Homework, due via email to dabel@post.harvard.edu Tuesday, February 7, 2012 at 9:00 PM EST

Please list the names of any students with whom you collaborated. We will assume that you collaborated with your group for the first problem, and so it is not necessary to list them if your only collaboration with them was on the first problem.

1. Write up the first problem from today’s class. Here is the problem reproduced:
   Use the Well Ordering Principle to prove that
   \[
   \sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6},
   \]
   for all nonnegative integers, \( n \).

2. Use the Well Ordering Principle to show that \(4^n + 6n - 1\) is divisible by 9 for all nonnegative integers, \( n \).

   Hint: find an expression for \(4^c + 6c - 1\) in terms of \(4^{c-1} + 6(c-1) - 1\).

3. The operators \(\neg\) and \(\lor\) are sufficient to define the rest of our operators as well. Using just \(\neg\) and \(\lor\) (and parentheses), write formulas involving \(p\) and \(q\) that are logically equivalent to
   (a) \(p \land q\)
   (b) \(p \lor q\)
   (c) \(p \rightarrow q\)
   (d) \(p \leftrightarrow q\)

4. Determine which of the following are equivalent to \((p \land q) \rightarrow r\) and which are equivalent to \((p \lor q) \rightarrow r\):*
   (a) \(p \rightarrow (q \rightarrow r)\)
   (b) \(q \rightarrow (p \rightarrow r)\)
(c) \((p \rightarrow r) \land (q \rightarrow r)\)
(d) \((p \rightarrow r) \lor (q \rightarrow r)\)

*Credit: Paul Bamberg / Fun and Games with Discrete Mathematics