HARVARD
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CS153: Compilers
Lecture 20: Dataflow analysis

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https://www.seas.harvard.edu/courses/cs153

Contains content from lecture notes by Steve Zdancewic and Greg Morrisett

Announcements

- HW5: Oat v.2 out
  - Due Tuesday 19 Nov
- HW6 will be released Tuesday 12 Nov
  - 3 weeks to complete

Today

- Dataflow analysis
- Liveness analysis
  - Worklist algorithm
- Generalizing dataflow analysis
  - Available expressions
  - Reaching definitions

Motivating Code Analyses

- There are lots of things that might influence the safety/applicability of an optimization
  - How do you know an expression is invariant?
  - How do you know if an expression has no side effects?
  - How do you keep track of where a variable is defined?
  - How do you know where a variable is used?
  - How do you know if two reference values may be aliases of one another?
- Today: algorithms and data structures useful for answering these questions

Moving Towards Register Allocation

- Oat compiler currently generates as many temporary variables as needed
  - The `%uids` that that you are very familiar with...
- Current compilation strategy:
  - Each `%uid` maps to a stack location
  - Yields programs with many loads/stores to memory
  - Very inefficient!
- Ideally, map as many `%uid`'s as possible into registers.
  - Eliminate the use of the `alloca` instruction?
  - Only 16 max registers available on 64-bit X86
  - `%rsp` and `%rbp` are reserved and some have special semantics, so really only 10 or 12 available
  - This means that a register must hold more than one slot
- When is this safe?

Liveness

- Observation: `%uid1` and `%uid2` can be assigned to the same register if their values will not be needed at the same time.
    - A `%uid` is"needed" if its contents will be used as a source operand in a later instruction.
- Such a variable is called "**live**"
- Two variables can share the same register if they are not live at the same time.

Scope vs. Liveness

- We can already get some coarse liveness information from variable scoping.
- Consider the following Oat program:

```
int f(int x) {
  var a = 0;
  if (x > 0) {
    var b = x * x;
    a = b + b;
  }
  var c = a * x;
  return c;
}
```

- Note that due to Oat's scoping rules, variables `b` and `c` can never be live at the same time.
  - `c`'s scope is disjoint from `b`'s scope
- So, we could assign `b` and `c` to the same `alloca`'ed slot and potentially to the same register.

But Scope is too Coarse

- Consider this program:

```
int f(int x) {
   int a = x + 2;
   int b = a * a;
   int c = b + x;
   return c;
}
```

`x` is live

`a` and `x` are live

`b` and `x` are live

`c` is live

- The scopes of `a`,`b`,`c`,`x` all overlap – they're all in scope at the end of the block.
- But `a`, `b`, `c` are never live at the same time.
  - So they can share the same stack slot / register

Live Variable Analysis

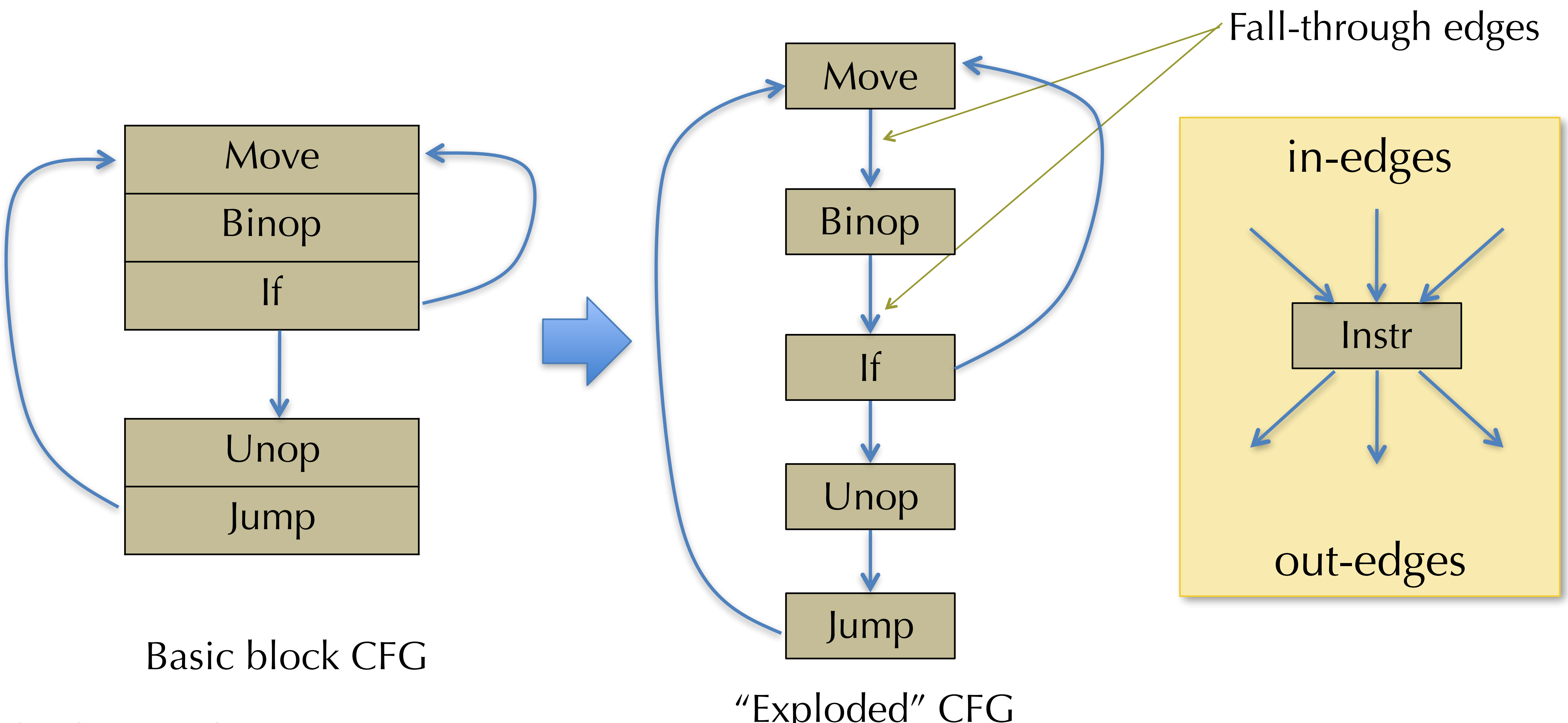
- Variable v is **live** at a program point if v is defined before the program point and used after it.
- Liveness is defined in terms of where variables are defined and where variables are used
- Liveness analysis: Compute the live variables between each statement.
  - May be conservative (i.e., may claim a variable is live when it isn't)
    - Safe approximation!
  - To be useful, it should be more precise than simple scoping rules.
- Liveness analysis is one example of dataflow analysis
  - Other examples: Available Expressions, Reaching Definitions, Constant-Propagation Analysis, …

Control-flow Graphs Revisited

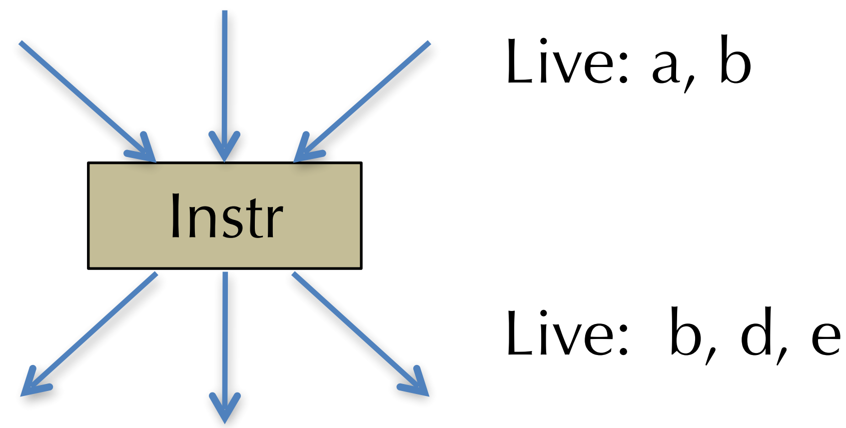
- Recall: a basic block is a sequence of instructions such that:
  - There is a distinguished, labeled entry point (no jumps into the middle of a basic block)
  - There is a (possibly empty) sequence of non-control-flow instructions
  - The block ends with a single control-flow instruction (jump, conditional branch, return, etc.)
- In a control flow graph (CFG), nodes are basic blocks
  - There is an edge from B1 to B2 if the control-flow instruction of B1 might jump to the entry label of B2
  - There are no "dangling" edges – there is a block for every jump target.
- Note: the following slides are intentionally ambiguous about the exact nature of the code in the CFGs
  - CFGs and dataflow analysis work for x86 assembly, imperative C-like source, LLVM IR, ...
  - Same general idea, but the exact details differ
  - e.g. LLVM IR doesn't have "imperative" update of %uid temporaries. SSA structure of the LLVM IR makes some of these analyses simpler.

Dataflow over CFGs

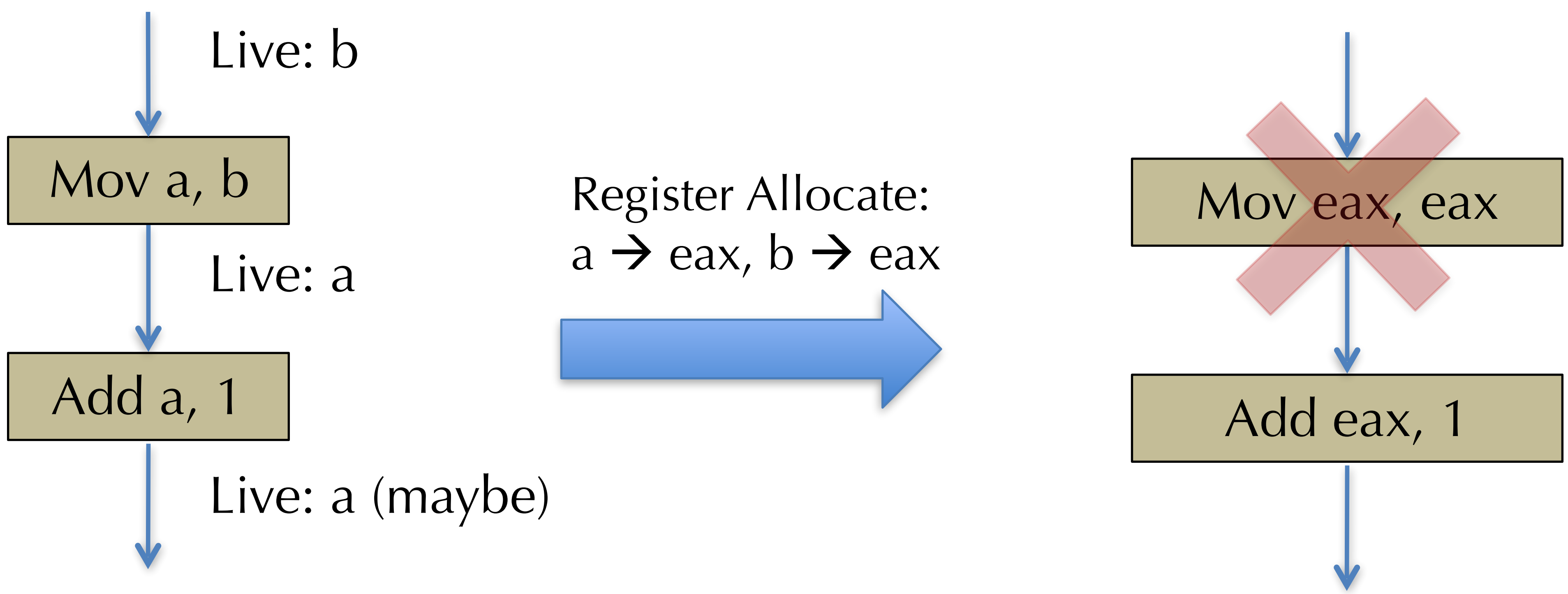
- For precision, it is helpful to think of the "fall through" between sequential instructions as an edge of the control-flow graph too.
  - Different implementation tradeoffs in practice…



Liveness is Associated with **Edges**



- This is useful so that the same register can be used for different temporaries in the same statement.
- Example: `a = b + 1`
- Compiles to:



Uses and Definitions

- Every instruction/statement **uses** some set of variables
  - i.e., reads from them
- Every instruction/statement **defines** some set of variables
  - i.e., writes to them

- For a node/statement s define:
  - use[s] : set of variables used by s
  - def[s] : set of variables defined by s
- Examples:
  - `a = b + c` use[s] = {`b`,`c`} def[s] = {`a`}
  - `a = a + 1` use[s] = {`a`} def[s] = {`a`}

Liveness, Formally

- Variable v is **live** on edge e if:
  - (1) there is a node n in the CFG such that use[n] contains v, and
  - (2) there is a directed path from e to n such that for every statement s′ on the path, def[s′] does not contain v
- Clause (1) says that v will be used on some path starting from edge e
- Clause (2) says that v won't be redefined on that path before the use

- Questions:
  - How to compute this efficiently?
  - How to use this information (e.g., for register allocation)?
  - How does the choice of IR affect this?
    (e.g. LLVM IR uses SSA, so it doesn't allow redefinition, which simplifies liveness analysis)

Simple, inefficient algorithm

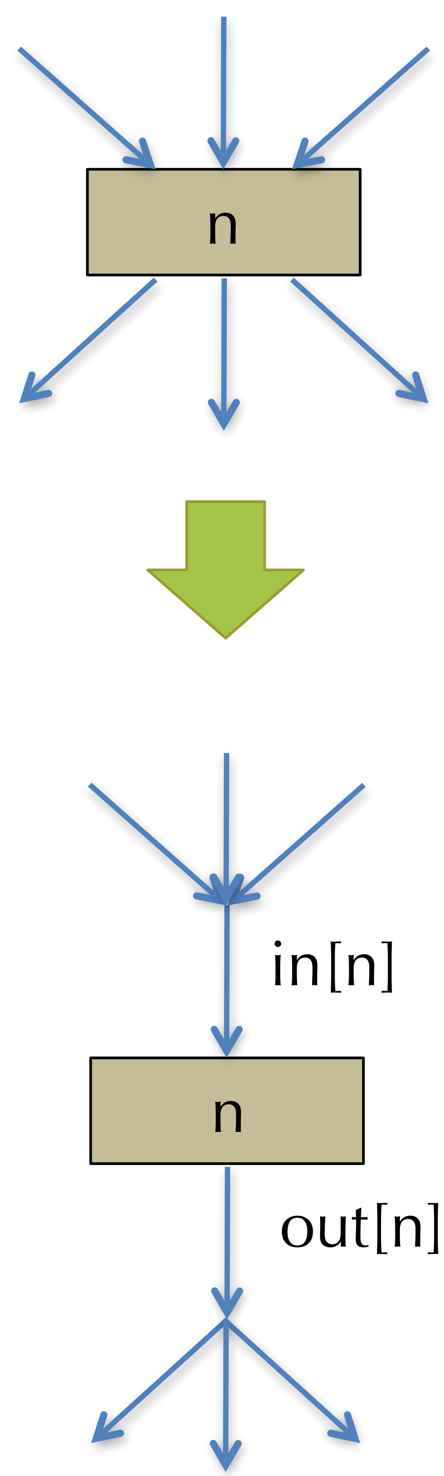
- "A variable v is live on an edge e if there is a node n in the CFG using it and a directed path from e to n that does not define v"
- Backtracking Algorithm:
  - For each variable v…
  - Try all paths from each use of v, tracing backwards through the control-flow graph until either v is defined or a previously visited node has been reached.
  - Mark the variable v live on each edge traversed.
- Inefficient because it explores the same paths many times (for different uses and different variables)

Dataflow Analysis

- **Idea:** compute liveness information for all variables simultaneously
  - Keep track of sets of information about each node
- Approach: define equations that must be satisfied by any liveness determination
  - Equations based on "obvious" constraints.
- Solve the equations by iteratively converging on a solution.
  - Start with a "rough" approximation to the answer
  - Refine the answer at each iteration
  - Keep going until no more refinement is possible: a **fixpoint** has been reached
- This is an instance of a general framework for computing program properties: dataflow analysis

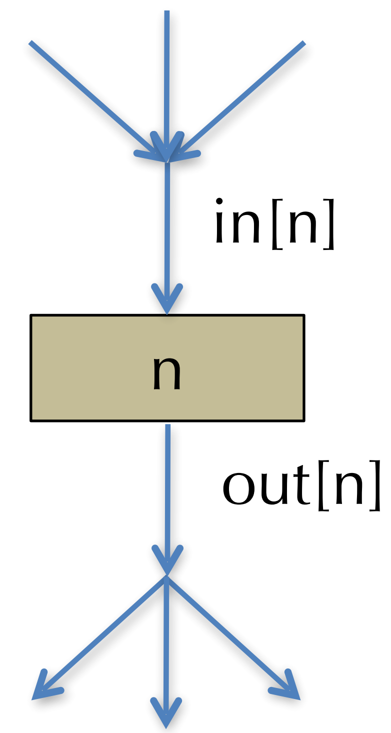
Dataflow Value Sets for Liveness

- Nodes are program statements, so, for each n, define the following sets:
    - use[n] : set of variables used by n
    - def[n] : set of variables defined by n
    - in[n] : set of variables live on entry to n
    - out[n] : set of variables live on exit from n
- Associate in[n] and out[n] with the “collected” information about incoming/ outgoing edges
    - i.e., out[n] is union of all liveness information on outgoing edges of n
- For liveness, what constraints are there among these sets?



Liveness Dataflow Constraints

- We have: $in[n] \supseteq use[n]$
  - "A variable must be live on entry to n if it is used by n"



- Also: $in[n] \supseteq out[n] - def[n]$
  - "If a variable is live on exit from n, and n doesn't define it, then it is live on entry to n"
  - Note: here '–' means "set difference"
- And: $out[n] \supseteq in[n']$ if $n' \in succ[n]$
  - "If a variable is live on entry to a successor node of n, it must be live on exit from n."

Iterative Dataflow Analysis

- Find a solution to those constraints by starting from a rough guess.
- Start with: $in[n] = \varnothing$ and $out[n] = \varnothing$
- They don't satisfy the constraints:
  - $in[n] \supseteq use[n]$
  - $in[n] \supseteq out[n] - def[n]$
  - $out[n] \supseteq in[n']$ if $n' \in succ[n]$
- Idea: iteratively re-compute in[n] and out[n] where forced to by the constraints
  - Each iteration will add variables to the sets in[n] and out[n] (i.e. the live variable sets will increase monotonically)
- We stop when in[n] and out[n] satisfy these equations: (which are derived from the constraints above)
  - $in[n] = use[n] \cup (out[n] - def[n])$
  - $out[n] = \cup_{n' \in succ[n]} in[n']$

Complete Liveness Analysis Algorithm

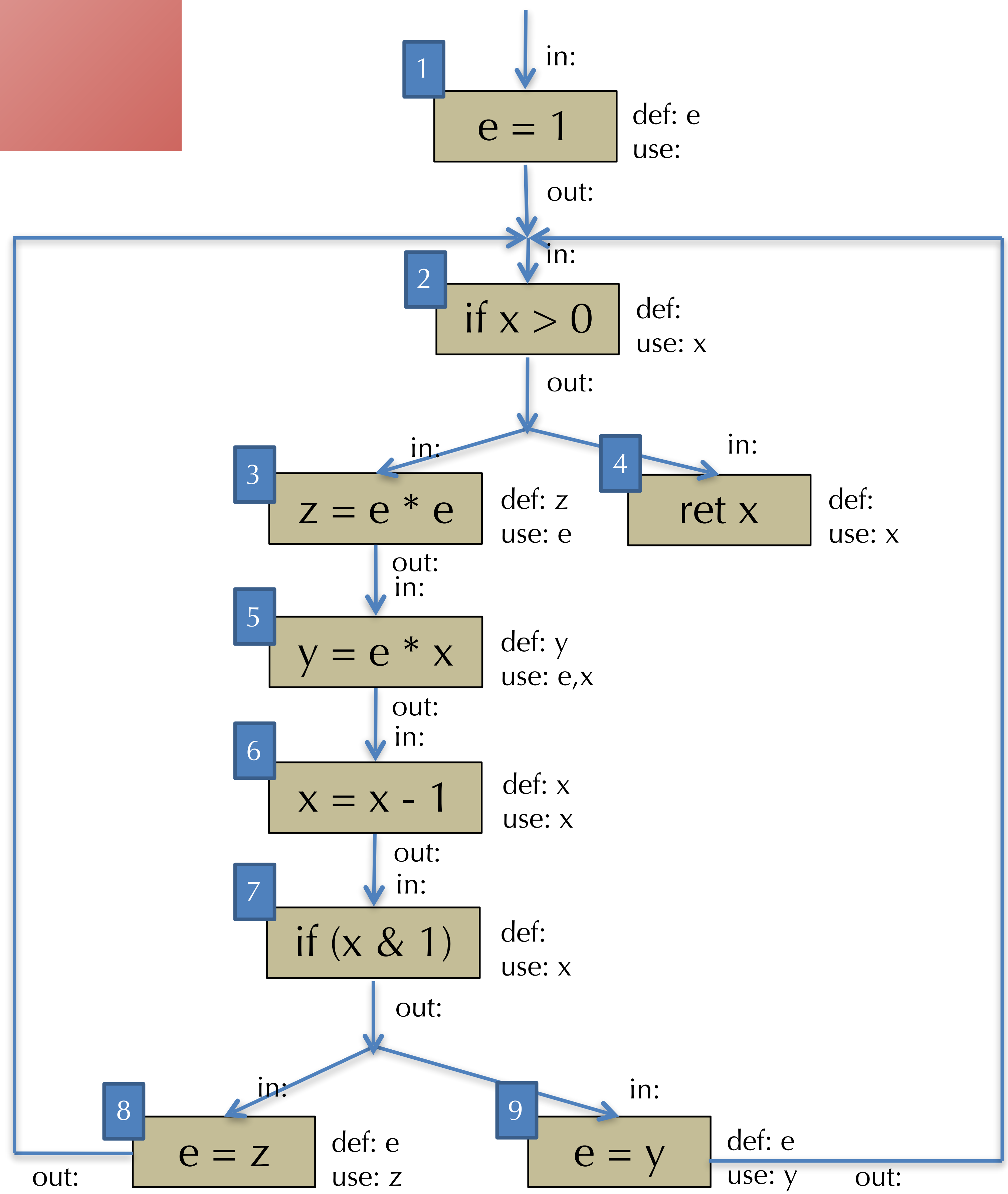
for all n, in[n] := ∅, out[n] := ∅
repeat until no change in 'in' and 'out'
    for all n
        out[n] := $\cup_{n' \in succ[n]}$ in[n']
        in[n] := use[n] ∪ (out[n] – def[n])
    end
end

- Finds a fixpoint of the in and out equations.
    - The algorithm is guaranteed to terminate… Why?
- Why do we start with ∅?

Example Liveness Analysis

- Example flow graph:

```
e = 1;
while(x>0) {
  z = e * e;
  y = e * x;
  x = x – 1;
  if (x & 1) {
    e = z;
  } else {
    e = y;
  }
}
return x;
```



Example Liveness Analysis

Each iteration update:

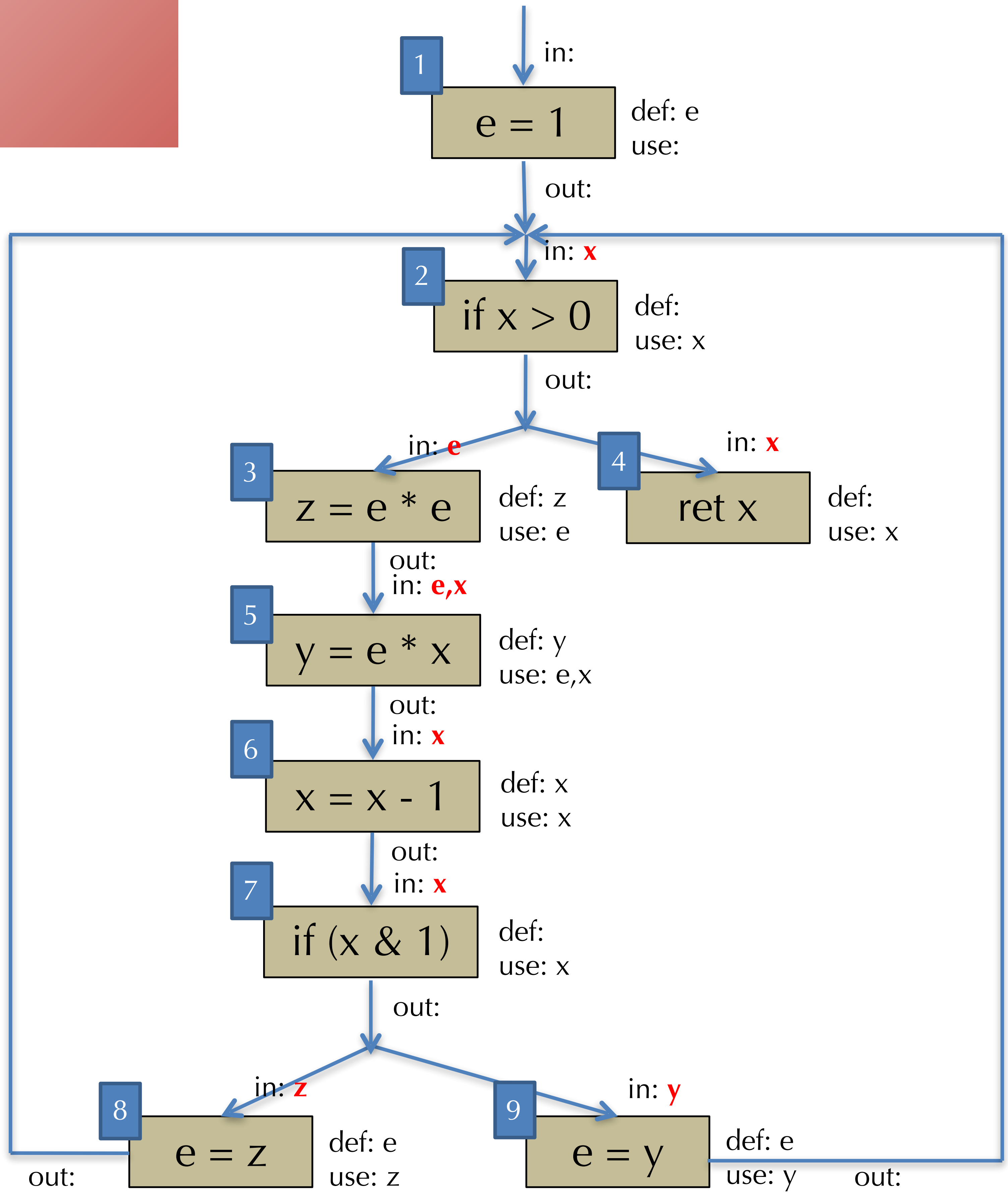
$\text{out}[n] := \bigcup_{n' \in \text{succ}[n]} \text{in}[n']$

$\text{in}[n] := \text{use}[n] \cup (\text{out}[n] - \text{def}[n])$

- Iteration 1:

in[2] = x
in[3] = e
in[4] = x
in[5] = e,x
in[6] = x
in[7] = x
in[8] = z
in[9] = y

(showing only updates that make a change)



Example Liveness Analysis

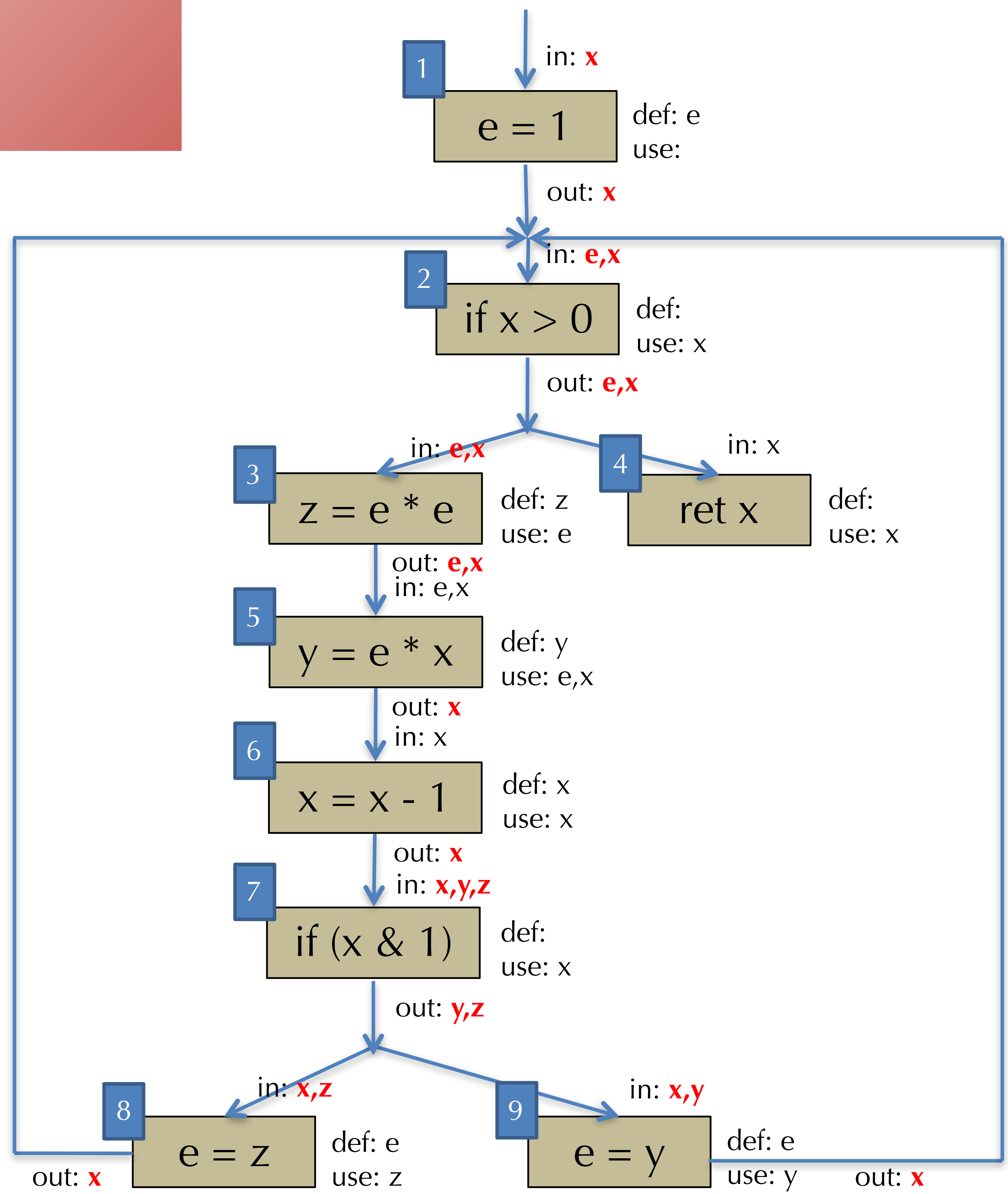
Each iteration update:

$out[n] := \bigcup_{n' \in succ[n]} in[n']$

$in[n] := use[n] \cup (out[n] - def[n])$

- Iteration 2:

out[1]= x
in[1] = x
out[2] = e,x
in[2] = e,x
out[3] = e,x
in[3] = e,x
out[5] = x
out[6] = x
out[7] = z,y
in[7] = x,z,y
out[8] = x
in[8] = x,z
out[9] = x
in[9] = x,y



Example Liveness Analysis

Each iteration update:

$out[n] := \bigcup_{n' \in succ[n]} in[n']$

$in[n] := use[n] \cup (out[n] - def[n])$

- Iteration 3:

out[1]= e,x

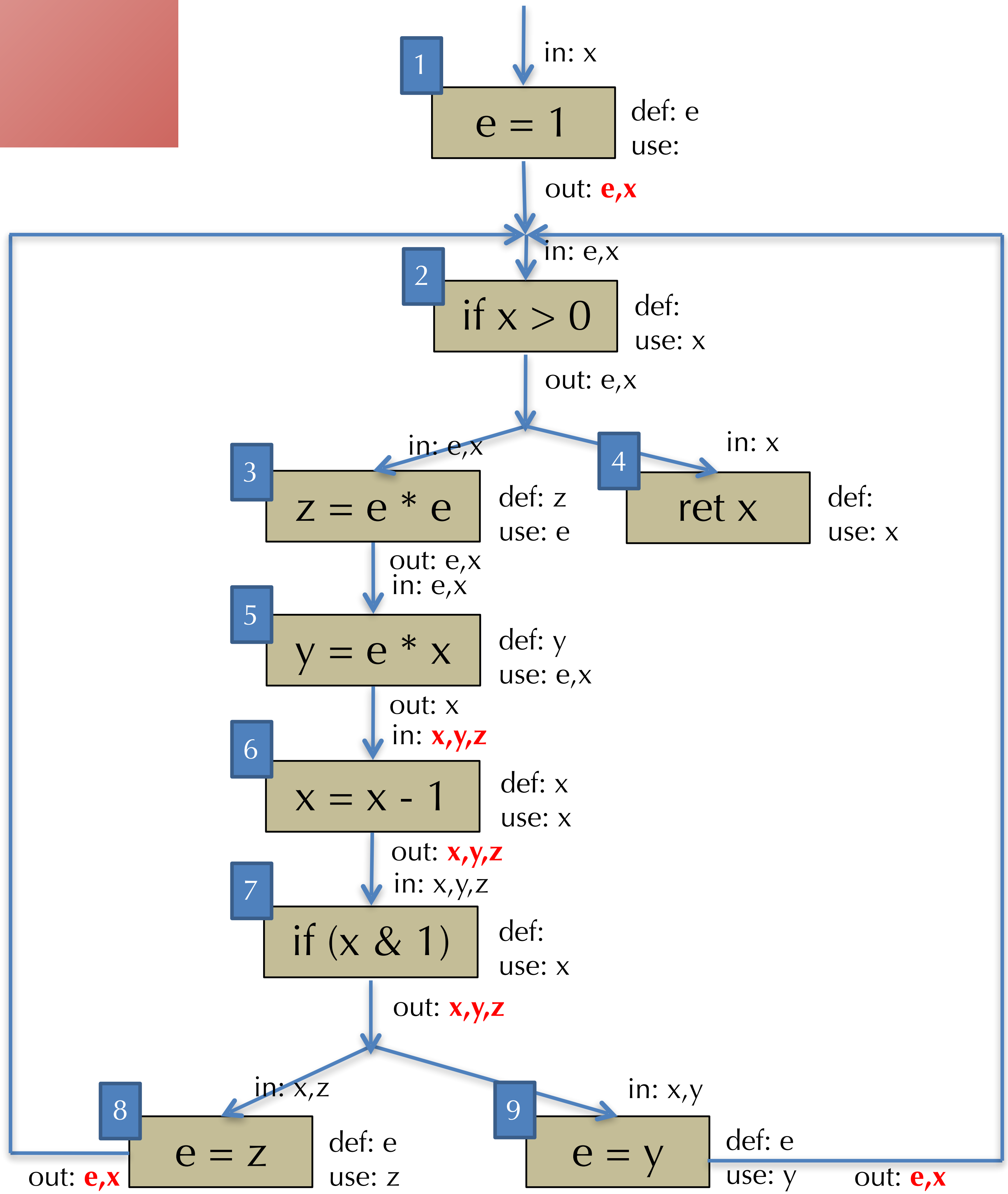
out[6]= x,y,z

in[6]= x,y,z

out[7]= x,y,z

out[8]= e,x

out[9]= e,x



Example Liveness Analysis

Each iteration update:

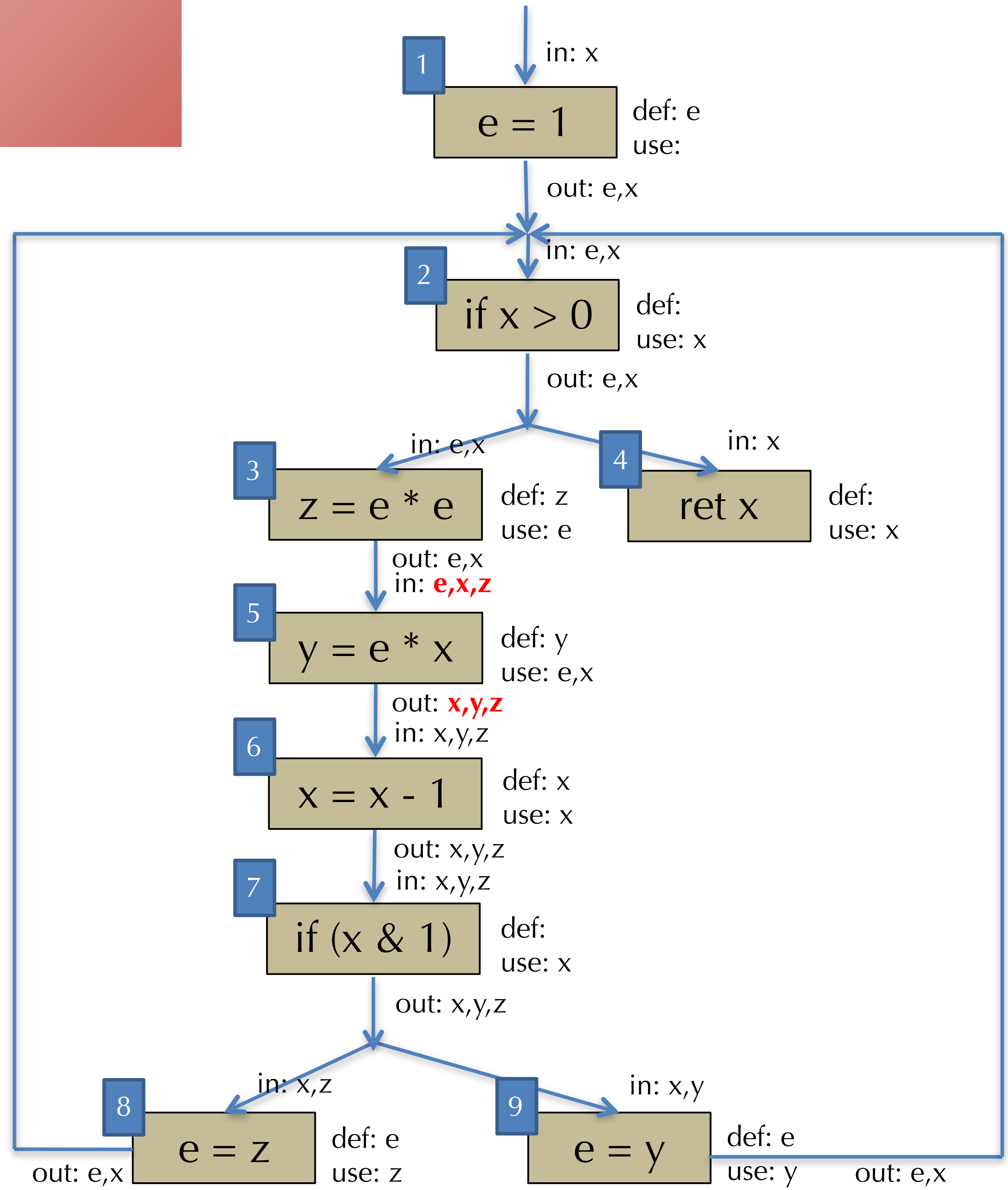
$\text{out}[n] := \bigcup_{n' \in \text{succ}[n]} \text{in}[n']$

$\text{in}[n] := \text{use}[n] \cup (\text{out}[n] - \text{def}[n])$

- Iteration 4:

out[5]= x,y,z

in[5]= e,x,z



Example Liveness Analysis

Each iteration update:

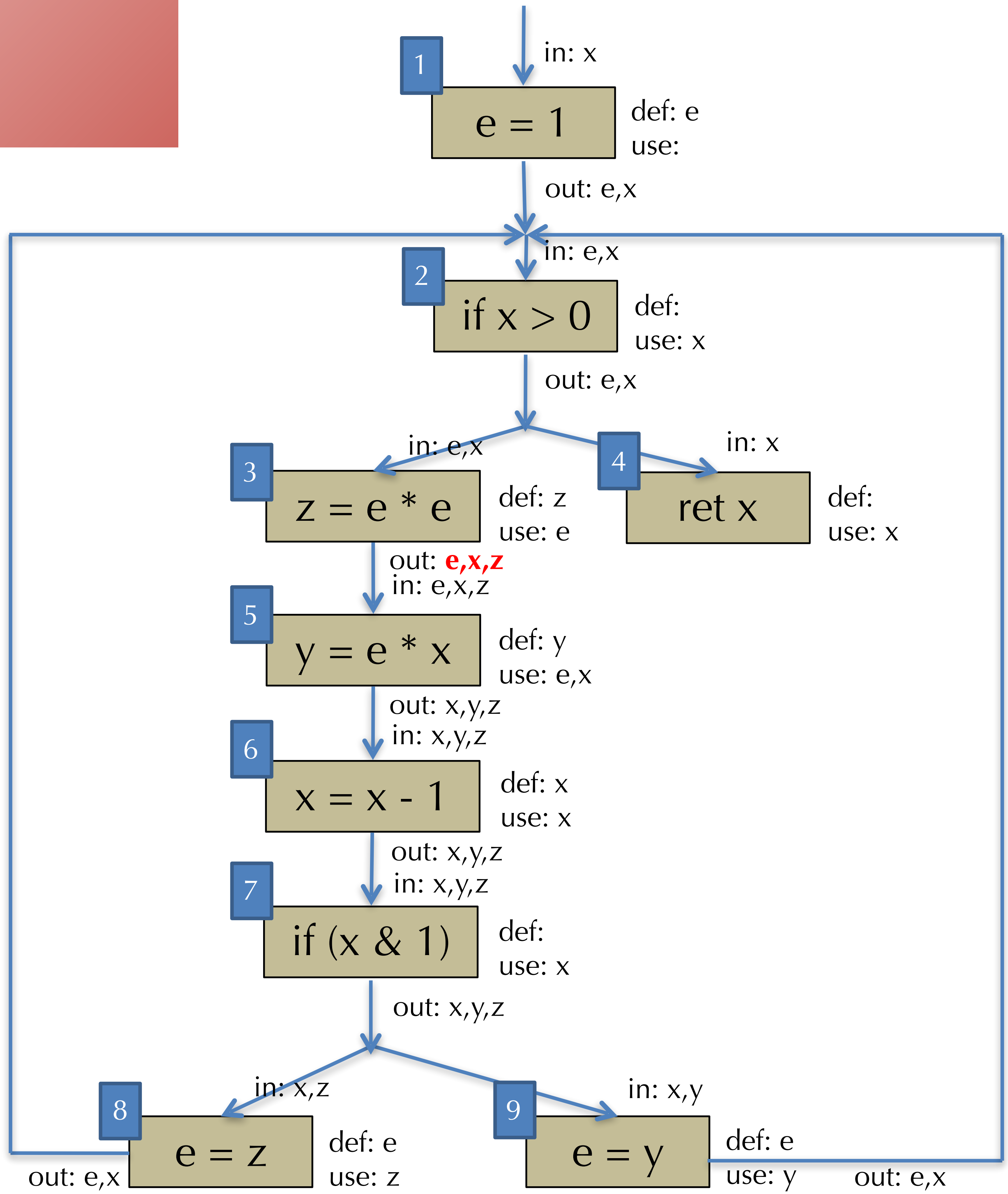
$\text{out}[n] := \bigcup_{n' \in \text{succ}[n]} \text{in}[n']$

$\text{in}[n] := \text{use}[n] \cup (\text{out}[n] - \text{def}[n])$

- Iteration 5:

out[3]= e,x,z

Done!



Improving the Algorithm

- Can we do better?
- Observe: the only way information propagates from one node to another is using:

$$out[n] := \cup_{n' \in succ[n]} in[n']$$

  - This is the only rule that involves more than one node
- If the in sets of a node's successors haven't changed, then the node itself won't change!
- Idea for an improved version of the algorithm:
  - Keep track of which node's successors have changed

A Worklist Algorithm

- Use a FIFO queue of nodes that might need to be updated.

```
for all n, in[n] := Ø, out[n] := Ø
w = new queue with all nodes
repeat until w is empty
    let n = w.pop()                              // pull a node off the queue
     old_in = in[n]                              // remember old in[n]
     out[n] := ∪n′∈succ[n] in[n′]
     in[n] := use[n] ∪ (out[n] – def[n])
     if (old_in != in[n]),                       // if in[n] has changed
          for all m in pred[n], w.push(m)        // add to worklist
end
```

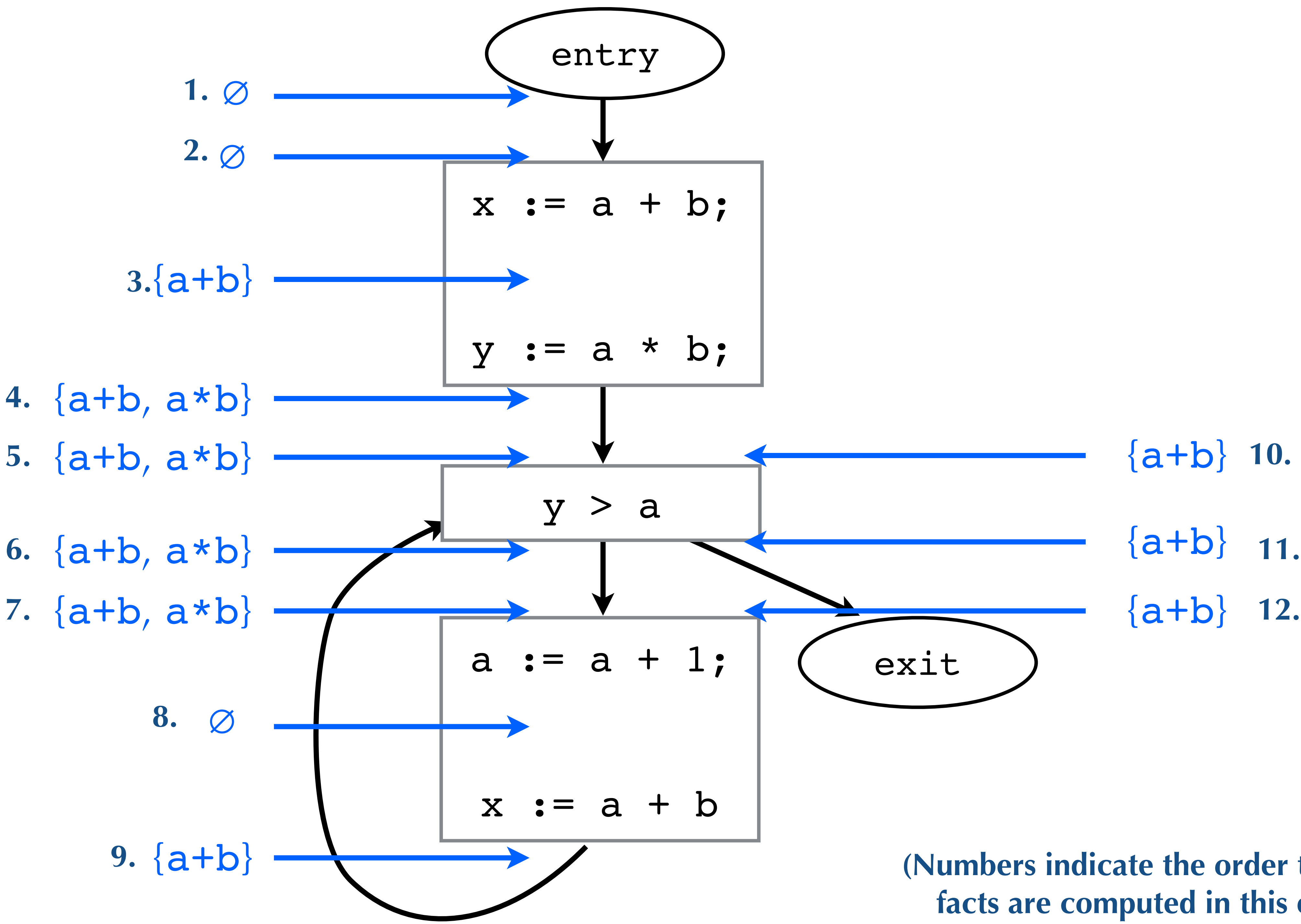
Generalizing Dataflow Analyses

- The kind of iterative constraint solving used for liveness analysis applies to other kinds of analyses as well
  - Available expressions analysis
  - Reaching definitions analysis
  - Alias Analysis
  - Constant Propagation

Available Expressions

- An expression `e` is **available** at program point *p* if on all paths from the entry to *p*, expression `e` is computed at least once, and there are no intervening assignment to `x` or to the free variables of `e`
- If `e` is available at *p*, we do not need to re-compute `e`
  - (i.e., for common sub-expression elimination)
- How do we compute the available expressions at each program point?

Available Expressions Example



(Numbers indicate the order that the facts are computed in this example.)

Reaching definitions

- A definition of a variable *v* is an assignment to *v*
- A definition of variable *v* reaches point *p* if
  - There is a path from the definition of *v* to *p*
  - There is no intervening assignment to *v* on that path
  - Also called **def-use** information

Common Framework: Gen-Kill

- Can think of all these dataflow analysis as computing **facts** at program points
  - in[n] is set of facts that hold immediately before before n
  - out[n] is set of facts that hold immediately before before n
- Each instruction n **gen**erates some facts, and **kill**s some facts
  - E.g., liveness: in[n] := use[n] ∪ (out[n] – def[n])
  - Generates use[n] and kills def[n]
- Analyses differ on:
  - Which facts we are computing and which facts instructions gen and kill
  - Forward or backwards
    - Forwards: compute out[n] using in[n]
    - Backwards: compute in[n] using out[n]
  - How to combine facts: may or must
    - Must: compute facts which must be true, by intersect-ing facts
    - May: compute facts that may be true, by union-ing facts

Comparing Dataflow Analyses

- **Liveness**:
  backward may analysis
  - Facts = variables that are live
  - gen[n] = use[n]
    kill[n] = def[n]
  - out[n] := $\cup_{n' \in succ[n]}$ in[n′]
  - in[n] := gen[n] ∪ (out[n] – kill[n])

- **Available Expressions**:
  forward must analysis
  - Facts = expressions that are available
  - gen[n] = expressions evaluated
    kill[n] = expressions containing a variable in def[n]
  - in[n] := $\cap_{n' \in pred[n]}$ out[n′]
  - out[n] := gen[n] ∪ (in[n] – kill[n])

- **Reaching Definitions**:
  forward may analysis
  - Facts = definitions (i.e., instructions that assign)
  - gen[n] = { n } if n defines variables
    kill[n] = { n′ | n′ defines a variable in def[n] }
  - in[n] := $\cup_{n' \in pred[n]}$ out[n′]
  - out[n] := gen[n] ∪ (in[n] – kill[n])