

COMPUTER SCIENCE 20, SPRING 2012  
DISCRETE MATHEMATICS FOR COMPUTER SCIENCE

Class #16 (Directed Graphs)

**Homework, due in hard copy Wednesday 3/7/2012 at 10:10am**

**Please write your TF's name on your homework, and list the names of any students with whom you collaborated.**

1. A directed tree is a special type of directed graph that has the following two properties:
  - all vertices in a directed tree have in-degree at most one.
  - any pair of two distinct vertices in a tree is connected by *exactly* one path (in other words, for every pair of vertices  $A$  and  $B$ ,  $A \rightarrow B$  XOR  $B \rightarrow A$ ).
- (a) Prove that if a graph  $G$  is a directed tree, then  $G$  is a DAG.
- (b) Prove that if a graph  $G$  is a directed tree, then it has exactly one vertex with in-degree zero.
- (c) Prove that all directed trees with  $n$  vertices have exactly  $n - 1$  edges.