ES202 Lecture notes #13.1
Concepts of filtering (Estimation of the State of a Dynamic Systems)

1. Recursive filtering vs batch processing

2. Simplest example of filtering a constant: interpretations as averaging out the noise, unweighted least square fit, probabilistic estimation in gaussian noise.

3. \( p(x_t/z_{\tau}, t_0 \tau t) \equiv p(x_t/Z_t) \)

4. One step of propagation: Graphically,

   \[ p(x_{t+1}/Z_t) = p(x_{t+1}/x_t, Z_t) p(x_t/Z_t) dx_t \]

   Analytically,
   \[ x_{t+1} = f(x_t, w_t); \quad p(x_0) \text{ given}; \quad w_t \text{ is white noise } (0, Q) \]
   \[ p(x_{t+1}/Z_t) = \int p(x_{t+1}/x_t, Z_t) p(x_t/Z_t) dx_t \]

   In the case of Gauss-Markov (linear system driven by Gaussian white noise)
   \[ \hat{x}_{t+1} = \Phi\hat{x}_t + D\hat{w}_t; \quad M_{t+1} = \Phi P_t \Phi^T + DQ_t D^T \]
   for \( p(x_t/Z_t) \sim N(\hat{x}_t, P_t), w_t \sim GWN(\hat{w}_t, Q_t), \) and \( p(x_{t+1}/Z_t) \sim N(\hat{x}_{t+1}, M_t) \) for the linear system
   \[ x_{t+1} = \Phi x_t + D w_t; \quad p(x_0) \text{ given}; \quad w_t \text{ is GWN}(\hat{w}_t, Q) \]

5. One step of Updating
Graphically Analytically

\[ z_{t+1} = h( x_{t+1}, v_{t+1} ) ; \quad p( x_{t+1} / Z_t ) \text{ given ;} \]
\[ v_{t+1} \text{ is white noise } (0, R_{t+1}) \]
\[ p(x_{t+1} / Z_{t+1}) = \frac{p(z_{t+1} / Z_t, x_{t+1})}{p(z_{t+1} / Z_t)} p(x_{t+1} / Z_t) \]

in the case of Gauss-Markov

\[ z_{t+1} = H x_{t+1} + v_{t+1} ; \quad p( x_{t+1} / Z_t ) \sim \mathcal{N}( \bar{x}_{t+1}, M_{t+1} ) ; \]
\[ v_{t+1} \text{ is GWN}(0, \quad R_{t+1}) \]
\[ \hat{x}_{t+1} = x_{t+1} + P_{t+1} H^T R^{-1} (z_{t+1} - H x_{t+1}) \]
\[ P_{t+1} = M_{t+1} - M_{t+1} H^T (H M_{t+1} H^T + R)^{-1} H M_{t+1} \]

6. Weighted least square fit and maximum posterior likelihood interpretation of the above

7. Filtering (estimating \( x(t) \) based on \( Z(t) \)), Predicting (estimating \( x(t) \) based on \( Z(\tau) \)), and Smoothing (estimating \( x(t) \) based on \( Z(\tau \tau<t) \)) merely require creative combination of the PROPAGATION and the UPDATING steps.

8. For continuous time filtering and estimation, no new concepts are introduced except for the mathematical device of a continuous time white noise process involving the use of Dirac delta functions. The formulas are simpler looking but no more practical than those in §4-5 above. see chs. 11-12 of Bryson and Ho.