ENGINEERING SCIENCES 50

INTRODUCTION TO ELECTRICAL ENGINEERING

Solution Set 5

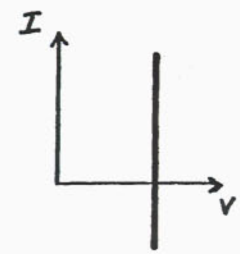
Problems 1.13, 1.19, 2.4, 2.15, 2.23, 2.26, 2.33, 3.2, 3.3, 3.15, 3.17, 3.32

(1.13) $I = \frac{25}{6} = 4.167A = Nq$
$= N(1.6 \times 10^{-19})$. $\therefore N = 2.6 \cdot 10^{19}$ /SEC

(1.19) $P_{TOTAL} = I(V_{12} + V_{23} + V_{34} + V_{45} + V_{56} + V_{61}) = 0$

THIS RESULT EXPRESSES ENERGY CONSERVATION. WHATEVER POWER IS DISSIPATED (IN BLOCKS WHERE VI IS POSITIVE) MUST BE BALANCED BY POWER BEING GENERATED (IN BLOCKS WHERE VI IS NEGATIVE).

(2.4)



(2.15) (a) THE VOLTAGE ACROSS EACH RESISTOR IS THE SAME; CALL IT V. THEN

$$P = \frac{V^2}{R_1} + \frac{V^2}{R_2} \Rightarrow V = \left(\frac{PR_1R_2}{R_1+R_2}\right)^{1/2}$$

THE POWER DISSIPATED IN R_1 IS THEN

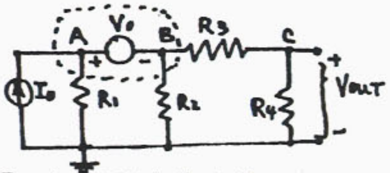
$$P_1 = \frac{V^2}{R_1} = P \cdot \frac{R_2}{R_1+R_2}$$

(b) LET THE CURRENT THROUGH BOTH BE I.

$$P = I^2R_1 + I^2R_2 \Rightarrow I = \left(\frac{P}{R_1+R_2}\right)^{1/2}$$

$$P_1 = I^2R_1 = P\frac{R_1}{R_1+R_2}$$

(2.23) (1)



SINCE WE HAVE A VOLTAGE SOURCE AS A BRANCH, WE MERGE NODE A & B FOR THE PURPOSE OF WRITING KCL EQUATIONS.

$$V_A = V_B + V_0 \quad \text{— (a)}$$

KCL AT A&B $\quad I_0 - \frac{V_A}{R_1} - \frac{V_B}{R_2} + \frac{V_C - V_B}{R_3} = 0 \quad \text{— (b)}$

KCL AT C $\quad \frac{V_B - V_C}{R_3} - \frac{V_C}{R_4} = 0 \quad \text{— (c)}$

ONLY THREE EQUATIONS ARE NECESSARY.

(2) SOLVING THE EQUATIONS GIVES

$$V_B = \frac{R_2(R_3+R_4)(I_0R_1 - V_0)}{(R_1+R_2)(R_3+R_4)+R_1R_2}$$

$$V_C = \frac{R_4}{R_3+R_4} \cdot V_B$$

$$= \frac{R_2R_4(I_0R_1 - V_0)}{(R_1+R_2)(R_3+R_4)+R_1R_2}$$

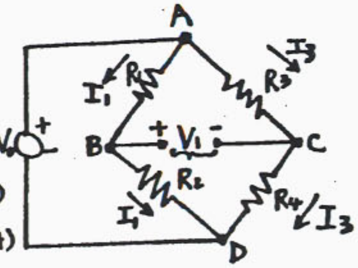
$$V_A = V_B + V_0$$

$$= \frac{R_1(R_3+R_4)(V_0+I_0R_2)+R_1R_2V_0}{(R_1+R_2)(R_3+R_4)+R_1R_2}$$

$$V_{OUT} = V_C$$

■

(2.26)



$$V_B - V_A = -I_1R_1 \quad \text{—(1)}$$

$$V_C - V_A = -I_3R_3 \quad \text{—(2)}$$

$$V_B - V_D = I_1R_2 \quad \text{—(3)}$$

$$V_C - V_D = I_3R_4 \quad \text{—(4)}$$

FROM (1) & (2)

$$V_B - V_C = I_3R_3 - I_1R_1$$

FROM (3) & (4)

$$V_B - V_C = I_1R_2 - I_3R_4$$

$$V_1 = 0 \Rightarrow V_B - V_C = 0$$

$$\Rightarrow \frac{I_3}{I_1} = \frac{R_1}{R_3} = \frac{R_2}{R_4}$$

HENCE $\quad R_4 = \frac{R_3}{R_1} \cdot R_2 = \frac{10K}{27K} \cdot 16K$

$$= \underline{5.9\ K\Omega}$$

■

(2.33) THE TOTAL RESISTANCE IS THE COMPOSITE OF TEST RESISTANCE AND THE RESISTANCE OF THE VOLTMETER IN PARALLEL.

∴ BY OHM'S LAW $\quad I_0 = \frac{V_{AB}}{27K // 10K}$

$$= \frac{2.16}{7.297K}$$

$$= \underline{0.296\ mA}$$

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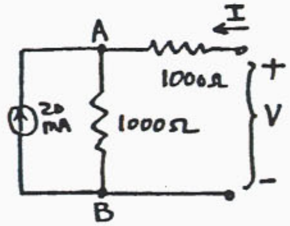
(3.2) WRITE A NODE EQUATON FOR NODE X:

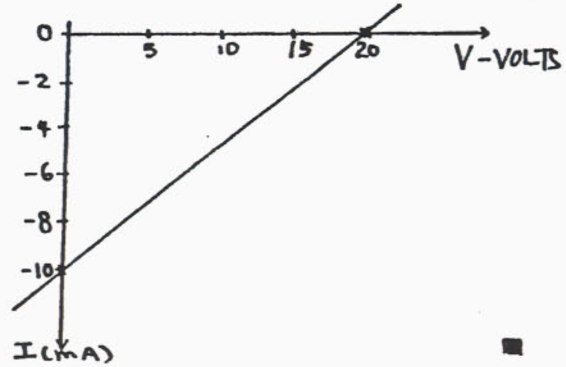


$$20mA + \frac{V_B - V_A}{1000\Omega} + I = 0$$

BUT $V_A - V_B = V - 1000I$

$$\therefore\ 20m - \frac{V - 1000I}{1000\Omega} + I = 0$$

$$\Rightarrow I = 0.5\times10^{-3}V - 10\times10^{-3}$$



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(3.3) WRITING KCL AT THE POSITIVE TERMINAL

$$I - \frac{V}{4K} - \frac{V-40}{4K} = 0$$

$$\Rightarrow I = \frac{V}{2000} - \frac{10}{1000}$$

THIS IS THE SAME RELATIONSHIP AS IN 3.2. SO THESE TWO SUBCIRCUITS ARE EQUIVALENT.

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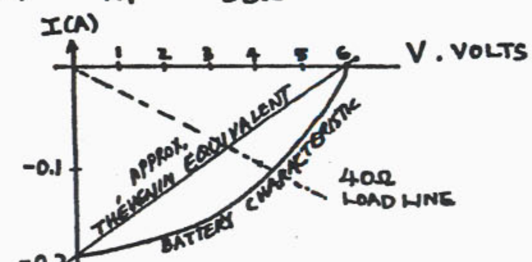
(3.15) AN IDEAL . VOLTMETER (AMMETER) IS AN OPEN (SHORT) CIRCUIT. THUS FROM FIG 3.38, $V_{oc} = 6V$ AND $I_{sc} = -0.19A$. IT FOLLOWS THAT THE CORRESPONDING THÉVENIN EQUIVALENT CIRCUIT IS CHARACTERIZED BY $V_T = V_{oc} = 6V$.

$$R_T = -\frac{V_{oc}}{I_{sc}} = -\frac{6V}{-0.19A} \approx 33\Omega$$ AN

I-V CHARACTERISTIC GIVEN BY BY EQ.(3-1)

$$I = \frac{V}{R_T} - \frac{V_T}{R_T} = \frac{1}{33\Omega}V \quad 0.19A.$$



THE TWO CHARACTERISTICS SHOWN DO NOT AGREE BECAUSE THE BATTERY IS NON-LINEAR AND HENCE HAS NO THÉVENIN EQUIVALENT, A THÉVENIN CIRCUIT IS ALWAYS REPRESENTED BY A LINEAR I-V CHARACTERISITC.

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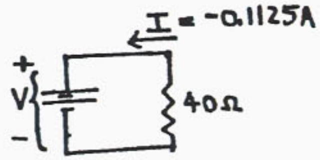
(3.17) $V_T = V_{OC} = \underline{6V}$



USING KVL

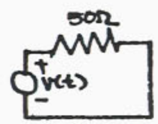
$V = -I(40)$

BUT $V = IR_T + V_{OC}$

$\therefore 40 + R_T = -\frac{V_{OC}}{I} = \frac{6}{0.1125} = 53.3\Omega$

$\Rightarrow \underline{R_T = 13.3\Omega}$ ■

(3.32) $P_{AV} = \frac{1}{T}\int_0^T \frac{v^2(t)}{R}\,dt$



$= \frac{1}{0.002}\int_0^{0.002} \frac{\left(\frac{0.3}{0.002}t\right)^2}{50}\,dt$

$= \frac{1}{0.002}\left[\frac{0.3^2}{0.002^2}t^3/3(50)\right]_0^{0.002}$

$= \frac{1}{0.002}\left[\frac{0.3^2(0.002)}{150}\right]$

$= \underline{0.6mW}$ ■