

COMPUTER SCIENCE 20, SPRING 2012
DISCRETE MATHEMATICS FOR COMPUTER SCIENCE

Class #2 (Proofs)

Homework, due in hard copy Friday 1/27/2012 at 10:10am

1. Prove by contradiction that if $ab = n$ where a , b , and n are nonnegative integers, then a or b (or both) must be less than or equal to \sqrt{n} .*
2. In this problem you will show that some power of 3 ends in the digits 001.
 - (i) Show that there are two powers of three that have the same last three digits. Hint: consider the first thousand powers, $3^1, 3^2, \dots, 3^{1000} = 3, 9, 27, \dots$, and the fact that the last three digits can't be 000 for any of these (why?).

(ii) If two powers of three have the same last three digits, we could write them as $3^m = a + 1000s, 3^n = a + 1000t$, where $m \neq n$, s and t are integers and a is some integer $0 < a < 1000$. Subtract these two equations and use the resulting equation to finish the proof that there exists a power of 3 ending in 001. Hint: 3 does not divide 1000, so in an equation of the form $1000x = 3y$ for integers x and y , we know that 3 must divide x (that is, $\frac{x}{3}$ is an integer).

*Credit: Albert R. Meyer / MIT 6.042