Homework, due via email to dabel@post.harvard.edu Tuesday, May 1, 2012 at 9:00 PM EDT

Please list the names of any students with whom you collaborated

At one time, the Guinness Book of World Records reported that the “greatest human calculator” was a guy who could compute 13th roots of 100-digit numbers that were powers of 13. What a curious choice of tasks . . .

1. Prove that \( d^{13} \equiv d \mod 10 \) for \( 0 \leq d < 10 \)

2. Now prove that \( n^{13} \equiv n \mod 10 \) for all \( n \).

3. (a) Compute \( 123456789 \mod 987654321 \).
   (b) Compute \( 100000000 \mod 3 \).

4. Let the capital letters A through Z be used to represent base-26 numbers. Convert NBA to base 10.

5. Let \( xy \equiv 5 \mod 8 \) for two distinct integers \( x \) and \( y \) such that \( x, y > 1 \). What is the smallest possible value of \( x + y \)?

6. Let \( x \equiv 2x \mod 6 \). What is \( x \mod 3 \)?

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\(^1\)Credit: Albert R. Meyer, MIT 6.042