start-parser.mly`